INTRODUCTION

TO THE TRUE

ASTRONOMY:

OR,

ASTRONOMICAL LECTURES,

Read in the

Astronomical School

OF THE

UNIVERSITY of *OXFORD*.

By *JOHN KEILL*, M. D. Fellow of the *Royal Society*, and Professor of *Astronomy* in that University.

The Works of the Lord are great, sought out of all them that have Pleasure therein, Psalm cxi. 2.

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To HIS GRACE

JAMES,

Duke of *CHANDOS*,

May it please Your GRACE,

AMONG *all the Mathematical Sciences which have been continually improved, and are daily improving in the World, the first Place has, as it were, by general Consent, been always given to* Astronomy. *And such has been either the good Fortune of the Science, or the Virtue of Mankind, that the greatest and most eminent Persons, in all Ages and Nations, have been Patrons and Encouragers of this Study above all others.*

M A Y *it please your Grace, therefore, to take this Book into your Protection, since whatever may be wanting, either in the Work or the Author, to recommend it to your Favour, will abundantly be supplied by the Dignity of the Subject.*

F OR *to whom can I so properly send a Treatise of the Stars and heavenly Motions, as to a faithful and zealous Servant of that heavenly King,* who knoweth the Number of the Stars, and calleth them by their Names? *So remarkable is your Grace's Zeal for the Seavice and Honour of God, that you took particular Care to adorn* his House, *before you would lay the Foundation of your* own. *Nor did your Care extend only to the Ornaments of the Temple, but likewise, and more especially, to the Decency of the Worship: You called Musick in to excite Devotion; Musick being the Delight and Employment of the heavenly Choir.*

Y OU, *my* L O R D, *are the publick and standing Mark of all Mens Admiration, the beautiful Pattern which all desire to imitate, though few can hope to equal. In publick Affairs, what Statesman more able? In domestical Management, what private Man more expert; in the constant stating and*

exact keeping of Accompts, nobody more provident, nobody more frugal: In Expences, nobody more liberal: In Largesses, nobody so magnificent.

SO *great is your Affection to Learning and the Learned, that while you make yourself Master of every Art, you give Matter and Encouragement to every Artist. To the particular Science which is the Subject of this Book, your Grace is so eminent, so beneficient a Patron, that in the stately and beautiful Structure of* Cannons, *Astronomers will find every Thing for the Improvement of their Knowledge; Instruments worthy of the Science, and an Observatory worthy of its Lord.*

THE *Book I now present is a Translation of those Astronomical Lectures, which were honoured with your Grace's Name at their first Publication, in the Language they were read in at the University of* Oxford. *The Version was made at the Request, and for the Service of, the Fair Sex, and particularly for the Service of the great Ornament of her Sex, the Duchess of* Chandos. *It is no Flattery to the Ladies, to say, that such of them as delight in Arts and Sciences, as to Quickness of Perception, and Delicacy of Taste, are equal, if not*

superior, to Men; and it is no Affront to the most refined of either Sex, to say, there is not a finer Genius than my Lady Duchess.

MAY *every Star in Heaven shed its kindest Influence on both your Heads. And may you long continue to enjoy that Affluence, which, like the Rays of the Sun, scatters Light and Warmth to all round You.*

THIS, *my* LORD, *is a general Wish, because it is for the general Good of Mankind, particularly of him who is, with the deepest Sense of Gratitude,*

Your Grace's

Most faithful and

Most humble Servant,

JOHN KEILL.

THE PREFACE.

AMONG all the Gifts and Benefits the most bountiful God has most plentifully bestowed on Mankind, those are in the first Place valuable which consist in the Improvements of the Mind by Arts and Sciences. And as among the Sciences there are none which *Astrenomy* comes behind upon the Account of its Antiquity, and the Pleasure that attends the Study of it, so it will yield to none of them on the Account of its Usefulness, and the Advantages it affords to human Life. By it we discover the wonderful Harmony of Nature, wherewith the Frame and Structure of all created Beings are linked and knit together, to constitute the great Machine of the Universe. *Astronomy* teaches us to observe and discover the Motions of the heavenly Bodies, and it weighs and considers the Vigour and Force by which they circulate in their Orbs. It is a Science, which the greatest Heroes from the Beginning of the World, have taken Pleasure to study and improve; so that it was always esteemed as a Science fit for Kings and Emperors to employ themselves in. On which Account the *Chaldean* Wise-men and Philosophers were always reverenced and favoured by the antient Kings, who thought it absurd that any should govern the World, who knew not what the World was.

THE Excellency of this Science appears from this, that there is no Knowledge which is attained by the Light of Nature, that gives us truer and juster Notions of the Supreme and Almighty GOD, the Maker

of both Heaven and Earth, than it does. None furnishes us with stronger Arguments by which his Existence is demonstrated; nothing shews more his Power and Wisdom than the Contemplation of the Stars and their Motions. That Prophet as well as King, the Holy *David*, tells us, that *The Heavens declare the Glory of God, and the Firmament sheweth his Handy Work.* And again, *The Heavens declare his Righteousness, and all the People have seen his Glory.*

MARCUS Tullius Cicero, who was guided only by the Light of his own Reason, had the same Sentiments. *Nothing*, says he, *is more evident, nothing plainer, when we look up to the Heavens, and contemplate the Bodies there, than that there is a Deity of most excellent Wisdom who governs them.* What is there that more ravishes the Mind of Man into an Admiration, Reverence and Love of God, than so many and so great Bodies endowed with heavenly Light, most beautiful to the Eye, and when contemplated, most delightful to the Understanding? Their mutual Intercourses, most regular Motions, their certain and determined Circulations, and their Returns and Periods settled by a divine Law, in an admirable Harmony, make manifest to us the immense Power, Wisdom and Providence of their Maker; which when we consider, we must necessarily acknowledge, reverence and celebrate the Author and Contriver of all these Things.

BESIDES *Astronomy*, with its sublime Speculations about so many and so large Bodies, and at such immense Distances, does wonderfully please and recreate the Mind.

ASTRONOMY, for the Certainty and Evidence of its Demonstrations, is not inferior to *Geometry*; its Usefulness is manifold, and the Amplitude of its *Subject* is so large, that it comprehends nothing less than the World itself. For as, among all the liberal Sciences, there are none that contemplate Objects more in Number, greater in Quantity, or at longer Distances from us, than *Astronomy*; so likewise there are none in which there still remain fewer Difficulties

to

to be explained, Objections to be anſwered, or Scruples to be removed, than there are in *Aſtronomy*, and no Science has yet attained ſo great a Degree of Perfection as it has.

In moſt of the other Arts there are ſeveral inextricable Labyrinths; many ſtrange Objections are raiſed, and unanſwerable Arguments, which do ſo confound the Mind, that like thick Clouds they ſtop all further Proſpect and Diſcovery. But the Motions of the heavenly Bodies are now certainly known, and their Cauſes demonſtrated, and the Reaſon of all the *Phænomena* of the Heavens are exactly underſtood.

The ſmalleſt Stars we can ſee, though they be at an unmeaſurable Diſtance from us, yet have their Longitudes and Latitudes exactly determined, their proper Places ſettled, and are all reduced into Catalogues; though at the ſame Time the Science of *Geography*, or a Deſcription of our own Habitation, is ſo imperfect, that we have an exact Determination of the Longitudes and Latitudes of but a very few Places, there ſtill remaining many *Unknown Lands* and Countries that have not as yet been diſcovered: And there are now great and far extended *Continents*, of which we ſcarcely know any Thing beſides their Coaſts and Shores. And what is ſtill more ſtrange, in our little Provinces and Counties which we daily travel over, there are many Towns and Cities whoſe Poſitions are ſtill uncertain, as is plain from the many *Geographical* Maps of them, which contradict each other.

The *Aſtronomers* foretel, for many Ages to come the Eclipſes of both Sun and Moon, their Quantities and Durations; the Conjunctions, Oppoſitions and mutual Aſpects of the Planets, and what will be the Diſtances of all the Stars from the Pole at any Time; whereas there is no Man ſo well ſkilled in *Meteorology* as can certainly foretel what will be the State and Condition of our Atmoſphere for the very next Day, and yet it reaches but a few Miles from us: We are unable to judge whether we ſhall have fair Weather or foul, calm or ſtormy, or even ſo much

much as to foresee from what Point the Wind will blow. And this is no Wonder, since the Causes from whence those Effects arise are unsearchable.

No *Philosopher* has ever yet discovered the Figures of the small Parts of Matter, or the Texture, Intervals, Form and Composition of the Parts of the most common Plant. Nor has any Physician yet found out the Reason of the Virtues and Operations by which their Medicines affect human Bodies. And even in all animated and vegetable Bodies, the Fountain and first Principle of Life and Action is unsearchable; and looks like a Mystery much beyond the Reach of our Understanding, which Knowledge perhaps, in this Life, is never to be attained. But *Astronomers* in their proper Science meet with no such Difficulties; they contemplate not the Natures, but the Motions of the celestial Bodies, and they clearly account for the *Phænomena* or Appearances that arise from thence: They not only determine what Sort of Motion the Planets have, and in how large a Compass they circulate, but they likewise shew us the crooked Tracts in the immense Regions of Space which the wandering Comets take: They can give us the *Geometrical* Properties of their Orbits, and the Laws which they observe in describing them. The *Astronomers* are not ignorant where or when the Planets are at their farthest Distance from the Sun, and participate the least of his Heat and Light; from whence they return, and are constantly quickened in their Motions by the Sun, who draws them towards himself, till they come to those Parts of Space where they make their nearest Approach to him, enjoy most of his Heat and Light, and are actuated by the greatest Force of their own Gravity.

Most of the Discoveries we have related were known to the *Astronomers* of former Ages. But our Times, and this our Country of *Britain*, have had the Happiness to produce a *Genius* of a divine Nature and extraordinary Qualities; I mean the Great Sir *ISAAC NEWTON*, who, besides his innumerable other wonderful Inventions, has discovered the Fountain

tain and Spring of all the celeſtial Motions, and the great Law, which is univerſally diffuſed through the whole Syſtem of Nature, which the Almighty and Wiſe Creator has commanded all Bodies to obſerve, *viz.* That every Particle of Matter attracts each other in a reciprocal duplicate Proportion of its Diſtance.

THIS Law is as it were the Cement of Nature, and the Principle of Union, by which all Things remain in their proper State and Order; it detains not only the Planets, but the Comets within their due Bounds, and hinders them from making Excurſions into the immenſe Regions of Space, which they would do if they were only actuated by a Motion once implanted in them, which naturally they would always preſerve according to the principal Law of Motion.

WE are obliged to the ſame Gentleman for the Diſcovery of the Law that regulates all the heavenly Motions, ſets Bounds to the Planets Orbs, determines their greateſt Excurſions from the Sun, and their neareſt Approaches to him. To this ſublime Genius we owe, that now we know the Cauſe why ſuch a conſtant and regular Proportion is obſerved, by both primary aud ſecondary Planets, in their Circulations round their central Bodies, in comparing their Diſtances with their Periods; and why all the celeſtial Motions are ſtill continued in ſuch a wonderful Regularity, Harmony and Order. The ſame incomparable Perſon, having a complete Knowledge of the Laws of Nature and Motion, has from them furniſhed us with a new Theory of the Moon, which accurately anſwers all her Inequalities, and accounts for them by the Laws of Gravity and Mechaniſm; ſo that now the Moon's Place, computed by the Rules of this new Theory, does not ſenſibly differ at any Time from what it is obſerved to obtain in the Heavens, which does exceed the Hopes and Expectations of our *Aſtronomers*; ſo that we have now a Proſpect of improving our Navigation, by finding from Obſervations of the Moon, the Longitude of a Ship at Sea: A *Problem* of great Uſe, whoſe Solution

lution is much to be desired, and for which there are very ample Rewards allowed.

There is nothing that does more shew the Force and Penetration of human Understanding, than these great and wonderful Discoveries. There is no more certain Way of comprehending the prodigious Bulk of the whole mundane Fabrick, or the amazing Beauty of so divine a Structure, and the infinite Wisdom of its divine Contriver, than by considering these Laws which are lately discovered. From them we learn to have a most noble and magnificent Notion of the whole System of Nature. Now we are assured, that this Earth we inhabit is but a small and inconsiderable Part of a glorious Fabrick; since there are almost infinite Worlds, crreated by a Supreme and an Almighty Being, which are prodigiously larger than ours, in the disposing and governing of which the same Being exercises his infinite Power and Wisdom. *It is he who spoke the Word, and the Heavens were made. He commanded, and they were created. He hath made them fast for ever and ever. He hath given them a Law which shall not be broken.*

Astronomy is not only useful, as it improves the Mind, and by its most delightful Speculations increases the Force and Penetration of the Understanding; but it is likewise a considerable Help to the perfecting of other Arts and Sciences. In how great Darkness would the *Geographers*, the *Chronologists* wander, were they not assisted with Light from *Astronomy*? To it is owing, that we know the Figure and Magnitude of the Earth, and find out the Situations and Distances of Places. We learn from it the true Measure of the Year, and can give an Account of Actions, according to the true Order of the Times in which they happened. Hence is evident how useful *Astronomy* is to human Affairs; for without it we could have no *Geography* nor *Chronology*, and consequently no certain Account of History.

But among all the Arts and Sciences, there is none that has received greater Improvements from

Astronomy

Astronomy than *Navigation* has done; for by our Knowledge in it we can carry our Ships through the vast Ocean in a right Course, though there is no Tract to be seen, and visit the utmost Regions of the Earth. Hence arise the Advantages of Trade and Commerce; so that whatever Things other Countries afford, that are either precious or delightful, we receive and enjoy without the Inconveniencies of intemperate Heats or Colds to which those Countries are liable. It is owing to our Skill in Navigation that our *British* Monarchs have obtained the Sovereignty of the Seas; so that there is no Nation, at what Distance soever, but what are kept from doing Injuries to our Countrymen by the Terrors of a *British* Fleet.

As the Art of Sailing does in a great Measure depend on the Knowledge of the Stars; so the impetuous and ambitious Desires of Kings and Princes to discover unknown and foreign Countries, inclined them to cultivate *Astronomy*. The first and chief of all the Sailors was *Neptune*, who, upon the Account of his Skill in this Art, was celebrated as God of the Ocean. His Son *Belus*, being an *Astronomer*, by his Knowledge therein, carried the Inhabitants of *Libya* into *Asia*, where he instituted Colleges of *Astronomers*; for *Diodorus*, in the first Book of his Histories, writes thus: *It is reported*, says he, *that the* Egyptian Belus, *the Son of* Neptune *and* Libya, *brought a Colony to* Babylon; *and there he instituted Priests, whom the* Babylonians *call* Chaldeans; *who, after ther Manner of the* Egyptians, *were to observe the Stars.* Before his Time, there was *Atlas* King of *Mauritania*, a great *Astronomer*, who first shewed us the Doctrine of the Sphere. And therefore *Virgil* introduces *Iopas* singing what *Atlas* had taught Mankind,

> ----- *Docuit quæ maximus* Atlas,
> *Hic canit errantem Lunam, Solisque labores.*

So *Uranus*, King of the Country situated on the Shore of the *Atlantick* Ocean, for his Skill in *Astronomy*,

nomy, is said to have been descended from the Gods. *Zoroaster*, a *Persian* Philosopher, is celebrated by all Antiquity as a skilful *Astronomer*. And the Honour and Dignity of this Science was had in so great a Reputation, as to be called the *Royal Science*, being that Kings were most delighted by it above all others. For the Kings of *Africk* and *Syria* first invented and improved it, and that long before it was known in *Greece*. This *Plato* acknowledges in his Dialogue, which he calls *Epinomis*. *The first*, says he, *who observed these Things was a* Barbarian, *who lived in an antient Country, where, upon the Account of the Clearness if the Summer Season, they could first discover them, such as* Egypt *and* Syria, *where the Stars are clearly seen, there being neither Rains nor Clouds to hinder their Prospect. And because we are more remote from this Summer Clearness of Weather than the* Barbarians, *we came later to the Knowledge of these Stars.* So *Lucian* tells us, *That the* Ethiopians *first took Notice of the heavenly Motions, and by finding the Causes of the Lunations they knew that the Moon had no proper Light of its own, but borrowed it from the Sun.* However, it is certain that *Astronomy* from the very Beginning was cultivated and improved by the Eastern Nations. For if we may believe *Porphyry*, when *Alexander* took *Babylon*, *Callisthenes* at the Desire of *Aristotle* carried from that City the Observations of 1903 Years, which brings the Beginning of these Observatios to 115 Years after the Flood, and 15 Years after the Building of *Babel*. *Pliny*, in his Natural History relates, that *Epigenes* affirmed, that the *Babylonians* had Observations of 720 Years, all graven upon Bricks. And *Achilles Tatius*, in the Beginning of his Introduction to *Aratus*'s *Phænomena*, informs us, "That the *Egyptians* were the first who measured the Heavens "and the Earth; and their Science in this Matter "was engraven on Columns, and by that Means delivered to Posterity. Yet the *Chaldeans* take the "Honour of the Invention to themselves, and "ascribe it to *Belus*." The *Greeks* had all their

Astronimical

Astronomical Learning from *Egypt*: For *Laertius* owns, that *Thales*, *Pythagoras*, *Eudoxus*, and many others, went to that Country to be instructed in the Sidereal Science. These Men were not only the first, but the greatest Philosophers that *Greece* produced; and from the same Author we know, that they who staid longest in that Country were most famous for their Skill in *Geometry* and *Astronomy* after they returned home: So *Pythagoras*, who lived in Society with the *Egyptian* Priests seven Years, and was initiated into their Religion, carried Home from thence, besides several *Geometrical* Inventions, the true System of the Universe; and was the first that taught in *Greece*, that the Earth and Planets turned round the Sun, which was immoveable in the Center; and that the diurnal Motion of the Sun and fixed Stars was not real, but apparent, arising from the Motion of the Earth round its Axis. At that Time nobody was esteemed as a Philosopher, but who was well acquainted with the Mathematical Sciences.

But these Sciences were soon neglected by the Philosophers that came after them, who much degenerating from their Predecessors, had so little Care and Concern for the Mathematical Sciences, especially *Astronomy*, that of all the Observations of Eclipses, for the Space of near 2000 Years, that were sent from *Babylon* by *Callisthenes*, *Ptolemy* could recover but a very few, the rest being lost by the Carelesness, Negligence, and want of Skill of those Men who should have preserved them. For these Pretenders to Philosophy, having no Concern for the useful Parts of it, spent their Time about Trifles and Disputes of no Value, and in endeavouring to find out Sophisms, whereby they would impose upon their own, and the common Sense of all Mankind: Such were *Zeno*'s Arguments against Motion, and most of the Philosophers Disputations against the Divisibility of Matter *in infinitum*; whereas a little Knowledge of *Geometry* would easily have dissolved all the Difficulties they could raise. But though *Astronomy* was thus banished out

of

of the Schools of the common Philosophers, yet it was received and cultivated by some, tho' but a few, especially by the *Pythagorean* Sect, which flourished in *Italy* many Years, among whom was *Philolaus* and *Aristarchus Samius*. The *Ptolemies* Kings of *Egypt*, were also great Patrons of Learning; they founded an Academy for *Astronomy* at *Alexandria*, which furnish us with great Men, the chief of whom was *Hipparchus*, who according to *Pliny*, *undertook a Business which would have been a great Work for a* GOD *to perform, that is, to number the Stars, and leave the Heavens for an Heritage to all that come after.* This Man foretold the Eclipses of both Sun and Moon for 600 Years; and upon his Observations is founded that precious Work of *Ptolemy*, which he called his μεγάλη σύνταξις, or his great Construction; for from them he gathered the Procession of the Equinoxies, and the Theory of the Planets.

WHEN *Egypt* was conquered by the *Saracens*, and *Alexandria* reduced under their Jurisdiction, the Conquerors took *Astronomy*, with the rest of the Liberal Arts, under their Protection, and took care that most Part of the Books concerning the Liberal Arts and Sciences should be translated from the *Greek* into their own *Arabian* Language.

THE *Saracens* passing from *Africk* into *Spain*, and having a Commerce with the Western *European* Nations, imparted to them the Science of *Astronomy*, which before was almost lost in *Europe*; so that about the Year 1230, at the Command of the Emperor *Frederick*, *Ptolemy*'s Almagest, or his great Syntaxis, was translated from the *Arabick* into *Latin*.

AFTER that Time, *Astronomy* received many Improvements from the Patronage of the greatest Princes, and the Labours of the most celebrated Philosophers; among whom, in the first Place, is to be named *Alphonsus* King of *Castile*, who is never to be forgotten, on the Account of the *Astronomical* Tables called after his Name. *Nicolaus Copernicus* was not only a diligent Observer, but also a Restorer of the antient *Pythagorean* System.

Syſtem Prince *William*, *Landgrave* of *Heſs*, who procured Quadrants and Sextants much larger than what were formerly uſed, to obſerve the true Places of the Stars: This Prince's Obſervations are publiſhed by *Snellius*. Sir *Henry Savill* was moſt ſkilful both in *Aſtronomy* and *Geometry*, who is ever to be honoured for his Munificence in founding our two Profeſſions of *Aſtronomy* and *Geometry* in the Univerſity of *Oxford*, and endowing them with ample Salaries; upon which Account, and many other Benefits he beſtowed on the learned World, he will always be had in Remembrance with the greateſt Reſpect. That Noble *Dane*, *Tycho Brahe*, who for his Skill in obſerving was ſuperior to all that went before him; and who, for the Furniture of his Obſervatory, exceeded even Princes and Kings: He publiſhed a Catalogue of 770 fixed Stars, which he had diligently obſerved. *John Kepler*, a moſt excellent *Aſtronomer*, by the Help of *Tycho*'s Labours, found out the true Syſtem of the World, and the Laws the Celeſtial Bodies obſerve in their Motions, with which he vaſtly improved *Aſtronomy*; his excellent Works are well known to the learned World, and will ever ſhew how much he is to be praiſed. *Galileus*, the *Lyncian* Philoſopher, who firſt applied a Teleſcope to the Heavens, and by its Means diſcovered a great many new ſurprizing *Phænomena*, as the Moons or Satellits of *Jupiter*, and their Motions; the various Phaſes of *Saturn*, the Increaſe and Decreaſe of the Light of *Venus*, the mountainous and uneven Surface of the Moon, the Spots of the Sun, and the Revolution of the Sun about his own Axis; all which were firſt obſerved by this great Philoſopher.

I ſhould much exceed the Bounds of a Preface, if I ſhould name the reſt of the great Improvers of our Art, with the Praiſes that are due to them; particularly *Hevelius*, who has given us a Catalogue of the fixed Stars much larger than *Tycho*'s, compoſed from his own curious Obſervations. The moſt illuſtrious Gentlemen, Meſſieurs *Hugens* and *Caſſini*, who firſt ſaw the Satellits of *Saturn*, and diſcovered

his Ring: *Gassendus*, *Horrox*, *Bullialdus*, *Ward*, *Ricciolus*, and many other *Astronomers* of great Renown. But we have one here, who on Account of his great Merits in *Astronomy* does excel them all, that is, the most eminent and learned Dr. *Edmund Halley*, *Savillian* Professor of *Geometry*, in this University, my most friendly Colleague, to whose Labours *Astronomy* owes many, and those not small Improvements: In him there shines out together (which I know not if they are to be found in any other Person to such a Degree) the greatest Dexterity in Practical *Astronomy*, and a most profound and exquisite Skill in *Geometry*, which will appear by his *Astronomical* Tables when published; for they will far excel all others that ever were, or perhaps ever will be published.

I could name many others of our own Countrymen, who have done much Service towards the Improvement of *Astronomy*; but we must not pass over in Silence the Labours of the celebrated Royal Professor, the late Mr. *John Flamsteed*, who with indefatigable Pains, for more than 40 Years, watched the Motions of the Stars, and has given us innumerable Observations of the Sun, Moon and Planets, which he made with very large Instruments, exactly divided by most exquisite Art, and fitted with telescopical Sights. Whence we are to rely more on the Observations he hath made than on those that went before him, who made their Observations with the naked Eye, without the Assistance of Telescopes. The said Mr. *Flamsteed* has likewise composed the *British* Catalogue of the fixed Stars, containing about 3000 Stars, which is twice the Number that are in the Catalogue of *Hevelius*; to each of which he has annexed its Longitude, Latitude, Right Ascension, and Distance from the Pole; together with the Variation of Right Ascension and Declination, while the Longitude increases a Degree. This Catalogue, together with most of his Observations, is printed ou a fine Paper and Character, at the Expence of the late Prince *George* of *Denmark*; but Mr. *Flamsteed*,

ſteed, before he died, had near finiſhed another Edition of them at his own Expence, which were publiſhed in three Volumes in Folio, 1725.

AMONG ſo many Helps and Advantages towards the Underſtanding of *Aſtronomy*, there was ſtill wanting an univerſal and complete Theory of the celeſtial *Phænomena*, explained according to their true Motions and phyſical Cauſes. But this Work has been lately performed, finiſhed, and publiſhed by the late Dr. *Gregory*, the great Honour of our Profeſſion, and my Preceptor, whom I ought always to remember with Gratitude, for it is owing to him if I have made any Advances in this Study.

IN the mean time it is to be acknowledged, that this Work does not ſeem to be ſuited to the Capacity of young Beginners; for it contains many Things which require an Inſight into deep *Geometry*, ſo as to be clearly underſtood; which Skill is ſeldom to be met with in young Men, who are for all that capable of learning the Elements of *Aſtronomy*. Beſides, the celeſtial Motions and their phyſical Cauſes are always jointly explained; which two Things, when they are to be learned by Beginners, diſtract them too much, and make the Doctrine difficult. Therefore I thought it more advantageous to the Learner firſt to explain the Motions, and give an Account of the *Phænomena* that ariſe from theſe Motions, which when once underſtood there will be an eaſy Admiſſion into the Knowledge of phyſical Cauſes.

FOR which Purpoſe I compoſed the following *Lectures*, which I read in the *Aſtronomical* School at *Oxford*, as my Duty obliged me: In them I have taken ſome Pains, that all the celeſtial Motions may be clearly explained, and the Reaſons of the *Phænomena*, which ariſe from thoſe Motions, be given. But particularly of thoſe which are to be underſtood by the Help of a few Propoſitions of the Elements of *Geometry*. And therefore I would adviſe our young Beginners, who deſire to learn *Aſtronomy*, that they would place *Euclid*'s Elements before them when they read theſe *Lectures*, and conſult them when

they find any Propoſitions quoted by us. Thoſe we chiefly uſe are but few in Number; ſuch are the 4th, 5th, 8th, 13th, 15th, 27th, 29th, 32d, and 47th of the firſt Element. The 16th, 18th, 20th, 31ſt, 35th, 36th, 37th of the 3d Element: Alſo the 4th, 5th and 6th of the 6th Element; beſides the Doctrine of Proportion, contained in the 5th Book. It were likewiſe to be wiſhed that the young Student of *Aſtronomy* were ſkilled in plain and ſpherical *Trigonometry*. But if there be any, as I believe there are ſome, who deſire to learn *Aſtronomy*, and yet are ignorant of *Trigonometry*; I require of them that they grant and allow us this Poſtulate; becauſe in every *Triangle*, either ſpherical or plain, there are three Angles and three Sides; of theſe ſix, having any three, one of which in a plane Triangle muſt be a Side, all the reſt may be found. It is *Trigonometry* that teaches us how to perform this, whoſe Uſe is apparent in all the Parts of *Aſtronomy*.

THERE are alſo ſome Things in our *Aſtronomy* which require a Knowledge of deep *Geometry*, as when we ſpeak of the Elliptick Theories of the Planets diſcovered by *Kepler*. But I would not have the Beginners or young Students trouble themſelves with theſe Particulars, ſo they may paſs them over.

I deſire alſo of them that are unacquainted with *Aſtronomy*, that after they have read the XI. and XII. *Lectures*, concerning the general Cauſes of Eclipſes, they would leave the reſt of that Doctrine till they are inſtructed in the ſpherical Inſtitutions, as they are explained by us in the XX. and XXI. *Lectures*; and then they may return to the remaining Parts of the Doctrine of Eclipſes, contained in the XIII. and XIV. *Lectures*.

THEY who underſtand what is here delivered may with much Advantage undertake to read that excellent Work of Dr. *Gregory*'s, and learn the phyſical Cauſes of the celeſtial Motions from thence.

THE CONTENTS OF THE LECTURES.

I. OF visible and apparent Motion. Page 1

II. Of the apparent Motion which arises from the Motion of the Spectator or Observer. 11

III. Of the System of the World. 16

IV. In which is proved that the System explained in the former Lecture is the System of the World. 26

V. Of the Solar Spots. Of the Rotation of the Sun and Planets round their Axes; and of the fixed Stars. 41

VI. Of

VI. *Of the Magnitude and Order of the* fixed Stars. *Of the* Constellations, *Catalogues of the* Stars, *and the Changes to which they are liable.* *Page* 47

VII. *Of the Motion of the* Earth *round the* Sun: *And also about her own* Axis, *whereby the apparent Motion of the* Sun *and Heavens are explained.* 59

VIII. *Concerning several other* Phænomena *or Appearances which depend on the Motion of the* Earth. 75

IX. *Of the* Moon, *its* Phases *and* Motion. 90

X. *Of the Inequalities in the* Lunar *Motions. The Face of the* Moon, *her Mountains and Valleys.* 100

XI. *Of the Obscurations or* Eclipses *of the* Sun *and* Moon. 109

XII. *Of the* Penumbra *and its Cone, the Height of the Shadow, and the apparent Diameters of the Shadows.* 118

XIII. *Of the Projection of the* Moon's *Shadow on the Disk of the* Earth. 127

XIV. *A new Method of computing Eclipses of the* Sun, *as they are to be observed from*

from any given Place on the Earth's *Surface.* Page 139

XV. *Of the* Phænomena, *or Appearances arising from the Motions of the* Earth, *and the two inferior Planets* Venus *and* Mercury. 155

XVI *Of the Motions of the three superior Planets,* Mars, Jupiter, *and* Saturn, *and the Appearances arising from them.* 171

XVII. *Of Comets.* 189

XVIII. *The Spherical Doctrine, or the Circles of the Sphere.* 205

XIX. *Of the Doctrine of the* Sphere. 218

XX. *Of the* Twilight, *and of the Refraction of the Stars.* 232

XXI. *Of the Parallexes of the* Stars. 245

XXII. *Of the Theory of the annual Motion of the* Earth. 268

XXIII. *Of the Motion of a Planet in an Ellipse, and the Solution of* Kepler's *Problem, about the cutting of the* Elliptick Area. 280

XXIV. *Sir*

XXIV. *Sir* Isaac Newton's *Solution of* Kepler's *Problem, and* Ward's *Elliptick Hypothesis, explained.* Page 296

XXV. *Of the Equation of* Time. 312

XXVI. *Of the Theories of the other Planets.* 327

XXVII. *Of the Stations of the other Planets.* 345

XXVIII. *Of the Division of* Time, *and its Parts.* 359

XXIX. *Of the* Kalendars, *of* Cycles *and* Periods. 369

XXX. *An* Appendix, *containing a Description and Use of both the* Globes; *together with some spherical* Problems *that are to be solved by a* Trigonometrical *Calculation.* 381

ASTRO-

ASTRONOMICAL LECTURES.

LECTURE I.

Of Visible and Apparent Motion.

ASTRONOMY being a Science in which are explained the Motions of Bodies that are at an immense Distance from us, and the Appearances which arise from these Motions: They who would learn this Science, must first be informed of the Manner how the Motions of distant Bodies become visible, and the Objects of our Senses. Lecture I.

What Bodies seem to be at Rest. AND first it is plain, that since the Eye looks upon such Bodies to be at Rest, which keep the same visible Distance, the same Position and Situation, not only in respect of other Bodies which we conceive to be at Rest, but also in Respect of the Eye that beholds them; those Bodies can only

Lecture I.

be perceived to move, which change their Diſtances and Poſitions, in reſpect to other Bodies, or to the Eye of the Spectator.

What in Motion.

But that we may explain this Matter, by its proper Principles, and draw it from its Origin, that is from an Explanation of the Manner of Viſion: It muſt be known, that the Writers of *Opticks* demonſtrate that every Body which is ſeen, has its Image painted in the Bottom of the Eye, upon that Coat which is called Reticular, or the *Retina*, whoſe Surface is Spherical concave. This Image is made by the Rays of Light which flow from the Viſible Object to the Eye, and are therein received and refracted. The Image of each Point is in that Place, where the innumerable Rays which come from that Point, and paſſing through the Humours of the Eye, do by Refraction meet on the *Retina*.

Of the Senſe of Seeing.

Plate I. Fig. 1.

Let A B, a Portion of the Periphery of a Circle, repreſent the outward Surface of the Eye; D G the Bottom or reticular Coat, which is formed by the Extremities of the Optick Nerves, and let C be the Center of the Eye; the Image of the Point F will be in the Line F C H, and therefore at H: So alſo the Image of the Point E will be in the Line E C L, at the Point L; for the Rays of Light will, by the pellucid and clear Coats and Humours of the Eye, be ſo refracted, that all thoſe Rays which come from F, and enter the Eye, will change their Direction, and turn toward H, where they will meet; and likewiſe all thoſe which come from E, being refracted in the Eye, will converge and meet again at L, where they will form the Image of the Point E; for by ſtriking on the nervous Fibres in theſe Points, they will excite the Senſe of Viſion.

There is a fine Experiment which confirms and demonſtrates this Doctrine. For if the Eye of an *Ox*, or any other Creature, juſt after its Death, be taken out of its Head, and the opake and black Coat call'd the *Choroides*, which covers the back Part of the Eye, be ſeparated, ſo that the thin and pellucid reticular Coat may appear; if this Eye be turned towards

Lecture I.

towards a Window, or any Object that is ſtrongly illuminated, we ſhall ſee with Pleaſure and Admiration, a fine Picture on the *Retina*, exactly repreſenting the Object in its proper Colours. We ſhall have the ſame Appearance, if inſtead of the Eye we take any convex-glaſs of a Teleſcope, and turn it towards the Object, and place a white Paper at a due Diſtance behind the Glaſs, we ſhall obſerve upon the Paper an exact Image of the Object, diſtinctly repreſented with its lively Colours.

How Motion is perceived by our Eyes.

IF therefore the Image H of the Point F remain unmoved on the ſame Point of the *Retina*, the Eye being likewiſe unmoved, the Object F will be at Reſt: But if the Point F be carried to E, its Image will thereby ſucceſſively paſs through different Parts of the *Retina*, and deſcribing the Space H L, will excite the Senſation of Motion. If the Point F be at a great Diſtance from us, and the Motion be made in a Plane, paſſing through the Eye, the Spectator will judge of the Magnitude of the apparent Motion, by the Magnitude of the Angle F C E.

IF in the Line C F there be another Object M, which is likewiſe at a great Diſtance from us, and this Object be carried from M to N, its Motion will appear to be the ſame with that of the Object F; for the Way of both will appear the ſame, the two Images having the ſame Path, and paſſing through the ſame Space in the Bottom of the Eye. If the viſible Point M be carried in the Line C F from M to F, ſuch a Motion cannot be perceived by the Spectator, the Image of M remaining unmoved all the while on the *Ratina*: And whatever Bodies are moved in Lines that paſs through the Center of the Eye, the Motions of ſuch Bodies are not to be obſerved by our Sight, nor can we any other Way diſcern ſuch Motions, but by the Increaſe or Diminution of the Splendor and viſible Magnitude of the Objects. I ſpeak here of diſtant Objects: For thoſe that are near us, though they move in Lines paſſing thro' the Eye, yet we may diſcern their Motions by the Change of Poſition and

Lecture I.

Situation which they hold in respect to other Bodies, whose Positions and Distances are known. Now whatever be the Path of the moveable Point F in the Plane F C E, whether it be in the right Line F E, or in the circular Arch F P E, or in any other curve Line F Q E; when it comes to the Line C E, its apparent Motion will always be seen to be the same, while the Angle F C E remains the same; but when the Angle F C E is increased or diminished, the visible Motion will be in like Manner increased or diminished, which therefore can be only measured by that Angle.

The Measures of Angles.

THAT therefore the apparent Motions of Bodies may be determined, we must here shew the Method by which *Geometers* and *Astronomers* find out the Measures of Angles; which though it is commonly known even to the meanest Artists, yet that we may omit nothing which will make what is to follow easily conceived by Beginners, we will here explain it in a few Words.

A Degree what. Scruples or Minutes.

EUCLID has demonstrated, that the Angles at the Center of any Circle are proportional to the Arches on which they stand, and therefore the Measures of Angles will be best known from those Peripheries or Arches which subtend them: On which Account the *Astronomers* divide the whole Periphery of a Circle into 360 Parts, which are called Degrees; and they divide each Degree into 60 other Parts, which are named Scruples, or First Minutes; each of those Minutes are again dived into 60 second Scruples or Minutes; and each Second is also supposed to be divided into Thirds, each Third into Fourths; and so on.

Plate I. Fig. 2.

By this Means they reckon no more Degrees or Parts in the greatest Circle than in the least that is; and therefore if the same Angle at the Center be subtended by two concentrical Arches, they count as many Degrees or Parts in the one as they do in the other; for these two Arches have the same Proportion to their whole Peripheries. For Example: Let A C B be an Angle, and from the Center C let there

Lecture I.

there be described two Arches A B, D E, subtending the Angle: There are as many Degrees and Minutes contained in the Arch A B, as there are in the Arch D E, although the Radius or Semidiameter of the Arch A B were only a Foot long, and the Radius of the other reached the fixed Stars. It is true indeed, that a Degree in the Arch A B is so much less than a Degree of the Arch D E, as its Radius C B is less than C E or C D: The Angle C is said to be of so many Degrees or Minutes as the Arch which subtends it contains of such Parts.

The Method of Measuring Angles.

THE Instrument by which Angles are observed, is a known Portion of the Periphery of a Circle, as a Quadrant, Sextant, or Octant, that is, the fourth Part, sixth Part, or eighth Part of the whole Periphery. If it be a Quadrant, the Instrument-Makers divide it into 90 Degrees, 90 being the $\frac{1}{4}$th of 360: If a Sextant, it is divided into 60, which is the $\frac{1}{6}$th of 360: If an Octant, it contains 45 Degrees or the $\frac{1}{8}$th of 360. They divide again each Degree into Minutes, and each Minute into Seconds, if the Instrument be large enough to shew such Parts. The Instrument-Makers fix to the Side of the Instrument Pins or Sights, by which they collineate to the Object, and they fasten likewise a Rule moveable about the Center upon the Plain of the Instrument, which Rule is likewise furnished with Sights, with which they observe Angles in this Manner.

Plate I. Fig. 3.

LET A and B be two Objects at a great Distance from us: And suppose the Observer at C, who is to measure the Angle A C B: Let the Instrument be turned, 'till the Object A can be seen through the Sights of the Side C D; and let the Plane of the Instrument be so moved round the Side C D, and the Rule round the Center, that the Object B may be seen through the Sights of the Rule: It is manifest from what has been said, that the Arch D E will give the Measure of the Angle A C B, and that the Arch A B will contain as many Degrees and Parts, as the Arch D E, which the Rule cuts off from the Instrument.

Lecture I.

The Horizon.

The Altitude of a Star.

The Pole of the Horizon.

MOREOVER *Astronomers* have other Bounds or Marks from which they reckon the *Arcual* Distances of Stars, and measure them with a like Instrument. These are chiefly the *Horizon*, which is formed by a Plane touching the Surface of the Earth where the Spectator stands, and is infinitely extended towards the Heavens; which it divides into two Hemispheres, or Parts sensibly equal, and seperates the Visible Heavens from the Invisible. And if we suppose a Circle perpendicular to this Horizon passing through any Star, the Arch of it comprehended between the Star and the Horizon is called the Altitude or Height of that Star. There is another Mark which is called the Pole of the Horizon, and is that Point which is directly over-head, through which a Line perpendicular to the Horizon will pass: And it is in this Line all heavy Bodies endeavour to descend, and according to which we stand upright. By this Method the Sailors at Sea find out the Height of the Sun by the Angle which is formed in the Eye, by Lines coming from the Sun, and from the Horizon. So likewise the *Astronomers* by Rules and Quadrants made on Purpose for that Use, observe the Angle which the Rays or Lines that come from the Sun or Stars, make with the Line that is perpendicular to the Horizon.

INSTEAD of plain Sights we now commonly make Use of *Telescopes*; for by their Means distant Objects are more certainly and exactly observed, than they can be by our simple View. The Manner of fitting *Telescopes* to Instruments, the Method of dividing the Arch, and the Contrivances for managing and moving the Instrument for Practice, we leave to the *Mathematical* Instrument-Makers to describe.

Plate I. Fig. 4.

The apparent Diameters of Bodies.

BY the Measure of Angles we likewise find the apparent Diameters of distant Bodies: Let A B be a Line which is seen by the Eye at C directly opposite to it, and suppose drawn from its Extremities A, B, right Lines A C, B C to the Eye; that Line A B is said to appear under the Angle A C B, which

which is called its Apparent Magnitude, and is said to be so many Degrees and Minutes as that Angle observed by an Instrument contains. After the same Way the Object D E, seen by the Eye F, is said to appear under the Angle D F E, and the Apparent Magnitudes of the Lines A B, D E, are to one another, as the respective Angles A C B, D F E. Lecture I. Fig. 5.

But if the Eye come near to the Object A B, so as to view it but from half the Distance, that is, from G; the Object will be from thence seen under twice the Angle it appeared under before: If the Eye come three times nearer, its apparent Magnitude will be near three times greater, provided the Angles be but small, and exceed not a Degree or two: And the apparent Diameters of such Objects do nearly increase, as the Distances from which they are viewed are diminished. *The apparent Diameters, the nearer we approach them, grow bigger.* Fig. 4.

By this Method, if we know the apparent Diameters of two Bodies, and the Proportion of their Distances from us, we can know from thence the Proportion their true Diameters bear to one another: For if their Distances be equal, their true Diameters will be as their apparent: And if their apparent Diameters are equal, their true Diameters will be proportional to their Distances. For Example: If the Angle A C B be equal to the Angle D E F, but the Distance C B triple of the Distance E F, the Line A B will be triple of the Line D E: But if the Distance C B be not only triple of the Distance *e f*, but also the Angle A C B be double of the Angle *d f e*, the Object A B will be sextuple of the Object *d e*: For if we suppose C M equal to *d f*, and an Object M N to appear under the Angle M C N or A C B; because the Angle M C N is double of the Angle *d f e*, M N will be double of *d e*; but because C B is triple of C M or *d f*, A B will be triple of M N, and consequently it will be six times bigger than *d e*. Hence, if the apparent Diameters of the Sun and Moon be equal, let the Sun be 100 times further from us than the Moon, the Sun must needs be 100 times in Diameter bigger than the Moon: We shall Plate I. Fig. 4, 5.

Lecture I.

afterwards demonſtrate, that the Sun's Diſtance from us is above 100 times greater than that of the *Moon*.

If we know the apparent Diameter of any Body, we can from thence exactly know, by the Help of *Trigonometrical* Tables, what Proportion the Diſtance of that Body bears to its true Diameter. For ſuppoſe the Object D E to be ſeen by the Eye at F under the Angle D F E. For Example of one Degree; then the Diſtance F E will be to D E the Diameter of the Object, as the Radius of a Circle is to the Tangent of the Angle D F E; that is, ſuppoſing D F E one Degree, as 10000 is to 174,5. The Sun appearing under an Angle of about half a Degree, or 30 Minutes, its Diſtance will be to its own Diameter as 10000 to 87: Hence, we are certain that the Sun's Diſtance from us, is nearly equal to 115 of its own Diameters. And if an Eye were placed in the Sun, to obſerve the Angle under which the Diameter of the Earth appeared from thence, we then ſhould be able to tell exactly the Diſtance of the Sun from us, in Diameters of the Earth, or in Miles.

The Advantages of Teleſcopes.

Since, as we have ſaid, the apparent Diameters of Bodies grow bigger, the nearer we come to them, and that they are increaſed almoſt in the ſame Proportion that the Eye approaches them; (for Example: If any Man were ten times nearer to the Moon than we are, and did there obſerve it, he would ſee the Moon ten times bigger in its Diameter and clearer than we do; in Diameter I ſay, for the Surface would appear 100 times larger than it does to us:) If here on Earth we ſhould take a Teleſcope which only increaſes the Diameter ten times, and look to the Moon with it, the Moon will have the ſame Appearance ſeen with ſuch a Teleſcope, as would appear to a Spectator ten times nearer it than we are. But if we ſhould uſe Teleſcopes (and ſuch there are) which magnify the Diameters of Objects 100 or 200 times, they will ſhew the Moon in the ſame Manner, Figure and Bigneſs, as it would appear in, at a Diſtance 100 or 200 times leſs than ours. Hence we can perceive with our Eyes

Eyes with what Face, and how large the Moon would ſhew itſelf, at the Diſtance of three Diameters of the Earth: As likewiſe we can diſcern how it would appear, if we approached it much nearer, and view it only at the Diſtance of 1000 Miles; for from thence we ſhould be able to diſcover in it vaſt Ridges of Mountains, deep Caverns, many Vales, and large open Fields. By the Means of Teleſcopes we ſtill aſcend higher in the Heavens, and we can approach the Planets, Commets and fixed Stars ſo near, that of ſuch immenſe Diſtances there remains only the hundredth, or two hundredth Part to have the whole Journey finiſhed; and from thence we can behold the Converſions of the Planets about their proper *Axes*; the Moons of *Jupiter* and *Saturn*, their Eclipſes; the Belts of *Jupiter*, the wonderfnl Ring of *Saturn*, and all the various Appearances and Shapes it takes. We could not paſs over, without taking Notice in this Place, theſe Advantages of the Teleſcope, ſince it is the chief Inſtrument by which we obſerve the Magnitudes of the Heavenly Bodies, and their apparent Motions.

Lecture I.

How the Motions of far diſtant Bodies, which are in themſelves equal, may appear unequal.

Since the Motions of diſtant Bodies are no other ways to be known, but by the Change of the Angle which is at the Eye that obſerves them; it will eaſily appear from thence, that though Bodies move equally and regularly, deſcribing equal Spaces in equal Times, their Motions notwithſtanding may ſeem to be very unequal and irregular. This will be beſt underſtood by an Example.

Plate I. Fig. 6.

Suppose a Body to be revolved in the Periphery of a Circle A B D E F G Q, and to move through equal Arches A B, B D, D E, E F, in equal Times; and let the Eye be in the Plane of the ſame Circle, but at a Diſtance from it, viewing the Motion of the Body from O: When the Body goes from A to B, its apparent Motion is meaſured by the Angle A O B, or the Arch H L, which it will ſeem to deſcribe; but in an equal Time, while it moves through the Arch B D, its apparent Motion is deter-

Lecture I. determined by the Angle BOD, or the Arch LM which is much leſs than the former Arch HL; and the Body, when it arrives at D, will be ſeen at the Point M of the Periphery NLM: But it takes the ſame Time to deſcribe DE which is equal to AB, or BD; and when it arrives at E, it is ſtill ſeen at the Point M; ſo that all the Time it is moving through the Arch DE, it appears almoſt immoveable, and, as it were, to ſtand ſtill. While the Body is continually going forward in its proper Orbit, and deſcribing the Arch EF, when it comes to F, the Eye in O will ſee it in L, and it will appear to have gone backward in the Arch ML. So alſo, while it moves from F to G, at its Arrival at G, it will be ſeen at H in the very ſame Place it appeared in when it was in A. So likewiſe, while it paſſes from G through I to Q, the Spectator's Eye at O will obſerve it, as if it had deſcribed the Arch HKN. And though it is ſtill going on in its Orbit, while it runs through the Arch QP, the Spectator will obſerve it all that Time near the Point N, in a *Stationary* State. After its paſſing by P, and going to A, it will appear to change again its Courſe, and deſcribe the Arch NKH with very unequal Motions.

Optical Inequality. THIS Inequality of Motion is called by *Aſtronomers* the *Optical Inequality:* Becauſe it is not really in the Bodies moved, but only apparent to the Eye which perceives it, ariſing from the Poſition of the Spectator: For the Body all the Time moves uniformly forward, and if the Eye were in the Center, it would ſee the Motion always perfectly regularly performed.

Plate I. Fig. 7.

If the Eye be placed within the Circle, the Motion that is equal, may appear unequal, IF the Eye were placed in any Point, as O, within the Orbit of the Body, but not in the Center, and there the Spectator remained immoveable, he would ſtill obſerve the Motions to be unequal, although the Body moved never ſo regularly; and when at the greateſt Diſtance from him, as at A, it would appear to be ſloweſt; when it comes neareſt, it would ſeem to move quickeſt. This is plain; for the Arches AB and CD being equal, they will be deſcribed in equal

equal Times. But the Angle DOC being greater than AOB, the Motion in D will appear swifter than that at A. But in this Case the Body will never appear to stand or to go backward, but always forward: And therefore, when a Spectator placed within the Orbit of another Body, and viewing its Motion, perceives it sometimes to go forward, then to stand still, and afterwards go backward, we may from thence conclude, that the Place of the Spectator is likewise moved.

Lecture I.

but the Body can never be seen to go backward, or to change its Course.

LECTURE II.

Of the Apparent Motion which arises from the Motion of the Spectator or Observer.

HITHRTO we have supposed the Spectator to have remained immoveable all the Time of the Observation: But if the Place of the Observer be likewise moveable, then there will be very different Appearances, and the Eye will perceive those Bodies to be at Rest which may have really a very quick Motion, and other Bodies may seem to be in Motion, which remain really at Rest: And not only these Appearances may be seen, but the Motion of Bodies may appear to be directly contrary to what they truly are; and Bodies which are really going *Eastward*, may appear to move towards the *West*. All which will be most easily declared and made plain from the Appearances observed by them who sail in a Ship:

They who sail in a Ship, perceive not the Motion of the Ship.

SUPPOSE a Ship carried by the Winds with a swift, but uniform Motion; the Passengers can neither perceive the Motion in the Ship, nor of any Thing

Lecture II.

Thing in it that keeps the ſame relative Place in the Ship: For ſince the Veſſel and all its Parts retain the ſame Situation and Poſition in reſpect of the Eye, their Images painted on the *Retina* will always abide in the ſame Place, and therefore they muſt appear unmoved. Hence it is, that though every Thing in the Ship goes as faſt forward as the Ship itſelf does; yet ſuch Motions cannot be perceived by a Spectator that ſits in the ſame relative Place, and who has the ſame common Motion with the Ship. But when the Spectator turns his Eyes towards the Shore, or upon Objects which are without the Ship, they will ſeem to be moved; for while the Ship goes forward, it carries along with it the Eye of the Spectator, by which Motion of the Eye the Poſition of external Objects in reſpect of itſelf will be changed, and their Images will ſucceſſively occupy different Places on the *Retina*; and therefore Objects without the Ship, which are really at reſt, will ſeem to be moved, whereas thoſe that are within the Ship, and really in Motion, will appear to be at Reſt.

But external Objects without the Ship will ſeem to move.

If, while the Ship is moving very faſt forward, a Ball of Lead, or any other heavy Metal, were let fall from the Top-maſt, the Paſſengers in the Ship will obſerve the Ball to fall perpendicularly downwards, and it will fall upon the Deck juſt by the Foot of the Maſt, after the ſame Manner as it would fall were the Ship at Reſt. But notwithſtanding this, the true Motion of the Ball is not in the Perpendicular, but in an Oblique Line, in which it deſcends; and a Spectator in another Ship which is at Anchor, will eaſily obſerve this Obliquity and Curvity of its Way, while it falls through the Air. The Reaſon of this Appearance is eaſily ſhewed: For according to the firſt and principal Law of *Natural Philoſophy*, a Body once put into Motion, endeavours to retain that Motion, and to continue moving in the ſame Direction. Now the Ball, while it was held at the Top-maſt, went forward with the Ship, and had its Motion communicated to it; and

The Motion of a Ball falling down in a Ship.

and therefore, after it is left to fall, it will retain the ſame Force to go forward as it did before; and at the ſame Time, its Weight carrying it downward, it will both go forward and deſcend: For the two Forces, one communicated by the Ship, and the other from Gravity, will not hinder or diminiſh one another, they not being contrary. It will therefore be moved as faſt forward, and as much downward as it would be, did the two Forces act upon it ſeparately at different Times. By theſe two Forces acting together, the Rectitude of its Way is only hindered, which it would have, did the Perpendicular and the *Horizontal* Forces act ſeparately; and the real Way of the Ball through the Air is a curve Line, exactly like that which a Body takes when it is thrown according to an *Horizontal* Direction: And in ſuch a Line it will be obſerved to move, by a Spectator placed near it in another Ship which is at Reſt. Beſides, ſince the Ball and Maſt are both moved forward with the ſame Velocity, they will always remain at the ſame Diſtance from each other, and therefore the Ball will touch the Deck juſt by the Foot of the Maſt. Moreover the Motion of the Ball forward is common to the Ship, and all its Parts, as likewiſe to the Paſſengers that are relatively at Reſt in the Ship. But we have before ſhewed, that the common Motion could not be obſerved by the Paſſengers in the Ship, and therefore it cannot be perceived neither while the Ball is falling. Wherefore the only Motion that can be ſeen, will be that which is impreſſed upon it by its Gravity, which is peculiar to the Ball, and by which it deſcends. And therefore the Paſſangers will ſee the Ball deſcending only in a perpendicular Line. Experiments have been often made, which demonſtrate that all we have ſaid is exactly true.

Lecture II.

The Motion of a Ball thrown within the Ship.

If any Perſon ſitting at the Ship's Head, ſhould throw a Ball towards the Stern with the ſame Velocity that the Ship goes forward, that Ball would neither go forward nor backward; and if there were no Gravity, it would remain immoveable: But

Lecture II.

But because Gravity acts upon it, it will really descend in a perpendicular Line, and a Spectator in a Ship at Anchor would observe it descending in a right Line. For the Force impressed upon it when it is thrown, will only destroy the first Force communicated to it from the Ship, to which the Projectile Force is contrary and equal. But for all this, the Passengers will not perceive this perpendicular and direct Motion; but they will see the Ball go towards the Stern with the same Force, as it really would have done, had the Ship been at Rest, and the Ball been thrown with the same Force to the Stern.

But if the Velocity with which the Ball is thrown toward the Stern should be less than that of the Ship, the real Motion of the Globe will be forward, in the same Direction in which the Ship goes, but slower than it; for the whole Motion communicated by the Ship will not be destroyed, and there will still remain a Part of its former Motion, by which it will be carried forward, though not so fast as before. But the Passengers will perceive no such Motion, but they will observe the Ball to be moved in a Line directly contrary to its real Motion, with that very Velocity that it would have, were it thrown when the Sip is at Rest: Hence it is plain, that Bodies may appear to have a Motion directly contrary to their real and absolute Motion.

An Objection.

But some may object, that the Ball thus thrown, will really hit the Stern of the Ship, and impress on it a considerable Blow, which it could not do, had it not a Motion towards the Stern. But this Difficulty is easily removed; for though they that are within the Ship see the Ball go and hit the Stern, a Spectator without, who is not in Motion, will observe that the Ball does not come upon the Stern and give it a Stroke, but that the Stern rushes upon the Ball, and acts upon it with all its Force: And the Force of the Stroke which each Body receives, is the same as if the Ship had been at Rest, and the Ball had fallen upon it with the same Velocity

Velocity that the Stern does really come againſt the Ball; for it is known from the Laws of Motion, that if there be any two Bodies, A and B, equal or unequal, the Force of the Stroke will be the ſame, whether B with a certain Degree of Velocity comes upon A, which is at Reſt; or if B ſhall be at Reſt, and A with the ſame Velocity ruſhes upon B; or if both Bodies move the ſame Way, but A, moving faſter by its greater Velocity, gives B an Impulſe, the Force of the Stroke will be the ſame as if B were at Reſt, and A came upon it with the Differnce of their Velocities, that is, by the Exceſs wherewith the Velocity of A is greater than that of B: Or laſtly, if A and B have contrary Motions, and hit one againſt the other, the Greatneſs of the Stroke will be the ſame as if one of them ſtood ſtill, and the other came againſt it with the Sum of their Velocities. In one Word, whatever the real Velocities of the Bodies may be, ſo long as their relative Velocities, or the Velocities by which they approach each other, remain the ſame, the Force of the Stroke will likewiſe remain the ſame. Hence it is, that in a Ship, however ſwiftly it may ſail, all our apparent Motions, and the Motions of every Thing in the Ship, do all appear to be the ſame that they would, where the Ship at Reſt; And it is obſervable, that *Flies* and other *Inſects* keep the ſome Motion in regard to one another, whether the Ship is at Reſt, or ſails uniformly forward with any Degree of Velocity, let it be never ſo great. And it is univerſally true, that all Bodies that are ſhut up in any one Place, preſerve the ſame Motions in regard to one another, and all Appeances will be the ſame, whether the Place remains unmoveable, or has a direct uniform Motion forward.

Lecture II.

Plate I. Fig. 8.

I have brought theſe Examples, that you may perceive how wide the Differences may be between real and apparent Motions, and how hard it is to judge of real Motions, by thoſe that are ſeen.

By

Lecture II.

By this it is evident, that if a Spectator were placed in *Jupiter*, or *Saturn*, or any other of the *Planets*, he can never be made ſenſible of the Motion of his own Habitation, no more than they who ſail in a Ship can perceive the uniform Motion of the Ship. Paſſengers who ſail in Ships may indeed be very ſenſible of the frequent Toſſings and ſudden Shocks the Ships receive from the Waves and Wind, which they find exceedingly troubleſome to them. But the *Planets*, which compoſe a Celeſtial Fleet, are not liable to any Storm; they without any Diſturbance or Commotion circulate in their Orbits, and ſail, as it were, in a moſt Pacifick Ocean, which is continually calm and ſerene.

LECTURE III.

Of the Syſtem of the World.

WE have ſhewed, that according to the different Situation and Motions of the Spectator, the Appearances of Things will be very various and different. That we may have a more diſtinct Knowledge of the Fabrick of the *World*, and that the admirable Beauty of the Univerſe, and the harmonious Motions of the Bodies therein contained, may be more eaſily underſtood, it will be requiſite, that that Divine and Immenſe Fabrick ſhould not be obſerved from one Point or Corner only: But as in viewing of large Palaces, we take the different Proſpects they afford from ſeveral Places; ſo here, to have a true and juſt Notion of the World, we muſt ſuppoſe it to be obſerved in different

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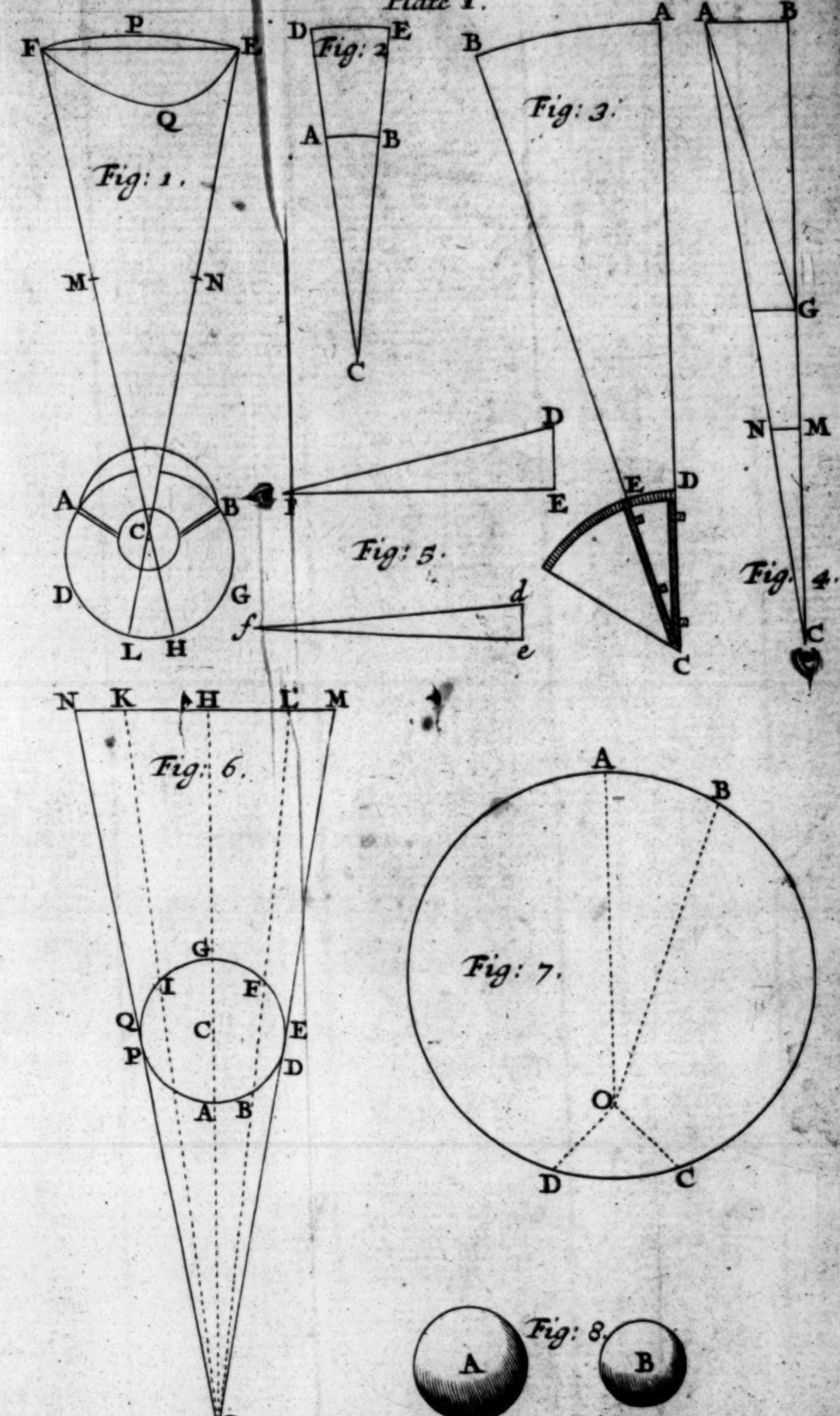
Plate I.



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Lecture III.

different Situations and Diſtances, that by contemplating the various Proſpects it gives us, and comparing them together, we may obtain at laſt a diſtinct Knowledge of this immenſe Palace of *God Almighty*, and have an *Idea* or Image of it impreſſed on our Minds, which is worthy of its infinitely wiſe *Architect*.

In order to underſtand therefore the Heavenly Bodies, their Motions, and Appearances, which are called *Phænomena*, we muſt feign ourſelves not to be Inhabitants of this *Earth*, and fixed to one Habitation, but ſuppoſe we have the Power of Travelling every-where, through the immenſe Regions of indefinite Space: And therefore we will ſometimes take Poſſeſſion of ſome immoveable Place; from thence we will transfer ourſelves to the *Sun*, to obſerve the Regularity and Harmony of the Motions which are to be ſeen from thence; afterwards we will take a Journey to ſome other of the *Planets*, that we may, from them, obſerve the apparent Motions of the Heavens; nor will we confine ourſelves within this *Planetary* Syſtem, but we will aſcend much higher in the Heavens, and view the World from a *Comet* or fix'd *Star*.

We may imagine ourſelves to have a free Courſe, or Paſſage, through all the Parts of the Univerſe, and of paſſing from one Star to another.

We, tho' from Heav'n remote, to Heav'n will move
With Strength of Mind, and tread th' Abyſs above:
And penetrate, with an interior Light,
Thoſe upper Depths, which Nature hid from Sight.
Pleas'd we will be to walk along the Sphere
Of ſhining Stars, and travel with the Year.
To leave this heavy Earth, and ſcale the Height
Of Atlas, *who ſupports the Heav'nly Weight:*
To look from upper Light, and thence ſurvey
Miſtaken Mortals wand'ring from the Way.

OVID's *Metamorphoſis*, Book XV.

Lecture III.

Now though our Bodies, by Reason of their Gravity towards the Earth, are detained, as it were, Prisoners in this earthly Mansion; yet nothing hinders, but that, with our Mind and Imagination, we may wander through all the Heavenly Regions, and from them contemplate, with the Eyes of our Reason, the whole System of *Nature*. Nor do I see how this Liberty of Imagination can be denied us, which was always allowed to the *Astronomers* of all Ages; for they, to observe the equal Motions of the Heavens, thrust the Spectator down to the Center of the Earth, and supposed that the Heavens were viewed from thence, as from the Center of a crystal *Globe*. An *Astronomer* thinks it no great Concession or *Postulatum*, that he can draw a Line from the *Sun* to the Center of the Earth; and from thence again to any *Planet* or Star. He divides the Heavens with his Circles, and marks out the Ways of the Planets; and indeed without such a Licence he could never have brought *Astronomy* to any Degree of Perfection.

As therefore it was a Custom among the *Astronomers*, to place the Eye in the Center of the Earth, to view from thence the apparent diurnal Revolution of the Heavens; which would from thence be seen an equable Motion: We will, on the contrary, carry the Spectator to some immoveable Place in the Heavens; that the real and absolute Motions may be observed from thence, as much as they can be, equable and uniform. For the *Astronomers* of all Sects do agree, that the Motions of the Planets are in themselves simple, regular, and uniform: But when the Heavens are viewed from the Surface of the Earth, or even from its Center; the Planets seem to be carried by very unequal Motions, and not to observe any regular Course; and therefore we may certainly conclude, that our Earth is not placed in the Center of their Motions. He therefore that would observe the real and proper Motions of these Celestial Globes, must first place himself

The Planets seen from the Earth, have unequal, and irregular Motions.

himſelf in the Center of the Sun, or in ſome Point or Space not far diſtant from it; and then let him conſider what will be the Appearance or *Phænomena* he will behold from thence.

Lecture III.

The Spectator is always in the Center of his own View.

AND, firſt, it is to be noted, that where ever the Spectator reſides, he will ſtil! be in the Center of his own View; for in an Indefinite Space, where there is nothing to bound our Proſpect, all Objects that are at a great Diſtance from us, though they be at immenſe Diſtance from one another, yet if they appear in the ſame right Line which paſſes through the Eye, will be ſeen at the ſame Point of Space; and all Bodies will appear equally remote, when their Diſtances from us become ſo great, that the Eye cannot eſtimate or judge of them: And conſequently the Spectator will look upon them all as placed in the Surface of a Sphere, which has the Eye for its Center, and whoſe Surface is at an immenſe Diſtance, in which Surface all the heavenly Bodies will ſeem to perform their Motions. Thus, though the *Moon* be many Millions of Miles nearer to us than the *Sun*, and he again much nearer than the fixed *Stars*, yet all appear as placed in the ſame concave Surface of the Heavens: And even the Clouds, which are but a few Miles above us, would be judged to be as far diſtant as the *Moon* and *Sun*, if they did not ſometimes cover them and obſcure their Light. In whatever Place therefore the Spectator reſides, whether it be in the *Earth*, or the *Sun*, or in *Saturn*, the furthermoſt of the *Planets*, or even in a fixed *Star*, that Place will be looked upon by its Inhabitants as the middle Point of the Univerſe, and the Center of the World; ſince it is the Center of that Spherical Surface in which all diſtant Bodies ſeem to be placed.

The Proſpect of the World from the Center of the Sun.

A Spectator therefore living in the *Sun*, when he looks towards the Heavens, will obſerve its Surface to be ſpherical-concave, and concentrical to his Eye; in which Surface he will obſerve an innumerable Multitude of *Stars*, which we call *Fixed*, every-where diſperſed throughout the whole Hea-

Lecture III. vens, which, like ſo many gilded Studs, with a bright Luſtre adorn the Firmament. Theſe *Stars* we call *Fixed*, becauſe, as ſeen from the *Earth*, they preſerve the ſame immutable Poſitions and Diſtances from each other; and ſo from the *Sun* likewiſe, they will appear always to retain the ſame Situations in reſpect of one another, nearly as they are obſerved to have when ſeen from the Earth. For their Diſtance either from the Earth or Sun is ſo great, that the little Change of Place, (however great it be, when compared to our common Meaſures) which is made by bringing a Spectator from the Earth to the Sun, will ſcarcely make any Change in the viſible Situation of the Stars. Now, though the fixed Stars ſeen from the Earth do always preſerve the ſame Diſtances, Poſitions and Situations in reſpect of one another, yet in reſpect of the Eye we obſerve them to change their Poſitions, and ſometimes they ſeem to mount higher in the Heavens, and to come more perpendicularly over us; then they deſcend again, and appear to turn round in Circles, ſome in greater, ſome in leſs, about an Axis which is the Axis of the Earth: And this Circumvolution of theirs is every Night to be obſerved from the Earth; but whoever would view them from the Center of the Sun, would perceive them abſolutely immoveable, and always abiding in the ſame Place of the Firmament. And this Appearance will be the ſame, whether the Stars do really reſt in the ſame Place; or whether the Heavens, in which the Stars are placed together with the Sun, revolved round the Axis of the Earth: For if there were really any ſuch Revolution of the Heavens, a Spectator in the Sun would have that Motion in common with the Stars; and therefore he could be no more made ſenſible of it, than a Paſſenger in a Ship can obſerve by his Senſes the Courſe and Motion of the Ship.

The immenſe Diſtance of the fixed Stars from the Sun.

The Stars change their Poſition in reſpect of the Spectator.

Besides the innumerable Stars at Reſt, there are Six other ſhining Globes to be obſerved, which perform their Circulations round the Sun, in very different

different Periods of Time: And therefore, they must have constantly variable Positions, and be always changing their Distances from one another, as well as from the quiescent Stars. These Globes or Stars are called *Planets*, which signifies *Wanderers*, and one of them is the *Earth*, the Place of our Abode. And even though we should suppose the Earth to be at Rest, and that the Sun did really move round it in the Space of a Year, yet a Spectator in the Sun would observe, that the Earth turned round about him; and would see it describe the same Circle in the Heavens, that we in the Earth observe the Sun to perform his Course in; as we shall afterwards demonstrate.

Lecture III.

The six Planets, or Wanderers.

THE Names and Characters, or Marks for the Planets are *Saturn* ♄, *Jupiter* ♃, *Mars*, ♂, the *Earth* ⊕, *Venus* ♀, *Mercury* ☿. These Characters were invented by the *Astronomers*, as Abbreviations in Writing. The Planets do all turn the same Way as the Sun from the *West* to the *East*, in Orbits which lie in Planes, which are not much inclined to one another, but nearly coinciding: So that the Planes of these Orbits in the Heavens, being little inclined to one another, make Angles with that Circle in which the Earth is seen to turn round the Sun, but of a very few Degrees. As all Planes that are not parallel, cut one another in right Lines; so the Planes of the Orbits in which the Planets move, cut one another in Lines that pass through the Sun's Center; and therefore a Spectator there placed will be in the Plane of each Orbit, and will observe that the Planets moving in the concave Surface of the Heavens, perform their Motions in great Circles, which divide the Heavens into equal Portions. Now the Eye being, in this Situation, in the Planes of all the Planets Orbits, can never by that Means judge of their different Distances from the Sun; for from thence they will all seem to be at the same Distance from him: And therefore to observe their different Distances, as well as Periods, it will be necessary that our Spectator should remove from the

The Planets turn round the Sun *from* West *to* East.

Lecture III. Sun, and riſe above the Planes of all the Orbits, in a Line perpendicular to the Plane of the Earth's Orbit; and, for Example's Sake, let us ſuppoſe him to riſe ſo high, as that his Diſtance from the Sun may equal the Earth's Diſtance from it, and let him there make his *Aſtronomical* Obſervatory: From thence he will not only obſerve the ſame fixed Stars in the ſame Poſition as before, but he will ſee both Sun and Planets in the Heavens: The Sun indeed will appear like the fixed Stars, immoveable; but the Planets will be ſeen to turn round in leſſer Circles about him, at very different Diſtances, and in different Periods: They who finiſh their Circuits ſooneſt, are ſeen neareſt to the Sun, and the Circles they move in are the leaſt; they who take a longer Time in revolving, deſcribe larger Circles, and are further removed from the Sun; and the Order of the Planets will be ſuch as is repreſented in Figure 1, *Plate* II. Where the Sun remains unmoved in the Center of all the Orbits; round about him ſix Planets make their Revolutions, *viz. Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn*, all from the *Weſt to the Eaſt*, according to the Order of the Letters A B C D. *Mercury* is next the *Sun*, and finiſhes his Courſe in three Months: *Venus*, in an Orbit ſomewhat larger, performs her Period in eight Months: Beyond the Orb of *Venus*, is that of the Earth, which revolves round the *Sun* in the Space of a Year: *Mars* takes two Years to complete his Circulation: And *Jupiter*, at a much greater Diſtance, does not finiſh his Revolution till after twelve Years: The furthermoſt and ſloweſt of all is *Saturn*, whoſe Orbit includes all the others, and requires not much leſs than thirty Years to complete his Courſe.

The Order of the Planets.

Plate II. Fig. 1.

The antient Pythagorean *Syſtem.*

THIS was the antient Syſtem of the World, which was at firſt introduced into *Greece* by the great *Pythagoras*, and his Diſciples, who had learned it from the wiſe Men of the *Eaſt*, to whom, as to an Univerſity, they then all reſorted for Inſtruction. 'Tis true, the other apparent Syſtem, which ſuppoſes

poſes the Earth immoveable, and the Heavens to revolve about it, was received among the vulgar and illiterate Part of Mankind; yet the Philoſophers retained the true Syſtem, 'till *Ariſtotle*, and the Philoſophers that came after him, degenerating from their Predeceſſors, and not being acquainted with true Philoſophy, embraced the common Syſtem of the Vulgar: So that the antient Syſtem was forgot, and not minded, 'till the Time of *Nicolaus Copernicus*, who again brought it to Life, and retrieved it from Oblivion, and eſtabliſhed it by ſolid Arguments and Reaſons: Whence this Syſtem is now called the *Copernican Syſtem*. After the Invention of Teleſcopes, the Secundary Planets, with many new and unthought of Appearances, were obſerved in the Heavens by the *Aſtronomers*, which did wonderfully inlarge the antient Syſtem, and confirmed it with invincible Demonſtrations.

The Planets are opake ſpherical Bodies.

IF a Spectator ſhould with a Teleſcope more nearly view the Planets, he will ſoon find, that they are ſpherical Bodies, and opake, like our Earth; having no proper Light of their own, but that they ſhine with the borrowed Light of the Sun; for that Side of them which is turned towards the Sun, is always illuminated: and it is by the reflected Light of the Sun, that they become viſible: But the Side oppoſite to the Sun, which the borrowed Rays cannot reach, remains dark and obſcure. And beſides this, as all opake Bodies do, the Planets caſt a Shadow behind them, which is always oppoſite to the Sun. The Line in the Planet's Body, which diſtinguiſhes the lucid Part from the obſcure, appears ſometimes right, ſometimes crooked: And it is ſometimes convex towards the ſplendid Part, and concave on the obſcure; ſometimes, on the contrary, it appears convex towards the obſcure Side, and concave towards the ſhining Face of the Planet, according to the different Situation of the Eye in reſpect of the Planet, and of the Sun which illuſtrates the Planet; which different Poſition is likewiſe the Cauſe why ſometimes

Lecture III. we see a greater, sometimes a lesser Portion of the illuminated Face; as it ought to be in spherical opake Bodies, which are exposed to the bright Light of the Sun.

The Secundary Planets. THREE of the Planets, *viz.* The *Earth*, *Jupiter*, and *Saturn*, have other lesser Planets, which continually accompany them; these are called Secundary Planets, Moons or Concomitants; for they constantly keep close to their respective Primaries, and always attend upon them in their Circulation round the Sun; and in the mean Time each of them performs his proper Revolution round his proper Primary.

The Earth *is accompanied by the* Moon. The *Earth* indeed has only the *Moon* to keep her Company, who never forsakes her in her annual Course round the *Sun*, and while she attends upon us, she performs proper Circulations of her own round the *Earth*, in the Space of a Month.

THAT the *Moon* appears so large to us, and shines so brightly beyond all the *Stars*, and in Bigness seems to equal the *Sun*, is owing intirely to her Nearness to the *Earth*; for a Spectator in the *Sun* would scarcely be able to observe her without a Telescope; and therefore, if she were as far removed from us as the *Sun*, she would be so small, as scarcely to be visible by an Eye that is not assisted by a Glass.

Jupiter's *four* Moons. JUPITER has four *Moons* that attend him, which at different Distances, and with different Periods, perform constant Circulations round him; that which is next to him, is no further removed than $2\frac{5}{6}$ of his own Diameters, and turns round in one Day, eighteen Hours and an Half. The second, at the Distance of $4\frac{1}{2}$ Diameters, describes its Orbit in the Space of three Days, and thirteen Hours. The third is removed from *Jupiter*, seven of his Diameters, and finishes his Circulation in seven Days, four Hours. The furthermost completes his Period in the Space of 16 Days, $16\frac{1}{2}$ Hours, at the Distance of 12 Diameters of *Jupiter*.

THESE

THESE Jovial Planets were first observed by that noble *Italian* Philosopher *Galileus*, by the Help of the Telescope which he first invented; and by them he increased the Number of the Celestial Bodies, and called them *Medicean* Stars, in Honour of the Dukes of *Tuscany*, with whose Name he dignified them. By the Benefit of these new-discovered Worlds, *Astronomy* and *Geography* have received many particular Advantages. Lecture III.

SATURN performs his Course round the Sun with no less than five Attendants, though most of them, by reason of their great Distance from the Sun, and the Smallness of their own Bodies, are not to be seen but by the Help of very long Telescopes: The acute Eyes of Mr. *Cassini*, the *French* King's *Astronomer*, were the first that reached all that have been already discovered; and but of late they have been seen in *Britain*, by Means only of that Telescope which was given to the ROYAL SOCIETY by the illustrious Mr. *Hugens*. The Distances of these Planets from *Saturn*, and their Periodical Times, are as followeth: The nearest completes his Revolution in one Day and $\frac{7}{8}$, and is distant from *Saturn*'s Center $4\frac{3}{8}$ of his Semidiameters. The second revolves about *Saturn* in 2 Days 17 Hours, and the Semidiameter of his Orbit is $5\frac{3}{8}$ of the Semidiameters of *Saturn*. The third finishes his Revolution in 4 Days and 12 Hours, at the Distance of eight Semidiameters. The fourth completes his Period in 16 Days, and is distant from *Saturn* 18 of his Semidiameters. The fifth and outermost takes $79\frac{1}{3}$ Days to finish his Course, and is 54 Semidiameters of *Saturn* distant from him. Saturn *has five* Moons.

BESIDES these Attendants, *Saturn* has an Ornament peculiar to himself; for he is dignified with a *Ring* which surrounds his Middle, and does nowhere touch his Body; but by an exact Libration and Equiponderancy of all its Parts, sustains itself like an Arch; and being thus suspended by *Geometry*, it is kept from falling upon his Body. The Diameter of this Ring is more than double of the Diameter Saturn's *Ring*.

Lecture IV. meter of *Saturn*; and though the Thicknefs of this Ring on the convex or concave Side be but fmall, yet its Breadth or Depth is fo great, that it takes up the Half of that Space which is between its outward Surface and the Body of *Saturn*, the reft of Space remaining void: So that in proper Situations we can fee the Heavens between the Ring and the Body. For what Purpofe this admirable Ring was made we know not; and, perhaps, we never may come to the Knowledge of it, fince we find nothing in Nature like it; but yet we cannot but admire the Infinite Majefty and Power of GOD, who, in this our Age, has difcovered and fhewed us new and unthought of Inftances of his Greatnefs.

LECTURE IV.

In which is proved that the Syftem Explained in the former Lecture, *is the true Syftem of the World.*

IT may perhaps be objected againft the Syftem of the World delivered in our laft Lecture, that we feigned and imagined our Spectator carried up into Heaven, and from thence to have feen with his Eyes the Motions, Situations, and Order of the Planets which we there explained. But this was only done in Imagination, and therefore being nothing but a Fancy or Fiction, the Syftem we have given upon that Suppofition, will be likewife only a Fiction or Hypothefis, and may not anfwer to the Reality of Things. Can there not, by the fame Liberty of Fancy, any other Order of the Planets be fuppofed, and another Syftem be given quite different from

from ours? Cannot we, relying upon our Senses, place the Earth in an immoveable Position, and suppose the Sun and Planets, and all the Stars, to move round it, as our Eyes testify to us that they do? And from such Positions cannot we explain all the Appearances of their Motions?

I answer, that although we fansied our Spectator raised up to the Heavens, and from thence to have looked upon the Sun and Planets, yet the Order, Motions, and Positions of the Planets, which would be seen upon that Supposition, and which we explained in the preceding Lecture, is no Fancy, or Fiction of the Imagination, but is as real, certain, and indubitable, as if a Spectator were there, and saw it with his Eyes. A true *Astronomer* feigns nothing without solid and sufficient Reasons; he takes Nature for his Guide and Rule, and lays his Foundations on Observations: He raises his System upon Physical Causes, and invincible *Geometrical* Demonstrations, with which, as with an indissolvible Cement, he joins and binds the whole Fabrick together. The *Hypotheses* of *Ptolemy* and *Tycho* may truly be called Fictions; for they have nothing in them but a bare Supposition, on which, without any Reason, they depend; and they distort and disorder the whole Frame of Nature. But the true *Astronomy* is the most antient of all; for it was preserved in the School of the *Pythagoreans*, to whom it was delivered by the first *Astronomers*, either *Egyptians* or *Chaldeans*: It has all its Parts fitly joined together in a most agreeable Harmony and Order; it leads us to the Knowledge of the Universe, and the wonderful Symmetry, Beauty, and regular Disposition of all the Bodies that compose it. There is nothing in Nature that does more shew the piercing Force of human Understanding, the Sublimity of its Speculations, and deep Researches, than true *Astronomy*. It raises our Minds above our Senses; and even in Contradiction to them, shews us the true System of the World: The Faculty of Reason by which

True Astronomy admits of no Fictions, or Hypotheses.

Lecture IV.

which we have made these great Discoveries in the Heavens, must needs be derived from Heaven, since no earthly Principle can attain so great a Perfection. And since the Origination of our Minds is from Heaven, it may be expected, that they will endeavour to return thither, and Heaven will become our final Habitation: We will here declare in few Words, some of the Ways by which the Mind arrived to the Knowledge of these heavenly Discoveries.

A Demonstration that the Planets turn round the Sun.

FIRST, it is certain that where-ever the Sun be placed, the Orbit of *Venus* does surround him, and includes him within itself; and therefore *Venus*, while she describes this Orbit, does really turn round the Sun; for *Venus* has been observed to be above or beyond the Sun, sometimes it has been seen below it, or between the Sun and us. That *Venus* ascends above the Sun, is plain from hence, that when she is in Conjunction with the Sun, that is, when she is seen from the Earth, near the same Part of the Heavens that the Sun is in, her lucid Face appears in a full and round Figure: For since all the Planets borrow all the Light with which they shine, from the Sun, it is necessary that that Face of hers should be lucid, which is towards the Sun, and that which is turned from him, be involved in Darkness; and therefore when she shines with a full and round Face, that Side of her which is towards the Sun, is also towards the Earth; and therefore, at that Time, she must be above the Sun; for in no other Position, could her illuminated Face be towards the Earth, when she is seen in Conjunction with the Sun. In the Figure, let S represent the Sun, T the Earth, and let *Venus* be in F or V, where she can be seen from the Earth, in the same Part of the Heaven that the Sun is; and she will appear to have a full and round shining Face; because that Side of her which is illuminated, and is towards the Sun, is likewise turned towards the Earth; and therefore the Place of *Venus*, in that Case, must necessarily be above the Sun. That *Venus* is also sometimes below

Plate II. Fig. 2.

low the Sun, or between the Sun and us, is evident from hence; that ſometimes, when ſhe is in Conjunction with the Sun, ſhe either quite diſappears, or if ſhe is viſible, ſhe appears horned, and takes exactly the Shape of a new Moon: And therefore that Face of hers which is towards the Sun, either is wholly turned from the Earth as in G; or only a very ſmall Part of the illuminated Face towards the Earth; and can be ſeen by its Inhabitants, as in H, in which Caſe ſhe aſſumes a horned Figure, as the Moon does; and, therefore at that Time, ſhe muſt of Neceſſity be placed between the Sun and us, and come lower or nearer to us, than the Sun is. Once *Venus* was ſeen within the Body or Diſk of the Sun; but there was but one Man who had the Happineſs to be Witneſs of the Sight, our Countryman Mr. *Horrox*, who in the Year of Chriſt 1639, obſerved it with his Teleſcope to enter upon the Body of the Sun like a black Spot. This is a Sight which can ſeldom be obſerved; for it will not be ſeen again in the Sun's Body, 'till the Year 1761, upon the 26th Day of the Month of *May* in the Morning; at which Time all our *Aſtronomers* will, no doubt, be buſy in making their Obſervations; for by them our Diſtance from the Sun can be nearly determined, which before that Time is not eaſily to be aſcertained. Beſides this, *Venus* is always obſerved to keep near the Sun, and in the ſame Quarter of the Heavens that he is; for ſhe never recedes from him beyond a certain Diſtance of about 45 Degrees; ſo that ſhe never comes in Oppoſition to the Sun, or to be ſeen in the *Eaſt* when he is in the *Weſt*; nay, ſhe never attains or arrives at a Quartile Aſpect with him, or to have a fourth Part of the Heaven between her and him, which would neceſſarily happen, did ſhe peform her Period round the Earth either in a longer, or ſhorter Time, than the Sun does.

Lecture IV.

The Appearances of Mercury *are like thoſe of* Venus.

After the ſame Manner *Mercury* always keeps himſelf in the Neighbourhood of the Sun, and never recedes from him ſo far as *Venus* does: He

Lecture IV.

hides himself so much in the Splendor of the Sun's Rays, that he is but seldom seen by us on the Earth; but since the Invention of Telescopes, he has been frequently observed, when in Conjunction with the Sun, to pass under his Disk like a black Spot, as *Venus* was seen by Mr. *Horrox*: The exceeding Brightness by which *Mercury* outshines all the Planets, does evidently prove him to be much nearer the Sun than any of the rest; for the nearer any Body is to the Sun, the greater is the Illustration it receiveth from him. From all this it is evident, that *Mercury* does likewise go round the Sun in a lesser Orbit, included within the Orbit of *Venus*; which therefore must necessarily be his Place, for no other can be assigned him.

The Orbit of Mars *includes the* Sun *within it, and likewise the* Earth.

MARS is not like *Mercury* and *Venus*; for he often comes in Opposition to the Sun, and appears to rise in the *East*, when the Sun sets in the *West*; and therefore his Orbit includes the Earth within it, and not only the Earth, but it necessarily includes the Sun likewise; for *Mars*, when he is seen near the Conjunction with the Sun, if he were between the Sun and Earth, would either quite disappear, or appear horned in the same Shape that *Venus* and the Moon have in that Position; but he always preserves a full, round, and shining Face, except near his *Quadrat Aspect*, that is, when there is about a fourth Part of the Heavens between the Sun and him; then he is observed to be somewhat gibbous, like the Moon, three or four Days before or after the *Full*,

Plate II. Fig. 3.

LET S represent the Sun, T the Earth, and the Circle MNPR the Orbit of *Mars*; it is plain that *Mars*, in both M and P, must shine with a full Face upon the Inhabitants of the Earth, because that in both these Positions his Face, which is towards the Sun, and by it illuminated, is likewise towards the Earth: But in N and R, he will appear a little gibbous or deficient from Full. Besides, *Mars*, when he is seen in Opposition to the Sun, looks almost seven times larger, in Diameter, than

when

when he is near to a Conjunction with him; and therefore he must needs be seven times nearer to the Earth in the one Position than in the other. From hence it is plain, that though the Earth lies within the Orbit of *Mars*, yet it is not near to the Center of his Orbit: But *Mars* always keeps nearly at the same Distance from the Sun, and therefore it is evident, that it is not the Earth, but the Sun, which *Mars* respects as the Center of his Motions: For *Mars* seen from the Earth appears to move very unequally, sometimes to go faster, sometimes slower; sometimes he scarcely seems to move at all, and sometimes he even goes with a backward Motion; whereas a Spectator in the Sun would always see *Mars* go forward in the same uniform Tenor: And therefore it is most evident that the Sun is the Center, and not the Earth, of *Mars*'s Motions. Again, since the same Appearances are observed in *Jupiter* and *Saturn*, as in *Mars*, (though the Disproportion or Difference of the Distances is not so great in *Jupiter*, as in *Mars*; nor so great in *Saturn*, as it is in *Jupiter*) and the Motions of these two Planets are no Ways uniform round the Earth; yet from the Sun their Motions will be seen to be regular and orderly: It is plain from hence, that the Sun and not the Earth, is in the Center of all the Orbits of the Planets. The Place of the Earth we have demonstrated to be without the Orbits of *Mercury* and *Venus*, and within the Orbit of *Mars*; and therefore its Place must needs be between the Orbits of *Venus* and *Mars*: And from thence it follows, that the Earth itself must turn round the Sun; for if it stood still, since it lies within the Orbits of the superior Planets *Mars*, *Jupiter*, and *Saturn*, we might indeed observe the Motions of those Planets from the Earth, to be very unequal and irregular; but they would never appear to stand still, or to go backward, so long as the Orbits themselves are quiescent, as we demonstrated in our first Lecture. Since therefore the Stations and Retrogradations of these Planets are observed from the Earth; and since

Lecture IV.

The Earth *is not in the Center of* Mars's *Orbit.*

The same Appearances of Jupiter *and* Saturn.

The Earth *moves in an Orbit round the* Sun.

Lecture IV.

since the Position of the Earth, or Place that it obtains in the System, is in the Middle of the moveable Bodies, having *Mercury* and *Venus* on one Side nearer to the Sun, and *Mars*, *Jupiter*, and *Saturn*, on the other Side more remote; it being of the same Nature as they are, must likewise have the same Sort of Motions; and as the Earth is in the middle Place between *Venus* and *Mars*, so its Period likewise, in which it performs its Course round the Sun, is also a Mean between the Periods of *Venus* and *Mars*, being greater than the one, and less than the other: For *Venus* describes her Orbit in eight Months; the Earth in a Year; but *Mars* takes near two Years to finish his Course.

A Demonstration of the Earth's Motion.

THERE is another Demonstration of the Earth's Motion drawn from Physical Causes, for which we are indebted to the admirable Discoveries of the incomparable Sir ISAAC NEWTON: He has demonstrated, that all the Planets gravitate towards the Sun: And Observations testify to us, that either the Earth turns round the Sun, or the Sun round the Earth, in such a Manner as that they describe equal *Area's* in equal Times. But Sir ISAAC has demonstrated, that whenever Bodies turn round each other, and regulate their Motions by such Law, the one must of Necessity gravitate to the other; and therefore, if the Sun in its Motion does gravitate to the Earth, Action, and Re-action being equal and contrary, the Earth must likewise gravitate to the Sun, He has likewise demonstrated, that when two Bodies gravitate to one another, without directly approaching one another in right Lines, they must both of them turn round their common Center of Gravity. The Sun and Earth therefore do both turn round their common Center of Gravity. But the Sun is so great a Body in respect of the Earth, which is but a Point, as it were, in comparison of the Sun, that the Common Center of Gravity, of the Earth and Sun, must lie within

the

the Body of the *Sun* itſelf, and not far from the Center of the *Sun :* The *Earth* therefore turns round a Point which is within the Body of the *Sun*, and therefore turns round the *Sun*. This Argument, drawn from Phyſical Cauſes, I take to be unanſwerable.

A wonderful Harmony obſerved between the Periods of the Planets, *and their Diſtances from the* Sun.

COMPARING the Periods of the *Planets*, or the Times they take to finiſh their Circulations, with their Diſtances from the *Sun*, we find they obſerve a wonderful Harmony and Proportion to one another; for the nearer that any *Planet* is to the *Sun*, the ſooner does he finiſh his Circulation, and his Motion is the quicker: And in this there is a conſtant and immutable Law, which all the Bodies of the Univerſe inviolably obſerve in their Circulations; *viz. That the Squares of their Periodical Times are as the Cubes of their Diſtances from the Center of their Orbits, about which they perform their Motions regularly.* The moſt ſagacious *Kepler* was the firſt who diſcovered this great Law of Nature in all the Primary *Planets*; afterwards the *Aſtronomers* obſerved, that the Secondary *Planets* did likewiſe regulate their Motions by the ſame Law; and that in the two Syſtems of Bodies revolving about *Jupiter* and *Saturn*, this Rule is conſtantly obſerved, that the Squares of their Periodical Times are as the Cubes of their Diſtances from their reſpective Primaries. Thus the *Moon* or *Satellite* that is next to *Jupiter*, is diſtant from *Jupiter*'s Center $2\frac{5}{6}$ of *Jupiter's* Diameters, and he performs his Period in 42 Hours. The outermoſt *Satellite* deſcribes his Orbit in 402 Hours. Say therefore, as 1764 the Square of 42, is to 161604, the Square of 402; ſo is $\frac{4913}{216}$ the Cube of $2\frac{5}{6}$, to a fourth proportional Number, which, by the *Golden Rule*, will be found nearly $\frac{450000}{216}$; out of which Number extract the Cube Root, and we have $\frac{76}{6}$ or $12\frac{2}{3}$ for the Diſtance of the furthermoſt *Satellite* from *Jupiter*. Now the Obſervations of all *Aſtronomers* confirm, that this is the true Diſtance of that *Satellite* from the Center of *Jupiter*, and the ſame Thing is to be obſerved

Lecture IV.

in all the rest, as likewise in the *Satellites* of *Saturn*.

THE Reason of this Law was unknown to *Kepler*, for he found it out only by Computation, comparing the Distances of the Planets with their Periods. But the Glory of investigating from its proper Cause, and demonstrating the Physical Necessity of this Law, was reserved for the Great Sir ISAAC NEWTON, who has demonstrated, that without a total Subversion of the Laws of Nature, no other Rule could take place in the Circulations of the Heavenly Bodies

Sir Isaac Newton *first discovered the Reason and Cause of this Harmony.*

SINCE therefore all *Astronomers* do unanimously agree, that the Law we have above explained, is constantly observed by 14 great Bodies, of which there are more than one that turn round a common Center, *viz.* five Primary *Planets* and nine Secondaries: And since the *Moon* turns round the *Earth*, if the *Sun* did likewise perform his Circuits round it, according to this Law of Nature, the *Moon* and *Sun* ought to regulate their Motions in the same Manner; and therefore since the *Moon* finishes her Course in 27 Days, and the *Sun* in 365, and the Distance of the *Moon* is known to be about 60 Semidiameters of the *Earth*; if we say, as 729 the Square of 27 is to 133225 the Square of 365, so is 216000 the Cube of 60 to another, which will be 39473251, the Cube Root thereof being 340, ought to express the Distance of the *Sun* from the *Earth*, provided he governed his Motion by the same Law that all other Bodies do. Now the *Astronomers* prove, by invincible Reasons, that the *Sun* is more than 30 times further from the *Earth* than 340 Semidiameters of the *Earth*; for it cannot well be supposed so little as 10000 Semidiameters: According to which Distance it could not turn round the *Earth* in less than 164 Years, if it observed the same Law which all other Bodies do.

NOW

Now it is certain that the *Sun* does either turn round the *Earth* or the *Earth* round the *Sun*, once in a Year. But if the *Sun* should be made to turn round the *Earth*, the universal Law of Nature would thereby be violated, the Harmony and Proportion of the Motions destroyed, and a Confusion and Disorder introduced into the Frame of the Universe. But if the Earth be made to go round the *Sun* in the Space of a Year, it will then perform its Circulation according to the same Law which the other *Planets* observe; and without the least Exception there will be a most beautiful Order and Harmony of Motions every-where preserved through the whole Frame of Nature.

Lecture IV. *If the* Sun *turned round the* Earth, *this Harmonical Rule would not universally hold.*

As we discover the mutual Relations and Likeness of Nature that is in the *Planets*, in that they are like our *Earth*, Opake, Spherical Bodies, which are illustrated and shine with the borrowed Light of the *Sun*, round whom they all circulate, as we have said, with a regular Harmony and Order: So likewise the *Sun*, and all the *fixed Stars* which shine with their own native Light, and remain immoveable in their Places, are to be considered as Bodies of the same Kind and Nature. The Reason why our *Sun* appears to us so great and bright in comparison of the *Stars*, whose weaker Light disappear as soon as the *Sun* begins with his Beams to refresh and illustrate our Habitation, is, that the *Earth*, at an immense Distance from all the rest of the *Stars*, keeps near to the *Sun*, round whom she always circulates; for a Spectator placed as near any of them as we are to our *Sun*, would see a Body as big and bright as the *Sun* appears to us, and every Way like our *Sun*. A Spectator as far distant from our *Sun* as the *fixed Stars* are from us, would observe our *Sun* as small as a *Star*, and, no doubt, would reckon the *Sun* as one of them in numbering the *Stars*. All the *fixed Stars* therefore are *Suns*, and the *Sun* differs in nothing from a *fixed Star*.

The Sun *and* fixed Stars *are Bodies of the same Nature.*

Lecture IV.

The Distance of the fixed Stars *from the* Sun *is immensely greater than that of the* Earth.

ALTHOUGH the *Earth* is at ſuch a Diſtance from the *Sun*, that if it were to be ſeen from his Body, it would appear no bigger than a Point; yet that Diſtance is ſo very ſmall in Compariſon of the exceeding great Diſtance of the neareſt *fixed Star*, that if the whole Orbit, in which we have ſaid the *Earth* moves round the *Sun*, were ſeen from a *Star*, it would appear likewiſe no bigger than a Point. For the Angle under which the whole Diameter of the *Earth*'s Orbit appears, is ſo very ſmall, that our quickeſt and moſt ſharp-ſighted *Aſtronomers* can ſcarcely obſerve it. They who have been moſt diligent in obſerving this Angle, (which they call the Parallax of the great Orbit, that is, of the Orbit in which the *Earth* moves) have always found it to be leſs than a Minute; and therefore the *fixed Stars* muſt be at leaſt 10000 times further from us, than we are from the *Sun*.

HENCE it follows, that though the *Earth* approaches nearer to ſome *fixed Stars* at one Time of the Year, than it does at the oppoſite Time, and that by the whole Interval of the Diameter of its Orbit, the *Stars* will not upon that Account ſeem bigger when it is neareſt to them, nor will the viſible Poſition of any two *Stars* be ſenſibly changed by the Motion of the *Earth*. For even here on *Earth*, if there be two Towers that are near to one another, if they are to be ſeen by a Spectator who is ten Miles diſtant from each, if this Spectator approach nearer to them only by one Pace, which is the ten thouſandth Part of the Diſtance, he only approaching by ſo ſmall a Space, can no Ways perceive the Towers bigger, or their Diſtance from one another greater. After the ſame Manner the *Earth* approaching a *fixed Star*, and coming nearer to it by the ten thouſandth Part of the Diſtance between the *Earth* and it, a Spectator on the *Earth*, upon the Account of ſo ſmall a Change of Place will not find thereby any ſenſible Difference either of the Magnitude or Poſition of the *Star*.

Lecture IV.

Hence it follows, that if the *Sun* were as far diſtant from us as the *Stars* are, that is 10000 Semediameters of the *Earth's* Orbit, it would appear 10000 times leſs, or under a ſmaller Angle than it does now. Now the Angle under which the *Sun* appears to us, is about half a Degree or 30 Minutes; if the *Sun* were removed as far from us as the *fixed Stars* are, he would be ſeen under an Angle of but the thouſandth Part of the three Minutes; that is, under an Angle no bigger than ten *Thirds*, which is altogether imperceptible; and no bigger would the Angle be under which a Spectator placed among the *fixed Stars* would obſerve the *Sun*.

The Angle under which the Sun *would appear ſeen from a* fixed Star.

Against this Poſition of ours ſome may object, that if the Diſtance of the *Stars* be ſo great, they themſelves muſt be vaſtly larger than our *Sun*; for they cannot, according to them, be leſs than a Sphere, whoſe Semidiameter equals the Diſtance between the *Sun* and us; for they aſſert, that the *Stars*, at leaſt thoſe of the firſt Magnitude, are ſeen under an Angle at leaſt of one Minute: But the Orbit of the *Earth* ſeen from the *fixed Stars* does not ſubtend a greater Angle; and therefore the Diameter of the *Stars* is not leſs than the Diameter of the *Earth's* Orbit. Now that Sphere whoſe Semidiameter equals the Diſtance between the *Sun* and *Earth*, is a Million of times greater than the *Sun*; conſequently the *fixed Stars* muſt be at leaſt a Million of times greater than our *Sun*: Since therefore there is ſuch enormous Difference in their Bigneſs, it cannot be ſuppoſed that the *Sun* and *fixed Stars* are Bodies of the ſame Kind.

Objection

But they who aſcribe ſuch immenſe Magnitude to the *fixed Stars*, are much deceived in their Meaſures; for their apparent Diameters are not near ſo great as they ſuppoſe them to be: For really theſe Diameters are ſo extremely ſmall, that if they be rightly obſerved, they appear like ſo many ſhining Points without any Breadth or Diameter at all; and there can be no Obſervations nice enough, by which

The fixed Stars *have no apparent Diameters, but look like Points;*

Lecture IV. which the Minuteneſs of their Diameters can be meaſured, or reduced to any determined Quantity. We obſerve about all-flaming and ſhining Bodies in the Night, a kind of Irradiation or luminous Appearance, by which their Diameters are ſeen a hundred or many more times bigger than they are; as it is plain by the Experiment of a Candle placed at a great Diſtance, whoſe Flame ſeems to be much larger than it is when we come near it. This Irradiation is much diminiſhed, if they are looked at through a ſmall Hole made with a Pin in Paper; but is eaſier and more nicely taken away, when they are ſeen through a Teleſcope, which deſtroys the adventitious Rays, and ſhews us the *fixed Stars* as ſo many lucid Points. Now though Teleſcopes do increaſe the Diameters of Bodies very much, yet the *fixed Stars* ſeen through them appear ſtill ſo ſmall that we have no Meaſure of any determined Magnitude, but what is greater than theirs ſeem to be; a Teleſcope magnifying a hundred times, ſhews them only like ſhining Points; and therefore I cannot but wonder why *Ricciolus* ſhould ſuppoſe the apparent Diameter of the *Dog-Star* or *Sirius* to be 18 Seconds; for if that were the Angle he ſubtended to the naked Eye, a Teleſcope which magnifies 200 times, would ſhew that *Star* under an Angle of 3600 Seconds or one Degree, ſo that he would appear to have a Diſk four times greater than the *Sun* and *Moon* have. And yet Obſervation teſtifies, that ſuch a Teleſcope ſhews us *Sirius* no bigger than a Point, at leaſt not larger than the Planet *Mars*: But *Mars* when he is neareſt us and biggeſt, ſubtends an Angle of about 30 Seconds; therefore, if the Diameter of *Sirius* magnified 200 times is but 30 Seconds, its apparent Diameter without magnifying will be the 200th Part of 30 Seconds, or $\frac{3}{20}$ of a Second, that is nine third Scruples or Minutes, or nearly equal to the *Sun* when it is ſeen at the Diſtance of the *fixed Stars*, as we ſhewed before: The *Sun* therefore and *Sirius* are nearly of the

which is demonſtrated by the Teleſcope.

the ſame Magnitude and Dimenſions, and may be eſteemed Bodies of the ſame Kind.

Lecture IV.

SINCE the *fixed Stars* appear ſo ſmall, and ſubtend at the Eye ſuch unperceivable Angles, ſome will wonder how they come to be at all ſeen, ſince there are Bodies which would ſhew themſelves under larger Angles, that are yet ſo ſmall as not to be looked at without a Microſcope: But all flaming and fiery Bodies can be ſeen at great Diſtances, even at ſuch from whence other Bodies, which have as large apparent Diameters, do quite diſappear, and become inviſible. Thus the Flame of a Candle in the Night-time is eaſily perceived at the Diſtance of two Miles, whereas in the Day-time an opake Object, though ſtrongly illuſtrated by the *Sun*, and ſix times bigger than the Flame of a Candle, is not to be obſerved with the naked Eye at that Diſtance. For the native Light that theſe fiery and flaming Bodies ſend forth, is ſtronger and more piercing, and acts upon the nervous Fibres of the *Retina* with a greater Force, than the Light reflected from opake Objects; for all Light is much weakened by Reflection. And it is upon the Account of this brick and ſtrong Light which flows from fiery Bodies, and which makes ſo ſenſible Impreſſions on the *Ratina*, that ſuch Bodies are judged to be ſo big in Compariſon of others which affect us with a weaker Light.

The fixed Stars *are luminous fiery Bodies.*

HENCE it is evident, that all theſe *Stars* which remain immoveable in the Heavens, are Bodies of a fiery Nature, as our *Sun* is, nor are they much leſs, nor much bigger than he is, and therefore are to be eſteemed as ſo many *Suns*. It is not to be imagined, that all theſe *Suns* are planted in one concave Surface of a Sphere, ſo as to be all equally diſtant from us; but it is more reaſonable to ſuppoſe that they are ſpread every where through the vaſt indefinite Space of the Univerſe, and that they are at great Diſtances from one another; ſo that there may be as great Diſtance between any two *Suns* that

The fixed Stars *are* Suns.

Lecture IV. that are next to one another, as there is between our *Sun* and the nearest *fixed Star*. Hence a Spectator who is near any one *Sun*, will only look upon him to whom he is nearest as a real *Sun*, and the rest he will consider as so many small shining *Stars* fixed in his Heaven or Firmament.

It is no Ways probable, that God Almighty, who always acts with infinite Wisdom, and does nothing in vain, should create so many *Suns*, and place them alone in indefinite Space, at such great Distances from each other, and not have made other Bodies, which he has placed near them, to be nourished, animated, and refreshed with the Heat and Light of these *Suns*: Those who affirm, that God created these great Bodies only to give us a small dim Light, must have a very mean Opinion of the Divine Wisdom. It is more reasonable to suppose, that every *Sun* is surrounded with a Company of *Planets* peculiar to himself, which in different Periods, and at different Distances, perform their Circulations round their proper *Sun*; and who knows but that some of these *Planets* may have Moons, and other Bodies, to attend them in their Circulations?

An Idea *of the Universe.*

Hence we may frame to ourselves an admirable magnificent *Idea* or Notion of the Vastness or Amplitude of the World, by imagining an indefinitely great Space of the Universe, in which there are placed innumerable *Suns*, which, though they appear to us like so many small *Stars*, yet are Bodies which are not behind our *Sun* either in Bigness, Light, or Glory; and each of them constantly attended with a Number of *Planets*, which dance round him, and constitute so many particular *Worlds* or Systems: Every *Sun* doing the same Office to his proper *Planets* in illustrating, warming, and cherishing them, that our *Sun* performs in the System to which we belong.

Hence we are to consider the whole Universe as a glorious Palace for an infinitely Great and every where

where present God; and that all the *Worlds* or System of *Worlds*, are as so many Theatres, in which he displays his divine Power, Wisdom and Goodness.

Lecture V.

LECTURE V.

Of the Solar Spots. Of the Rotation of the Sun *and* Planets *round their* Axes: *And of the* fixed Stars.

UPON the Account of the great Distance of the *Sun* from us, the Convexity of his Body cannot be perceived by our Sight; nor is this a Wonder, since the *Moon*, which is much nearer to us, cannot be seen as a globular Body: But both *Sun* and *Moon* shew themselves to us, as if they were circular Planes, which we call Disks: And a Point in the Middle, which is really in the Surface, is said to be the Center, or the Apparent Center of the Disk.

The Convexity of the Sun *and* Moon *cannot be perceived by our Eyes.*

If the *Sun* through all its Parts were every-where equally bright, and shined with the same Lustre, he might turn round his own *Axis*, and that Rotation no Ways be perceptible, not to be discovered by our Senses. But since in the most clear and lucid Body of the *Sun*, and in its purest Flame, many black Spots have several times been observed, it is by their Motion that we discover the Rotation of the *Sun* round its *Axis*: For these Spots have been observed to have first appeared near the Margin of the *Sun*, and then by Degrees to have crept towards the Middle, or Center of his Disk; and from thence going still forward, have arrived at the opposite Side or Edge of the Disk, where they have set or disappeared;

Dark Spots on the Sun's *Body.*

Lecture V.

ed; and some of them after setting, being hid or absconded in the opposite Side of the *Sun* for the Space of about fourteen Days, have again appeared in the Margin, and shewed themselves, taking the same Course as before. Let the Circle A G H D represent the *Sun*'s Disk; we often observe some dense and obscure Substances, like our thick and terrestrial Clouds, to appear in the Limb or Edge at A, which by Degrees move towards B, and at last arrive at the Middle of the Disk; after which, still going forward, they shew themselves in the opposite Point of the Circumference at D; where after a little Stay, they at last vanish and disappear.

Plate II. Fig. 4.

Some Spots rise and set, and in 27 Days are seen again where they first appeared.

Some of these Spots have been observed to rise in the Limb, and having traversed the Disk, and set, after the Space of twenty-seven Days, have again been seen to rise where they first appeared; and the Time they take to move through the Surface of the *Sun* opposite to us, during which Time they are invisible, is just and equal to the Time in which they pass over the visible Disk. The Motion of these Spots in the Margin, at A or at D, appears to be slow; when they come towards the Middle they seem to move quicker, and to describe larger Spaces: Their Figures likewise change; for in the Limb they are seen more contracted and narrow, near the Center they look broader and larger; these Appearances exactly answer to the Motion of some dense and dark Bodies, which swim upon the Surface of the *Sun*, and are whirled by the vertiginous Motion of the *Sun* round his *Axis*.

The Sun *turns round his* Axis.

Some have imagined, that these Spots do not stick upon the Body of the *Sun*, but that they are at some Distance from him, and that they perform their Circulations round the *Sun* in the same Manner as *Jupiter*'s *Satellites* circulate round *Jupiter*. But such are easily refuted; for if the Spots were not on the Body of the *Sun*, but made their Circuits at a Distance from him, they could not be seen half the Time of their Period in the Body of the *Sun*; for

for ſuppoſe the *Sun* in A be ſeen from the *Earth* at B, under the Angle of 30 Minutes: If the Spot were not on the *Sun's* Body, but deſcribed the Circle F E G at ſome Diſtance from him; it could not be obſerved to paſs under the *Sun*, 'till it came to E, where the right Line B D, touching the Diſk, does cut the Orbit; and if we draw another Tangent to the Diſk B G C, it would only be ſeen on the Surface of the *Sun*, while it deſcribed the Arch E G, which is much leſs than half the Periphery, and is deſcribed in much leſs Time than half the Period, in which the Spot performs its Circuit. Now we know by Obſervation, that thoſe Spots which finiſh an entire Circulation, (for ſome of them have been obſerved to make four or five complete Revolutions before they were diſſolved, each of which was 27 Days) have taken $13\frac{1}{2}$ Days from the Time of their appearing, or riſing on the *Eaſtern* Limb, to their Setting on the *Weſtern*: And therefore, ſince the Spots are ſeen for half the Time of their Period upon the Body of the *Sun*, it is plain their Orbits muſt lie on the *Sun's* Surface.

Plate II. Fig. 5.

The Spots are on the Surface of the Sun.

MANY Spots ſeem to be generated in the very Middle of the Diſk, where they firſt begin to appear: Others again ſeem to be diſſolved, and to vaniſh there; ſometimes ſeveral ſmall ones gather together and make a large Spot; and ſometimes a large Spot is ſeen to be divided, and cut into many leſſer ones. The great *Italian* Philoſopher *Galileus* firſt diſcovered them with his Teleſcope. Afterwards *Scheinerus* did more accurately obſerve them, who has publiſhed a large Volume about them. When he obſerved them, there were then 50 viſible in the *Sun*: From the Year 1650 to the Year 1670, there were rarely ſeen above one or two together; from that Time again many have been obſerved together; nor does their ſeem to be any certain Period of Time, or Law for their appearing, and Vaniſhing or Diſſolution.

Spots generated in the Body of the Sun, *and again are there diſſolved or diſſipated.*

HISTORIANS tell us, that the *Sun* has been obſerved for a whole Year to have appeared pale, without that Luſtre and Brightneſs it uſually gives; and

The Sun *has ſometimes appeared pale for a Year together.*

Lecture V.

and that it hath not imparted its Heat and Light with the ſame Force and Vigour that it ordinarily does. It is probable, that this Weakneſs of Light did ariſe from a great Multitude of Spots which at that Time did beſet the *Sun*, and covered a large Portion of his Surface: For now we frequently obſerve Spots which are larger or broader, not only than *Europe* or *Africk*, but what even equal, if they do not exceed, the Surface of the whole *Terraqueous* Globe.

THE Motion of the Spots is from the *Weſt* to the *Eaſt*, and by obſerving the *Sun* nicely, we find that its *Axis* is not perpendicular to the Plane of the *Earth*'s Orbit, which is called the Plane of the *Eliptick*: But it is inclined, ſo as to make an Angle with the *Axis* of the Orbit, or with a Perpendicular to the Plane of the Orbit which paſſes through the Center of the *Sun*, which Angle is in Quantity about ſeven Degrees: And therefore the Æquator of the *Sun*, that is to ſay, the Circle which is in the Middle between the two Poles, cuts the Plane of the Orbit in a right Line, which Line produced will interſect the Orbit in two Points; and whenever the *Earth* comes to theſe two Interſections, the Tracts which the Spots deſcribe will appear as right Lines, ſince the Eye of the Spectator is in the Plane of their Motion: But in all other Poſitions of the *Earth*, the Plane of the *Sun*'s Æquator is either above or below the Eye of the Spectator, and the Lines in which the Spots are ſeen to move, appear crooked and ſeem to be *Ellipſes*.

The Axis *of the* Sun *is inclined to the Plane of the* Eclip-tick.

SINCE the moſt bright Body of the *Sun* is not without its Spots and Blemiſhes, we are not to imagine that the *Planets* are clear, and without their Stains and Marks. *Jupiter*, *Mars* and *Venus*, when looked at through a Teleſcope, have ſeveral very remarkable ones; and it is by their Motions, that we conclude the Rotation of the *Planets* round their *Axes*, after the ſame Manner, and by the ſame Argument, that we proved the Rotation of the *Sun*,

The Planets *likewiſe have Spots.*

Sun. The Body of *Venus* performs its Revolution round its *Axis* in the Space of 23 Hours. *Mars* finishes his Rotation in 24 Hours and 40 Minutes. The *Earth* in a Day, which we collect from the apparent Revolution of the Heavens, and of all the *Stars* round it in that Time. All these Bodies in their Rotations go the same Way from the *West* to the *East*.

Lecture V.

They turn round their Axes.

Jupiter, besides Abundance of Spots, has some Bands or Girdles which surround him; they are parallel to one another, but they neither preserve the same Magnitude nor Distance but sometimes they increase, sometimes they diminish their Breadth sometimes they approach each other, sometimes they recede from one another, and undergo several Changes. in the Year 1665, M. *Cassini* discovered a very large Spot in *Jupiter*'s Disk, which he observed continually for the Space of two Years, and determined accurately its Figure and Position in respect to his Girdles; but this Spot vanished in the Year 1677, and was not again seen 'till the Year 1679: Afterwards for the Space of almost three Years, it continually shewed itself, and then by Degrees withdrew itself from our Sight, and has since several times appeared and disappeared. In a Word, from the Year 1665, when it was first observed, to the Year 1708, it has eight times appeared and vanished. By its Revolutions we conclude, that the Body of *Jupiter* moves round his *Axis* in the Compass of 9 Hours and 56 Minutes.

Jupiter's *Belts.*

The Changes he undergoes.

It is probable that our *Earth* enjoys a more constant and settled State and Condition than *Jupiter*; for we observe greater Changes in his Surface, than what would happen to the *Earth*, if the Ocean should leave its Place, and overwhelm the Land, and give us a new Place, and a new Figure of the Sea, by changing the Places of Land and Water. But this is upon Supposition that these Spots are inherent in the Body of *Jupiter*: But if they are only Clouds and vapours swimming in his Atmosphere, then indeed the Inhabitants of

Jupiter

Lecture V.

Jupiter would have a more constant and permanent State of Weather, than what any Part of the *Earth* enjoys: I leave it to the *Philosophers* to determine which of these two Opinions is the most probable.

MERCURY does always keep so near the *Sun*, with so great a shining Lustre in all its Parts; and the Heaven, while he is seen, is so much illuminated, that there can be no Observations made to discover his Spots. And *Saturn* is so much further removed from us, more than the other *Planets*, that his Spots are not to be discerned: Yet it is probable that they, as well as the other *Planets*, have each a Rotation round an *Axis*, that all the Parts of their Surfaces may frequently have imparted to them the Light and cherishing Heat of the *Sun*, and may again be withdrawn from him, to receive such Changes as are proper and convenient for the Nature of each *Planet*.

LECTURE

LECTURE VI.

Of the Magnitude and Order of the fixed Stars. *Of the* Constellations, *Catalogues of the* Stars, *and the Changes to which they are liable.*

THE *fixed Stars* appear to be of different Bignesses, not because they really are so, but because they are not all equally distant from us. Those that are nearest will excel in Lustre and Bigness; the more remote *Stars* will give a fainter Light, and appear smaller to the Eye. Hence arise the Distribution of *Stars*, according to their Order and Dignity, into *Classes*; the first Class containing those which are nearest to us, are called *Stars* of the first Magnitude; those that are next to them are *Stars* of the second Magnitude: The third Class comprehends them of the third Magnitude, and so forth, 'till we come to the *Stars* of the sixth Magnitude, which comprehend the smallest *Stars* that can be discerned with the bare Eye. For all the other *Stars*, which are only seen by the Help of a Telescope, and which are called Telescopical, are not reckoned among these six Orders. Although the Distinction of *Stars* into six Degrees of Magnitude is commonly received by *Astronomers*; yet we are not to judge, that every particular *Star* is exactly to be ranked according to a certain Bigness, which is one of the Six; but rather in reality there are almost as many Orders of *Stars*, as there are *Stars*, few of them being exactly of the same Bigness and Lustre. And even among those *Stars* which

The Distinction of Stars *according to their Magnitude.*

Lecture VI. which are reckoned of the brighteſt Claſs, there appears a Variety of Magnitude; for *Sirius* or *Arcturus* are each of them brighter than *Aldebaran* or the *Bull's* Eye, or even than the *Star* in *Spica*; and yet all theſe *Stars* are reckoned among the *Stars* of the firſt Order: And there are ſome *Stars* of ſuch an intermedial Order, that the *Aronomers* have differed in claſſing of them; ſome putting the ſame *Stars* in one Claſs, others in another. For Example: The little *Dog* was by *Tycho* placed among the *Stars* of the ſecond Magnitude, which *Ptolemy* reckoned among the *Stars* of the firſt Claſs: And therefore it is not truly either of the firſt or ſecond Order, but ought to be ranked in a Place between both.

The Conſtellations. ASTRONOMERS not only mark out the *Stars*, but that they may better bring them into Order, they diſtinguiſh them by their Situation and Poſition in reſpect to each other; and therefore they diſtribute and divide them into *Aſteriſms* or *Conſtellations*, allowing ſeveral *Stars* to make up one *Conſtellation*. A *Conſtellation* is a Syſtem of ſeveral *Stars* that are ſeen in the Heavens near to one another: And for the better diſtinguiſhing and obſerving them, they reduce the *Conſtellations* to the Forms of certain Animals, as *Men*, *Bulls*, and *Bears*, &c. or to the Image of ſome Things known, as of a *Crown*, a *Harp*, a *Balance*, &c. The Antients took theſe Figures from the Fables of their *Religion*: And the modern *Aſtronbmers* do ſtill retain them, that they may avoid the Confuſion which would ariſe by making new ones, which would much perplex them, when they compared the modern Obſervations with the old ones.

Their Antiquity. THE Diviſion of the *Stars* by Images and Figures is of great Antiquity, and ſeems to be as old as *Aſtronomy* or *Philoſophy* itſelf: For in the moſt antient Book of *Job*, *Orion*, *Arcturus*, and the *Pleiades* are mentioned; and we meet with the Names of many of the *Conſtellations* in the Writings of

of the first Poets, *Homer* and *Hesiod*. For it was necessary for the Advancement of *Astronomy* so to distinguish the *Stars*, and to bring them into some Order.

The same Face of the Starry Firmament is observed from all the Planets.

SINCE the Distance of the *Stars* is immensely great, it is no Matter in what Place of our System the Observer resides, whether in the *Sun*, in the *Earth*, or even in *Saturn*, the outmost of all the *Planets*; for Spectators in each of those Places will see the same Face of the Heavens, the same *Stars*, with the same Magnitude, and the same Figure of the Constellations; and the Heavens which surround and involve them all, will have the same Face.

The Regions of the Heavens.

ASTRONOMERS divide the *Starry Firmament* into three Regions, the Middle of which comprehends those *Stars* which have their Situation near the Planes of the Orbits in which the *Planets* move; this Part of Heaven they call the *Zodiack*, because the Constellations there placed, seemed for the most Part to represent some *Animal* or living Creature. In this Space the *Planets* are always to be seen, and none of them ever transgress its Bounds: Upon each Side of this *Zone* lie the other two Regions of the Heavens, one of which is called the *North*; and the other, the *South* Part of the Heavens.

The Images noted by the Antients, are 48.

THE Antients divided the visible Firmament into XLVIII. Images, twelve of which filled the *Zodiack*, and they give their Names to the twelve Signs, or the Portions into which it is divided. Their Names are the *Ram*, the *Bull*, the *Twins*, the *Crab*, the *Lion*, the *Virgin*, the *Balance*, the *Scorpion*, the *Archer*, the *Goat*, the *Water-Bearer*, and the *Fishes*.

IN the *Northern* Region there are XXI. Images, *viz.* the *Lesser Bear*, the *Great Bear*, the *Dragon*, *Cepheus*, *Bootes*, the *Northern Crown*, *Hercules*, the *Harp*, the *Swan*, *Cassiopeia*, *Perseus*, *Andromeda*, the *Triangle*, *Auriga*, *Pegasus* or the *Flying-Horse*, *Equuleus*, the *Dolphin*, the *Arrow*, the *Eagle*, *Serpentarius*,

Lecture VI.

rius, and the *Serpent*: Afterwards they added to them two others, *viz.* that of *Antinous*, which was made of the *Stars* that are not included within any Image, and are near the *Eagle*: And the Constellation called *Berenice*'s Hair, consisting of *Stars* which are near the *Lion*'s *Tail*.

Upon the *South* Side of the *Zodiack* there are fifteen *Asterisms*, which were known to the Antients, *viz.* the *Whale*, the *River Eridanus*, the *Hare*, *Orion*, the *Great Dog*, the *Lesser Dog*, the *Ship Argo*, *Hydra*, the *Cup*, the *Crow*, the *Centaur*, the *Wolf*, the *Altar*, the *Southern Crown*, and the *Southern Fish*: To these are lately added XII. more Constellations, which are not to be seen by us who inhabit the *Northern* Regions, because of the Convexity of the *Earth*; but in the *Southern* Parts they are very conspicuous. These are the *Phœnix*, the *Crane*, the *Peacock*, the *Indian*, the *Bird of Paradise*, the *Southern Triangle*, the *Fly*, the *Chameleon*, the *Flying Fish*, the *Toucan*, or *American Goose*, *Hydrus* or *Water Serpent*, *Xiphias* or the *Sword Fish*.

The Stars *without Forms.*

Without the Compass of the Constellations or Images, there are several *Stars*, which cannot be reduced to any of the Forms mentioned; and these are called *Unformed Stars*, out of which some great *Astronomers* have made new Constellations, as *Charles*'s *Heart*, and *Sobieski*'s *Shield*.

The Milky-way.

The *Galaxy*, or *Milky-way*, is also to be reckoned among the Constellations: This is a broad Circle of a whitish Hue like Milk; in some Places it is double, but for the most Part it consists of a single Path, and goes round the whole Heavens. The great *Galileus*, with his Telescope, discovered that the Portion of the Heavens which this Circle passes through, was every where filled with an infinite Multitude of exceeding small *Stars*; which though they cannot, by Reason of their Smallness, be seen distinctly by the naked Eye, yet with their Light they all combine to illustrate that Region of the Heavens where they are, and diffuse through it a shining Whiteness.

By

By the Help of Images, the antient *Astronomers* have been able to distinguish and mark out the *Stars* of the Firmament, and with great Care and Industry they have digested them into Catalogues, which they have delivered down to Posterity. These Catalogues have been much increased and corrected by our modern *Astronomers*; and now they not only comprehend the *Stars* visible by the naked Eye, but also many that are not to be observed or seen without a Telescope.

Lecture VI.

HIPPARCHUS the *Rhodian* about 120 Years before the Birth of *Christ*, was the first among the *Greeks* who reduced the *Stars* into a Catalogue. *Daring, according to Pliny, to undertake a Thing, which seemed to surpass the Power of a Divinity, that is, to number the* Stars *for Posterity, and to reduce them to Rule; having contrived Instruments by which he marked the Place and Magnitude of each* Star. *So that by this Means we can easily discover, not only whether any of the* Stars *perish, and others grow up; but also whether they move, and what is their Course, and also if they grow bigger or wax less; by which Means he has given to Posterity the Possession of the Heavens, if any of them have Subtilty enough to comprehend them,*

Hipparchus *first composed a Catalogue of the* Stars.

HIPPARCUHS, from his own proper Observations, and those of the antient *Astronomers* who lived before him, inserted into his Catalogue 1022 *Stars*, and annexed to each of them their proper Longitude and Latitude, which they had at that Time. *Ptolemy* inlarged *Hipparchus*'s Catalogue only with four *Stars*, numbering 1026. And after *Ptolemy*, *Ulug Beighi*, the Grandson of the great *Tamerlain*, observed again the *Stars*, and reduced 1017 of them into a Catalogue. In the 16th Century, and that which followed, *Astronomy* was courted by many Admirers and Suitors; among whom we may chiefly reckon *Regiomontanus* and *Copernicus*. But the noble *Danish Astronomer Tycho Brahe*, in adorning and perfecting this Science, did far surpass the Labours of all that went before him; who procured very large and exquisitely well contrived Instru-

Ptolemy.

Ulug Beighi *made a new Catalogue.*

As likewise Tycho Brahe.

Lecture VI. ments for observing the Heavens; and particularly he determined the Places of 777 *fixed Stars*, and reduced them into a Catalogue, their Places being all calculated from his own proper Observations. *Kepler* indeed, in his *Rudolphin* Tables, gives us a Catalogue of the *Stars*, he calls *Tychonick*, in which he has put down 1163 *Stars*; but all of them, except the 777 observed by *Tycho*, were taken partly from *Ptolemy*, and partly from other Authors. For *Tycho* in his own Catalogue sets down no *Star* which he had not observed himself, by his own Instruments, and calculated the Place from his proper Observations.

The Prince of Hess. ABOUT the same Time with *Tycho* lived *William* Prince of *Hess*, who likewise observed the *Stars*: He had two *Mathematicians* to assist him, *Rothmannus* and *Byrgius*, with whom, by thirty Years continued Labour, he computed the Places of 400 *Stars*, all founded on their own Observations, and inserted them in a Catalogue.

Ricciolus. THE Jesuit *Ricciolus* inriched the Catalogue of *Kepler* with 305 *Stars*, by which Means their Number was increased to 1468; but this Catalogue was not founded on his own Observations; for he, and his Companion *Grimaldi*, did not take the Places of above 101 *Stars* with their own Instruments; and he took all the rest from *Tycho*, *Kepler*, and other Authors: But it is surprising, that *Ricciolus* should insert in his Catalogue several *Stars*, which were plainly visible in *Tycho*'s Time, and duly observed by him, but which in *Ricciolus*'s Days had vanished, and were not to be seen; and yet they are preserved in his Catalogue, as if he himself had then observed them.

BARTSCHIUS, in his Book about the four Foot-Globe published at *Strasburgh* in 1635, in Quarto, tells us, That *Bayerus* had described in his *Uranometria*, the Places of 1725 *Stars*, and he also boasts, that he himself had painted in his Globe 1762 *Stars*, but he does not tell us by whom, or in what Year they were observed,

THE

THE *Stars* near the *Antarctick Pole*, not to be seen in our Climate, were first accurately observed by my Collegue Dr. *Edmund Halley*; who being animated with a great Love of this Siderial Science, undertook a long, and no less dangerous Voyage to the Island of St. *Helena*, that he might there take the Position of the *Stars* which are within the *Antarctick Circle*; and he published a Catalogue of 373 of them, whose Places he adapted to the End of the Year 1677.

Lecture VI.

Dr. Halley *first exactly observed the* Southern Stars.

THE illustrious *John Hevelius* of *Dantzig*, a Man of prodigious Industry, and unwearied Diligence, being well furnished with very exact Instruments, and with all the Tools that are proper for an *Astronomer*, did again assault the *Stars* with his Instruments, and computed the Places of 1553 of them from his own proper Observations; and so he composed a new Catalogue, which contained 1888, *viz.* 950 known to the Antients, and which were to be seen at *Dantzig*; 603 new ones, which no one before had ever rightly observed; and to them he joined 335 others round the *Antarctick Pole*, taken out of Dr. *Halley*'s Catalogue, which lie always hid under the *Horizon* of *Dantzig*.

Hevelius.

BUT the largest and most complete Catalogue of the *Stars* * is shortly to be expected from the Labours of that most excellent Observer Mr. *John Flamsteed*, late Royal Professor of *Astronomy* at *Greenwich*; the Number of *Stars* inserted in this Catalogue reach to 3000. And as *Hevelius* doubled by his Observations the Number of *Stars* observed by *Tycho*: So our *British Astronomer* has as far outdone *Hevelius*, having, by his Observations, doubled the *Stars* that were observed by him: We are so much indebted to this *Astronomer* for the Increase of the Knowledge we have of the *Celestial Bodies*, that there is not the least *Star* in the Heavens to be seen, whose Place and Situation is not better known, than the Position of many Cities

Mr. Flamsteed *has made the most copious and most exact Catalogue.*

* *This Catalogue was published Anno 1725.*

Lecture VI.

Cities through which Travellers do daily paſs. Nor is it any Wonder, that the *Aſtronomers* ſhould take ſo much Pains, and ſo obſtinately watch the *fixed Stars*, to determine their Places; for without the exact ſettling of their Poſitions and Places, they could never have found out the Ways of the *Planets*, nor have deſcribed their Orbits: For it is upon the Obſervations of the *fixed Stars*, as upon immoveable Pillars, that the whole Science of *Aſtronomy* is erected, and by them it is ſuſtained.

The Number of the Stars.

Of the 3000 *Stars* inſerted in Mr. *Flamſteed's* Catalogue, there are many that cannot be ſeen without a Teleſcope; ſo that it is ſeldom that even a very good Eye can reckon more than 100 together in the Heavens: This will certainly ſurprize a great many Perſons; for in the Winter, in a clear Night, without *Moon-ſhine*, at firſt Sight they ſeem to be innumerable. But this Appearance is only a Deception of our Sight, ariſing from their vehement and ſtrong Twinkling, or Scintillation, while we look upon them confuſedly, and without reducing them to any Order; but he who will diſtinctly view them, will find not one but what are obſerved by the *Aſtronomers*, and inſerted in their Catalogues: And if any one will take a Globe of the larger Size, and compare it with the Heavens, he will rarely find any *Star* in the Heavens, that is not marked upon the Surface of that Globe.

In the mean Time I muſt acknowledge, that the Number of the *Stars* is really vaſtly great, and almoſt infinite; for whoever will view the *Stars* with a good Teleſcope, will find every where a prodigious Number of them altogether indiſcernable by the naked Eye, eſpecially in the *Milky-way*; where they are ſo thick, that though they cannot be ſeen ſeparately, yet they give that Region of the Heavens, where they are placed, a Luſtre above all the reſt.

Many Stars *imperceptible to the naked Eye.*

The famous Dr. *Hook*, Profeſſor of *Geometry* in *Greſham-College*, directing his twelve-foot Teleſcope to the *Pleiades* or the ſeven *Stars*, though there are now only ſix to be ſeen with the naked Eye, did,

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in that ſmall Compaſs, count 78 *Stars*; and making uſe of longer and more perfect Teleſcopes, he diſcovered a great many more of very different Magnitudes: See his *Micrography*, pag. 241. *Antonius Maria de Reita*, in his Book which he calls *Radius Sidereomyſticus*, pag. 197, affirms, that he has numbered in the ſingle Conſtellation of *Orion* 2000 *Stars*.

Lecture VI.

The Matter of the Heavens is corruptible and changeable.

FROM what we have ſaid, in the preceding Lecture, it plainly appears, how falſe and ill-founded were the Notions of the antient *Philoſophers*, who having too favourable an Opinion of the Heavenly Regions, granted them ſome Privileges without any reaſonable Ground: For they affimed, that the Heavens were incaple of any Change, that the Celeſtial Matter was of a different Kind from any we have in our *Earth*, and that its Firmneſs did far exceed that of the moſt durable Diamond; for that is ſtill corruptible, and may be changed into other Sorts of Matter, and undergo ſeveral Tranſmutations: But the Form of the Heavenly Matter, according to them, is permanent and eternal. We have ſeen in the *Sun* and *Planets*, that there are frequently new Bodies produced and generated; others again are corrupted and periſh, and the Faces of the *Planets* undergo many Changes. Thoſe *Alterations* are not peculiar to our *Earth*, or our Planetary Syſtem; the Principle of Generation and Corruption is much further diffuſed, it reaches even the moſt diſtant *fixed Stars*, and all the Bodies of the Univerſe are under its Dominion; there is nothing but our Mind, and our Spiritual Part, that are exempted from its Juriſdictſon: For the Heavenly Bodies, as well as the Terreſtrial, are changeable and periſh.

The fixed Stars *liable to the Principle of Generation and Corruption.*

Several *Stars* which were obſerved by the Antients, are now no more to be ſeen, but are deſtroyed, and we have known ſome new ones come in the Heavens unknown to them; which likewiſe in due Time will vaniſh, and diſappear. There are alſo ſome *Stars* which for a Time are extinguiſhed, and become inviſible, but after a certain Period they reaſſume

Some Stars *vaniſh or periſh, and again appear.*

Lecture VI. assume their former Lustre. Of these *Stars*, the most remarkable is that which is in the Neck of the *Whale*, which for eight or nine Months of the Year withdraws itself from our Sight, and for the other three or four Months is constantly changing its Lustre and Bigness. It is probable, that the greatest Part of the Surface of this *Star* is covered with Spots and dark Bodies, some Part thereof remaining lucid; and while it turns about its *Axis*, does sometimes shew its bright Part, sometimes it turns its dark Side to us: But the very Spots themselves of this *Star* are liable to Changes: for it does not every Year appear with the same Lustre; sometimes it resembles a *Star* of the second Magnitude, in other Years it can scarely be reckoned among *Stars* of the third Order: Nor are the Times of its visiting us, always of the same Duration; for in some Years after three Months it takes its Leave of us; in others we enjoy its Light for the Space of four Months; nor does its Increase or Decrease always answer the Difference of Times.

New Stars.

MOREOVER we are assured from the Observations of *Astronomers*, that some *Stars* have been observed which never were before, and for a certain Time they have distinguished them by their *superlative Lustre*; but afterwards decreasing, they by Degrees vanished, and were no more to be seen. One of these *Stars* being first seen and observed by *Hipparchus*, the Chief of the *Astronomers* of the Antients, set him upon composing a Catalogue of the *fixed Stars*, that Posterity might by it learn whether any of the *Stars* perish, and others are produced afresh.

A new Star *in* Cassiopeia

AFTER several Ages, another new *Star* appeared to *Tycho*, and the *Astronomers* that were contemporary with him; which just like the new *Star* in *Hipparchus*'s Time induced him likewise to make a new Catalogue of the *fixed Stars*. This *Star* made its Appearance in the Constellation *Cassiopeia*, and was first observed about the Middle of *November* 1572, and never changed its Place all the Time it

was

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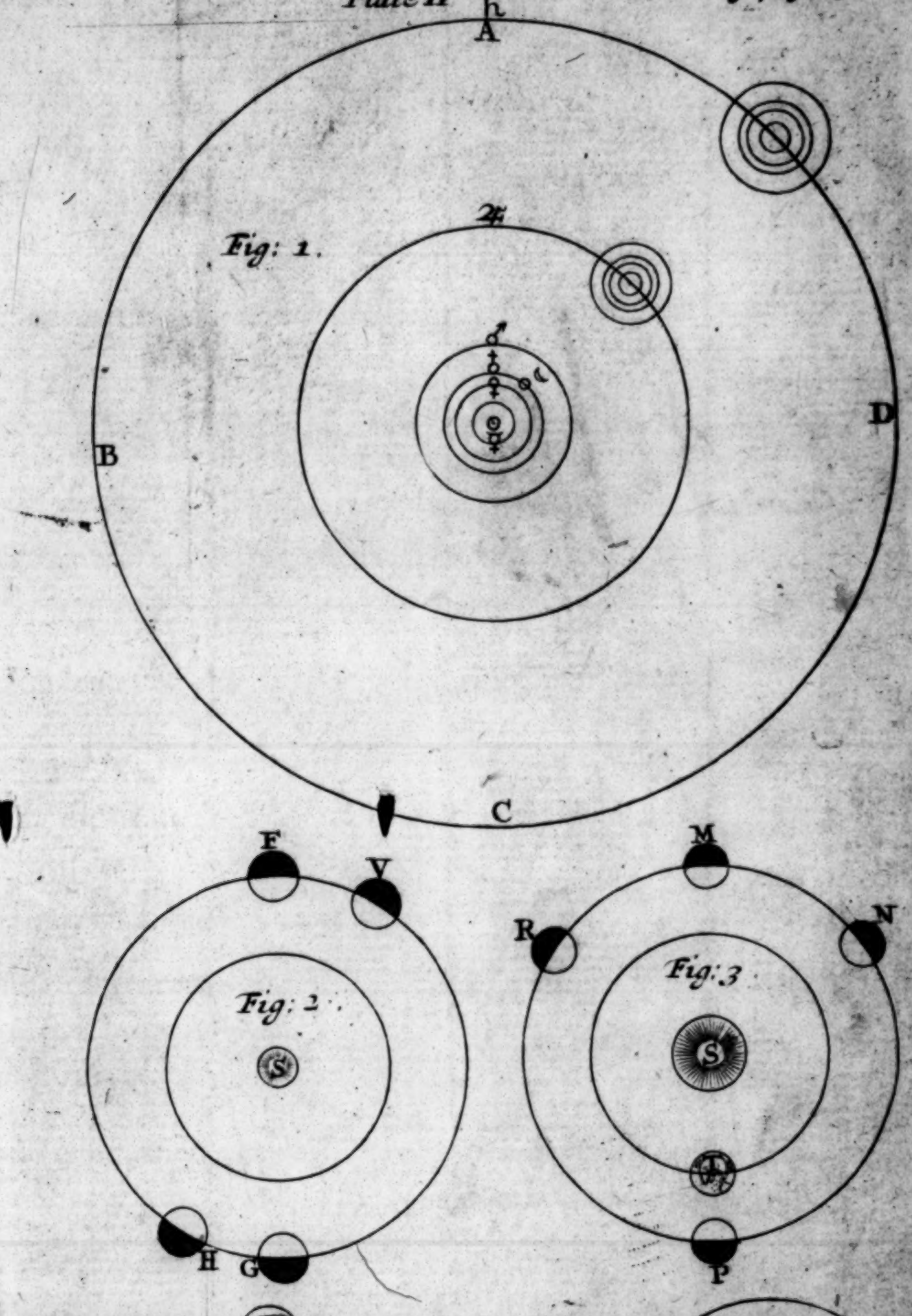
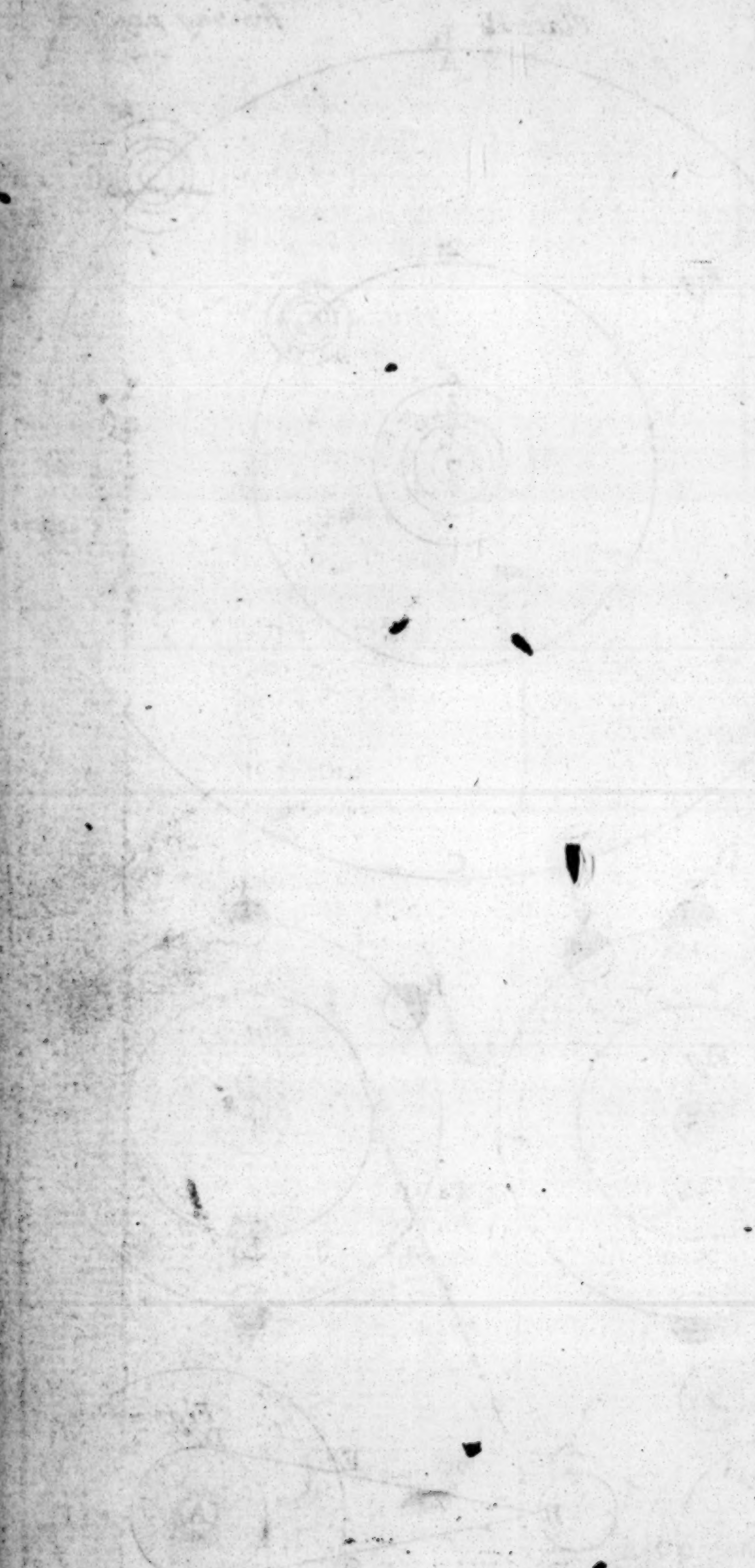
Plate II fronting page 58





was visible, which was for the Space of sixteen Months; but yet by Degrees it diminished, and at last became invisible. Its Magnitude exceeded that of *Syrius* or *Lyra*, which are the brightest of the *fixed Stars*, and even vied with *Venus* when she is nearest to us: So that sometimes it could be seen in fair Day-light or Sun-shine; but at last it continually lost something of its Splendor, 'till it quite disappeared, and it has never been seen since. *Leovicius*, from the Histories of those Times, tells us, that in the Time of the Emperor *Otho*, about the Year 945, a new *Star* appeared in *Cassiopeia*, just such a one as was seen in his Time in the Year 1572. And he brings us another antient Observation that there was likewise seen in the *Northern* Region of the Heavens, near the Constellation *Cassiopeia*, in the Year 1264, an eminently bright *Star*, which kept itself in the same Place, and had no proper Motion. It is probable, that these two *Stars* might have been the same with that which was seen by *Tycho*, and that in about 150 Years the same *Star* may again make its Appearance.

A new Star *in the* Swan's *Breast.*

In the Year 1600 and the following, *Kepler* observed another new *Star* in the *Swan*'s Breast, which remained visible for many Years; and in *Hevelius*'s Time looked like a *Star* of the third Magnitude; but at last became invisible from the Year 1660 till the Year 1666, when it was again observed by *Hevelius* as a *Star* of the sixth Magnitude just in the same Place as it was at first observed in; where it now appears.

We are assured by the Catalogues of the *fixed Stars*, that many Stars have been observed by the Antients, and some even by *Tycho*, which are now become invisible, and particularly in the *Pleiades* or *seven Stars*; there were formerly counted seven, but now we can reckon but six; and so it was in *Ovid*'s Time, who in his third Book of the *Fasti*, has this Verse;

Quæ septem dici, sex tamen esse solent.

The

Lecture VI.

The celebrated Mr. *Montanere*, Professor of *Mathematicks* of the University of *Bononia*, in his Letter written to the Royal Society, dated *April* 30, 1670, has these Words: " There are now wanting in the Heavens two *Stars* of the second Magnitude, in the Stern of the Ship *Argo*, and its Yard; *Bayerus* marked them with the Letters β and γ. I and others observed them in the Year 1664, upon the Occasion of the Comet that appeared that Year: When they disappeared first I know not; only I am sure, that in the Year 1668, upon the 10th of *April* there was not the least Glimpse of them to be seen; and yet the rest about them, even of the third and fourth Magnitudes, remained the same. I have observed many more Changes among the *fixed Stars*, even to the Number of a Hundred, though none of them are so great as those I have shewed.

It is no ways improbable, that these *Stars* lost their Brightness by a prodigious Number of Spots, which intirely covered, and, as it were, overwhelmed them. In what dismal Condition must their *Planets* remain, who have nothing but the dim and twinkling Rays of the *fixed Stars* to enlighten them?

LECTURE

LECTURE VII.

Of the Motion of the Earth *round the* Sun, *and also about her own* Axis; *whereby the Apparent Motion of the* Sun *and Heavens are explained.*

HAVING taken a curfory View of the Univerfe, and explained thofe Things which we have difcovered concerning the *fixed Stars*, we will now come to confider more accurately our own folar Syftem; for our *Aftronomy* is chiefly concerned about the Motions of the Bodies that are contained in it, and about the Appearances or *Phænomena* that arife from thofe Motions.

AND, firft, it is reafonable that we fhould begin from the Motion of the *Earth*, which is our own Seat and Habitation, that is, from our own Motion; fince we are to be the Spectators of all the Appearances which are here to be explained, and particularly, becaufe from our Motion arifes the apparent Motion of the *Sun*, which if not firft known, the Appearances and Motions of the *Planets* can neither be explained nor computed. *We are to begin with the Motion of the* Earth.

WE have in the preceding Lectures demonftrated, that the *Sun*, which is by far the biggeft and moft noble Body of the Univerfe, does poffefs himfelf of the Center, from whence he every Way diffufeth upon all the *Planets* his enlivening Beams and Warmth; and that they, as it were, dance round him at different Diftances and Periods. The *Earth* which we inhabit is to be reckoned as one of them, who goes round the *Sun* in the Space of a Year; *The* Sun *in the Center of our Syftem.* *The* Earth *turns round the* Sun.

Lecture VII.

The same Order and Position of the Stars *is seen from the* Sun *and* Earth.

and at the same Time turns round her own *Axis* every twenty-four Hours. Now, since the Distance of the *fixed Stars* is immensely great, in Comparison of the Distance of the *Earth* from the *Sun*, the Starry Firmament will have the same Face, and there will be the same Situation, Order, and Magnitude of the *Stars*, whether they be viewed from the *Sun*, or from the *Earth*: But since all distant Bodies appear as if they were in the Heavens, a Spectator in the *Sun* will observe the *Earth* to describe a Circle in the Starry Firmament; and because the Plane of the *Earth*'s Orbit passes through the *Sun*, the Circle which the *Earth* describes, will appear to be a great Circle in the Heavens.

The Motion of the Earth *seen from the* Sun. Plate III. Fig. 1.

LET S represent the *Sun*, ABCD the Orbit of the *Earth*, in which the *Earth* is carried from the *West* to the *East* in the Compass of a Year. A Spectator in S looking upon the *Earth* at A, will refer it to the Star ♈, as if it were in the same Point of Space with the *Star*: But when the Earth is brought to B, the Spectator in S will see the *Earth* in the Heavens just by the Star ♋; when the *Earth* is gone forward to C, it will be seen from the *Sun* in ♎; and when it is come to D, it will appear in ♑; and when it is returned to A, having finished its Period, it will be seen again in ♈.

HENCE, if the Plane of the *Earth*'s Orbit be imagined extended to the Heavens, as far as the *fixed Stars*, it will cut the starry Firmament, or the concave spherical Surface, in which all the *Stars* appear, in that very Circle in which a Spectator in the *Sun* would see the *Earth* to revolve every Year:

The Ecliptick *and its Division into twelve Parts or Signs.*

This Circle is called the *Ecliptick*, and divided by the *Astronomers* into twelve equal Parts which are called Signs; each of them takes its Name from that Constellation, which, at the Time the Names were imposed, was situated near the Portion of the *Ecliptick* it denominates. These Signs or Portions are the *Ram* ♈, the *Bull* ♉ the *Twins* ♊, the *Crab* ♋, the *Lyon* ♌, the *Virgin* ♍, the *Balance* ♎, the

Scorpion

Scorpion ♏, the *Archer* ♐, the *Goat* ♑, the *Water-Bearer* ♒, and the *Fishes* ♓.

Lecture VII.

LET us now bring our Spectator from the *Sun* to the *Earth*, and let him be carried by it round the *Sun*, and let us imagine that the *Earth* is in C: A Spectator sees the same Face of the Heavens, and the very same Constellations, as we have said he did before, while he was in the *Sun*; the only Difference will be, that, as before he imagined the *Earth* in the Heavens, and the *Sun* in the Center, he will now suppose the *Sun* to be in the Heavens, and himself with the *Earth* in the Center, it being really the Center of his own View. Therefore the *Earth* being in C, the Spectator will see the *Sun* at the Star ♈; and the Spectator being carried along with the *Earth*, and participating of the annual Motion, which is common to them both, he will observe all the Parts of the *Earth*, and all the Bodies fixed on its Surface, to keep the same Position in regard to one another, and to his own Eye, and always to remain at the same Distance from him; and there he cannot by his Eye, perceive either his own Motion, or that of the *Earth*. But looking to the *Sun*, and observing him, when the *Earth* comes to D, he will see the *Sun* at the *Star* ♋, and will perceive that he has changed his Place among the *Stars*, and has moved from ♈, by ♉, ♊, to ♋: And while the *Earth* goes on in its Progress, and goes to A, the *Sun* will be seen from thence to have moved through the Signs ♋, ♌, and ♍: And again while the *Earth* describes the Semicircle A B C the *Sun* will appear to have moved, in the concave Surface of the Heavens, through the six Signs ♎, ♏, ♐, ♑, ♒, ♓. And therefore an Inhabitant of the *Earth* observes the *Sun*, which is really immoveable, to go through the same Circle in the Heavens, and in the same Space of Time, that a Spectator in the *Sun*, would see the *Earth* describe.

The Apparent Motion of the Sun *seen from the* Earth.

HENCE

Lecture VII.

[illegible] seem: y^t Ear: turn from west to East y^t Heav: any place will seem: ascen above a more west^n part of y^e Eclip^t y^t a more eastern

HENCE arises the apparent Motion of the *Sun*, by which it is observed to creep every Day by little and little towards the *Eastern Stars*; so that if any *Star* near the Ecliptick does at any Time rise with the *Sun*, after some few Days the *Sun* will be got more to the *East* of the *Star*, and the *Star* will rise before the *Sun*, and will likewise set before him. So likewise a *Star* which is *Eastward* of the *Sun*, and is seen after the *Sun* sets, at a considerable Distance from him, in the Space of some few Days will set with the *Sun*, and will no more be seen after the *Sun* goes down. This Motion of the *Sun*, which is contrary to the apparent diurnal Revolution of the Heavens from *East* to *West*, was esteemed to be real by the Followers of *Ptolomy*, who maintained, that the *Sun* and all the *Stars* had two Motions, contrary to one another, the one common with the Heavens from *East* to *West*, in the Space of 24 Hours; the other proper, and peculiar to each, and was from the *West* to the *East*; which Course the *Sun* finished in the Space of a Year. But we have shewed, that there is no such real Motion in the *Sun*, and that it is only apparent, arising from the Motion of the *Earth*.

The Sun *will have just such Motions, when it is observed from other* Planets.

THE Inhabitants of all the other *Planets* will observe just such Motions in the *Sun*, and for the very same Reasons that we do in our *Earth*: And the *Sun* will be seen from every *Planet*, to describe the same Circle, and in the same Space of Time that a Spectator in the *Sun* would observe the *Planet* to do. For Example: An Inhabitant of *Jupiter* would think that the *Sun* turns round him, and would see him describe a Circle in the Heavens in the Space of twelve Years; that Circle would not be the same with our Ecliptick, and the Motion of the *Sun* would not be through the same *Stars*, which he appears to us to pass by. And upon the same Account the *Sun* seen from *Saturn* will appear to move in another Circle, distinct from either of the former, and will not seem to finish his Period in less

Time

Time than thirty Years. Since therefore it is impoſſible that the *Sun* can have all theſe Motions really in itſelf, and there can be no Reaſon ſhewn, why any one of them ſhould belong really to the *Sun* more than the reſt, we may ſafely affirm that there are none of them real, but that they are all apparent, and ariſe from the Motions of the reſpective *Planets*. Lecture VII.

BESIDES this annual Circulation of the *Earth* it has alſo a *virtiginous Motion* round its *Axis*, from the *Weſt* to the *Eaſt*, in twenty-four Hours: The two Points in which the *Axis* meets with the Surface of the *Earth*, are called the *Poles* of the *Earth*; and if this *Axis* be indefinitely produced to the Heavens both Ways, it will mark in the Heavens two Points, which are called the *Poles* of the Heavens: Every Point on the Surface of the *Earth*, except the *Poles*, will deſcribe the Circumference of a Circle bigger or leſs, according as it is further diſtant or nearer to one of the *Poles*. The *Poles* are the only two Points which have no Verticity: This plainly follows from the Nature of a vertiginous Motion. Any Place on the Surface of the *Earth*, which is equally diſtant from both the *Poles*, deſcribes by its Rotation a great Circle, which is called the *Æquator* of the *Earth*, or the *Æquinoctial Circle*; and the reſt deſcribed by Points nearer to one of the *Poles*, are leſſer Circles, and are called Parallels. *The Gyration of the* Earth *round its* Axis. *The* Poles. *The Æ*quator, *or* Æquinoctial *and Parallels.*

IF we imagine a Plane to paſs over that Point of the *Earth*'s Surface on which the Spectator ſtands, and to touch the Globe of the *Earth* there; this Plane, extended as far as the Heavens, will divide the Heavens into two Parts, and its Section with the Heavens will make a Circle, which is called the *Horizon*; and it will ſeparate or diſtinguiſh the viſible and open Part of the Heavens, from that which is inviſible, and which the Opakeneſs and Convexity of the Earth hides from us. This *Horizon* which we have deſcribed is properly the Senſible *Horizon*: The *The* Hori-zon. Plate III. Fig. 2. *The Senſible and Rational* Horizon.

Lecture VII. The Rational *Horizon* being a diſtinct Circle which paſſes through the Center of the *Earth*, and is parallel to the Senſible which touches the Surface. But theſe two Circles, though they are diſtant from one another by the Semidiameter of the *Earth*, yet in the Heavens they may be reckoned as coinciding; for that Semidiameter is but a Point, in Compariſon of the Diſtance of the Heavens.

The Rotation of the Earth *produces an apparent Revolution of the Heavens round the* Earth.

SINCE the *Earth* turns round its *Axis*, the Spectator, ſtanding on its Surface, much likewiſe turn round with it the ſame Way, that is, towards the *Eaſt*: And therefore all the Bodies in the Heavens, which are placed in the *Eaſt*, and were not to be ſeen by Reaſon of the Plane of the *Horizon* was above them, will become viſible, when by the Rotation this Plane ſubſidies, and comes under them. So likewiſe the oppoſite Part of this Plane towards the *Weſt*, riſing above the *Stars*, will hide them from the Sight of the Spectator, and all the *Stars* in the *Weſt* will become inviſible. Hence it is, the *Stars* of the *Eaſtern* Side of the *Horizon* will appear to riſe above the *Horizon*, becauſe the *Horizon* deſcends below them; and the *Stars* on the *Weſtern* Side will appear to ſet, or go below the *Horizon*, becauſe the *Horizon* does really get above them. Hence ariſes that apparent Motion of all the Bodies of the Univerſe that do not adhere to the *Earth*, wherewith the whole ſtarry Firmament, and every Point of the Heavens ſeem to revolve about the *Earth* from *Eaſt* to *Weſt*; every Point deſcribing a greater or leſſer Circle, as it is more remote, or nearer to one of the Celeſtial *Poles*: And theſe Celeſtial *Poles* which are made by the Production of the *Earth*'s *Axis* to the Heavens, are the only Points in the Heavens, which appear to be immoveable.

ALTHOUGH every Place on the Surface of the terraqueous Globe, is illuſtrated by all the *Stars* whicy are above the *Horizon* of that Place, or rather when the *Horizon* is under the *Stars*; yet the Illumination made by the *Sun* is ſo great, and

and the Reflection of its Light by the Atmosphere so strong, that the *Sun*, when he is above the *Horizon*, does with his Presence quite extinguish the faint Light of the *fixed Stars*, and produces Day: When the *Sun* withdraws himself, and goes below our *Horizon*, or, more properly, when our *Horizon* gets about the *Sun*, he then gives Leave to the *Stars* to shine and appear, at which Time it is Night. Now since the *Earth* is an opake spherical Body, at a great Distance from the *Sun*, one Half of it will always be illuminated by the *Sun*, while the other Half remains in Darkness: And the Circle which distinguishes the illuminated Face of the *Earth* from the dark Side, is called the Circle of the Intersection of Light and Shadow; a Line drawn from the Center of the *Sun*, to the Center of the *Earth*, is always perpendicular to the Plane of this Circle.

Lecture VII.

Whence Day-light. Whence Night.

The Circle bounding Light and Darkness.

If the *Axis* of the *Earth* had been placed in a Position perpendicular to the Plane of the Ecliptick, then in that Case the Plane of the *Earth*'s Æquator had coincided with the Plane of the Ecliptick, or the Plane of the *Earth*'s Orbit; and the Circle bounding Light and Darkness, would have always passed through the *Poles* of the *Earth*, and cut the Æquator and all its Parallels into equal Portions: And therefore, in that Case, the *Sun* and all the *Stars* would have remained as long above the *Horizon*, as they would have lain hid under it, and the Days would have been constantly equal to the Nights. But now as the Case is, the *Axis* of the *Earth* is not perpendicular to the Plane of the Ecliptick, but is inclined to that Plane, and makes with it an Angle of 66½ Degrees; and therefore the Plane of the *Earth*'s Æquator cannot coincide with the Plane of the *Ecliptick*, but these two Planes make with one another an Angle of 23½ Degrees.

The Axis *of the* Earth *is not perpendicular to the Plane of the Ecliptick.*

If the Plane of the *Earth*'s Æquator be imagined to be produced as far as the *fixed Stars*, it will there make a Circle which is called the Celestial Æquinoctial; which is exactly over the Terrestrial

The Æquinoctial of the Heavens.

Lecture VII. Æquinoctial: This Circle makes with the Ecliptick in the Heavens, an Angle likewise of $23\frac{1}{2}$ Degrees.

The Parallelism of the Earth's Axis.

THE *Earth* in its Revolution round the *Sun* does in such a Manner proceed in its Orbit, that it keeps its *Axis* parallel to itself; that is, if a Line be drawn parallel to the *Axis* while it is in any one Position, the *Axis* in all other Positions or Parts of the Orbit will always be parallel to that same Line, and it will never change its Direction, but always look towards the same Point of the Heavens: And this will necessarily be, if the *Earth* have no other Motion but that round the *Sun*, and the other round its own *Axis*. For suppose any Body, whose Center is carried in the Line A B, and in A we should mark any Diameter C D, which is inclined in any Angle to the Line A B; if this Body have no other but a progressive Motion in the Line A B, when it comes to the Point B, the Diameter C D will be in the Situation *c d*, and its Position will be parallel to the former Position C D: Now if there should be impressed upon this Body a Rotation round C D as an *Axis*, all the Diameters of the Body will constantly change their Position by this Rotation, except the *Axis*, which will remain in its former State: The Points in the *Axis* being the only Points in the Body which have no Rotation. But this *Axis*, as was shewed, did before the Rotation always preserve a Position parallel to itself; therefore after the Rotation is impressed upon the Body, the *Axis* will still keep parallel to itself.

Plate III. Fig. 3.

HENCE it is evident, that there is no Need of a third Motion for the *Earth*, as some have imagined it must have, to make it keep its *Axis* parallel to itself: For to this Effect there is nothing more required, than that it should have only the former two, with which alone it will necessarily keep its *Axis* parallel to itself.

SINCE the Plane of the Æquator does not coincide with the Plane of the Ecliptick, these two Planes must cut one another in a right Line; and while

while the *Earth* turns round the *Sun*, the common Section of these two Planes will likewise always remain parallel to itself, for the same Reason, as we shewed in the Position of the *Earth*'s *Axis*: And therefore this Section or Line, in which the Planes cut one another, will always be directed towards two opposite Points of the Ecliptick, and will always look to the same Points of the Universe. Lecture VII.

A great Circle in the Heavens passing through the two Celestial *Poles*, and the common Section of the Æquator and the Ecliptick, is called the *Equinoctial Colure*: Another Circle, cutting the former in the *Poles* at right Angles, is called the *Solstitial Colure*, which passes through the Points where the Æquator and Ecliptick are at the greatest Distance from one another, and cuts likewise both these Circles at right Angles; and therefore does likewise pass through the *Poles* of the Ecliptick, or that Point which is every-where equally distant from the Ecliptick. The four Points in which these two *Colures* intersect the Ecliptick, are called the four *Cardinal Points*; because when the *Sun* is seen in them, he determines the four Seasons of the Year. The two Intersections of the *Equinoctial Colure* with the Ecliptick, are called the *Equinoctial Points*; the other two, being the Intersections of the *Solstitial Colure* with the Ecliptick, are called the *Solstitial Points*.

The Æquinoctial Colure. And the Solstitial Colure.

The Cardinal Points.

Suppose now the Eye of a Spectator to look from afar, obliquely upon the Orbit of the *Earth*; it will then appear, or have a Representation of an Oval Figure, according to the Rules of Perspective; and in the Middle of this Oval the *Sun* will keep. Through the Center of the *Sun* S, draw the right Line ♈ S ♎ parallel to the common Section of the Æquator and the Ecliptick, which will meet with the Ecliptick in two Points ♈, ♎: And when the *Earth* seen from the *Sun* is in either of these Points, the right Line S ♈ or S ♎, which joins

Plate III. Fig. 4.

Lecture VII.

The different Illumination of the Parallels, according to the Place of the Earth *in the Ecliptick.*

The Appearances when the Earth *is in* Libra, *and the* Sun *is seen in* Aries.

joins the Center of the *Earth* and *Sun*, will coincide with the common Section of the Æquator and Ecliptick; and will then be perpendicular to the *Axis* of the *Earth*, or of the Æquator, because it is in the Plane of the Æquator: But the same Line is also perpendicular to the Circle which bounds the Light and Darkness; and therefore the *Axis* of the *Earth* will be in the Plane of that Circle, which will therefore pass through the *Poles* of the *Earth*, and will cut the Æquator and all its Parallels into equal Parts. When the *Earth* therefore is in the Beginning of ♎, the *Sun* will be seen in ♈, in the common Section of the Æquator and the Ecliptick; and therefore it will appear in the Celestial Æquinoctial, and will not be seen to decline to either of the *Poles*; but being exactly in the Middle between both, it will then, by its apparent diurnal Revolution, describe the Celestial Æquinoctial. In this Position of the *Earth*, the *Sun* will exactly illuminate the *Earth* from *Pole* to *Pole*; and, as we said, the Circle bounding Light and Darkness will cut the Parallels exactly into equal Parts; and every Point of the *Earth*, being carried round by the vertiginous Motion, will remain as long in the obscure Side, as it was in the Light or illuminated Portion of the *Earth*'s Surface: And therefore at that Time, through the whole Globe of the *Earth*, the Day will be equal to the Night: From hence the Circle which that Day the *Sun* seems to describe in the Heavens, has obtained the Name of *Æquinoctial*.

The Appearances when the Earth *is in* Capricorn.

The *Earth* in its annual Motion going by Degrees through ♏ and ♐ towards ♑, and the common Section of the Æquator and the Ecliptick remaining always parallel to itself, it will no longer pass through the Body of the *Sun*; but in ♑ it makes a right Angle with the Line S P, which joins the Centers of the *Sun* and *Earth*: And because the Line S P is not in the Plane of the Æquator, but in that of the Ecliptick; the Angle B P S, which the *Axis* of the *Earth* makes with it, will not now

be

ye Sun is Vert.l to a plac when it is in ye Zenith of yt place yt is in ye pole of its Horizon & Conuq when ye finitor passes thro ye Horizon of yt place

Lecture VII.

be a right Angle, but oblique; these two Lines making an Angle of $66\frac{1}{2}$ Degrees, which is the same with the Inclination of the *Axis* to the Plane of the Ecliptick. Let the Angle SPL be a right Angle, and the Circle bounding Light and Darkness will pass through the Point L; and then the Arch BL, or the Angle BPL, will be $23\frac{1}{2}$ Degrees, that is equal to the Complement of the Angle BPS to a right Angle.

Let the Angle BPE be a right Angle, and then the Line PE will be in the Plane of the Æquator: Therefore because the Arches BE and LT are equal, each of them being Quadrants; if the common Arch BT be taken away, there will remain TE equal to LB equal to $23\frac{1}{2}$ Degrees. Take EM equal to ET, and through the Points M and T, describe two parallel Circles TC, MN; the one is called the *Tropick* of *Cancer* ♋; the other the *Tropick* of *Capricorn* ♑. And the *Earth* being in this Situation, the *Sun* will shine perpendicularly upon the Point T, and then it will seem to approach the nearest that it can come to the *North Pole*: And the Circle, which by the apparent diurnal Revolution of the Heavens, the *Sun* seems to describe, will be directly over the Circle TC in the *Earth*, which Circle is therefore called the Celestial Tropick of *Cancer*. Now upon the Account of the Revolution of the *Earth* round its *fixed Axis*, all the Points of the Parallel TC will in their Turns pass by the Point T, and will be directly under the *Sun*; and therefore the *Sun* will be vertical to all the Inhabitants that are under the Tropick of ♋, when he comes to their Meridians. While the *Earth* is in this Position, it is manifest, that the Circle which bounds Light and Darkness reaches beyond the *North Pole* B to L; but towards the *South* it falls short of the *South Pole* A, and reaches no further than F. Through L and F let two Parallels to the Æquator he described: These two Circles are called the the *Polar Circles*; this

The two Tropicks.

Lecture VII.

is called the *Arctick Polar*, the other the *Antarctick.* And while the *Earth* is in P, all that Tract of it which is included within the *Polar* Circle KL, continues in the Light, notwithstanding the constant Revolution round the *Axis*; and the Inhabitants there enjoy a continual Day. On the contrary, those that lie within the *Antarctick* Circle remain in continual Darkness, having all Night without any Day. Besides it is likewise manifest, that all the Parallels between the Æquator and the *Arctick* Circle, are cut by the Circle bounding Light and Darkness, into unequal Portions; the largest Portions of these Circles remaining in the Light, and the smallest in Darkness: But those Parallels which are towards the *Antarctick* Circle have their greatest Portions in Darkness, and the least in the Light; and the Difference of these Portions will be greater or less, according as the Circles are nearer to the Pole, or to the Æquator. Therefore in this Position of the *Earth*, when the *Sun* is seen in ♋, the Inhabitants of the *Northern Hemisphere* will have their Days at the longest, and their Nights the shortest; and the Season of the Year will be Summer: But in the *Southern Hemisphere* the Inhabitants will have their Nights longest, and their Days shortest; and they will be in their Winter Season.

The Arctick *and* Antarctick *Circles.*

When the Days are longest. When shortest.

And in every Place the Length of the longest Days will be the greatest, and the Nights the shortest, according as the Place is further removed from the Æquator, and comes nearer the *North Pole.* We see likewise, that of all the Parallels, there is only the Æquator which is cut into equal Parts by the Circle bounding Light and Darkness, they being both of them great Circles: And therefore it is only the Inhabitants of the *Earth* that live in the Æquator, that have their Days constantly equal to their Nights, throughout the whole Year.

While the *Earth* goes on from ♑ by ♒, ♓, to ♈, in which Time the *Sun* is seen to pass

through

through the Signs ♋, ♌ and ♍, he will appear to return by little and little towards the Æquator; and when the *Earth* is arrived at ♈, the *Sun* will appear in ♎, where the common Section of the Æquator and the Ecliptick always keeping parallel to itself, will pass through the Center of the *Sun*; and then the Sun will appear in the Celestial Æquinoctial: At which Time the Days will again be equal to the Nights, to all the Inhabitants of the *Earth*; just after the same Way as it was when the *Earth* was in ♎, in that Position the Circle bounding light and Darkness passing through the *Poles*.

Lecture VII.

The Appearances when the Earth *is in* Aries.

THE *Earth* moving on through ♈, ♉ and ♊, the *Sun* will be seen to go in the Ecliptick through ♎, ♏ and ♐, and will appear to decline from the Æquator towards the *South*; so that when the *Earth* is really in ♋, the *Sun* will appear among the *Stars* near the Constellation ♑. And whereas the *Axis* B A does not change its Inclination, but does always retain its Parallelism, the *Earth* will have the same Aspect and Position in respect to the *Sun*, that it had when it was in ♑; but with this Difference, that when the Tract within the *Polar* Circle K L was in continual Light while the *Earth* was in ♑; now the *Earth* arriving at ♋, that same Tract will be altogether in Darkness, and the Beams of the *Sun* cannot reach it. But the opposite Space withing the Circle F G, will be in a continual Illumination, and at the *Pole* A there will be no Night for the Space of six Months.

The Appearances when the Earth *is in* Cancer.

HERE likewise of the Parallels between the Æquator and the *North Pole*, the illuminated Portions are much less than the Portions which remain in Darkness; the contrary of which happened in the former Position. So likewise the *Sun* at Midday will appear vertical, or directly over Head to all the Inhabitants that live in the Tropick M N, so that it will appear to have descended towards the *South* from the Parallel T C to the Parallel M N,

Lecture VII.

Inhabitants towards each Pole have the Sun 47 Degrees nearer their Vertex at one time of the Year than another.

through the Arch CQN, which is 47 Degrees. Therefore the Inhabitants of all Places of the *Earth* that are beyond the Tropicks, towards either of the *Poles*, have the *Sun* in their Summer 47 whole Degrees nearer to their Vertex or to the Point directly over their Heads, than in the opposite Time of Winter. This Change of Situation in respect to the *Snn*, does not arise because the *Earth* is raised or depressed, but on the contrary, because it is no where depressed, and no where raised; but with its *Axis* keeps the same immutable Position, in respect of the Universe, only going round the *Sun* which is placed in the Center of its Orbit, and the *Axis* thereof retaining the same Inclination to the Plane of the Orbit, and the same Situation in respect to any other fixed Line.

How all these Appearances may be represented to the Eye.

All we have here said will appear evident to our Eyes, if we light a Candle in a dark Room, and take a small Globe of two or three Inches Diameter, in which we must mark the *Poles*, the Æquator, some Parallels, and some Meridians, or Circles passing from *Pole* to *Pole*: Then we must so hold this Globe before the Candle, that its *Axis* may not be perpendicular to the Plane of the Table on which the Candle stands; but let it be inclined to it, in an Angle nearly of 66½ Degrees: Then place the Globe in such a Manner, that one of its *Poles* may point directly *Northward*; and let the Light of the Candle first reach from *Pole* to *Pole*; that is, let the Circle bounding Light and Shadow first pass through the two *Poles* of the Globe: Then let the Position of the *Axis* be well observed, and then move the Globe round the Candle with your Hand, in a Circle parallel to the *Horizon*, holding it so that the *Axis* may always point the same Way, and retain the same Inclination to the *Horizon*. This done, you will see that the Flame of the Candle will in the same Manner illuminate this Globe, as the *Sun* actually does the *Earth*: and the *Poles* of the Globe, its Æquator and Parallels, will undergo the same

ſame Viciſſitudes of Light and Darkneſs, which we have now explained.

Lecture VII.

The like Appearances from any other Planet that turns round its Axis.

THE like *Phænomena* or Appearances may be obſerved from any other *Planet* that turns round its *Axis*. For Example: *Jupiter* performs his Gyration in the Space of ten Hours; and therefore a *Jovian*, or an Inhabitant of *Jupiter*, will ſee the whole Heavens, and even our *Earth* together with the *Sun*, to have a rapid Motion round his Body in the Space of of ten Hours: But the *Axis* of *Jupiter* is very nearly perpendicular to the Plane of his Orbit, and therefore the Circle bounding Light and Darkneſs in *Jupiter*, does always nearly paſs through his *Poles*; and therefore the Days and Nights in the *Planet* are almoſt conſtantly equal. Hence it ſeems that *Jovians* enjoin an uniform temperate Seaſon, without being uneaſy at the approaching Heats of the Summer, or the Colds of the Winter.

The Axis of the Ecliptick.

IF through the Center of the *Sun* or *Earth* (it is no Matter which, for theſe two Points at the Diſtance of the *Stars* will ſeem to coincide) there be raiſed a Line which is perpendicular to the Plane of the Ecliptick, and this Line be produced to the Heavens, it is called the *Axis* of the Ecliptick; and the two Points, which this Line on both Sides produced, does tend to in the Heavens, are called the *Poles* of the Heavens. Now if we imagine great Circles to paſs through theſe *Poles*, and by every *Star* or *Planet*, they will all be perpendicular to the Plane of the Ecliptick. Theſe Circles are called *Secondaries* of the *Ecliptick*, or *Circles* of *Longitude*. And an Arch of one of theſe Circles intercepted between any *Star* and the Ecliptick, is called the *Latitude* of that *Star*, or its Diſtance from the Ecliptick; which may be either *North* or *South*, according as the *Star* is upon the *North* or *South* Side of the Ecliptick. So alſo an Arch of the Ecliptick between the firſt Point of ♈, or its Interſection there with the Æquator, and the Point were the Circle of Longitude paſſing through a *Star*, cuts the Ecliptick, is called the *Longitude* of that *Star*.

The Poles.

The Secondaries of the Ecliptick.

The Latitude of a Star.

The Longitude of a Star.

AFTER

After the same Manner if there be conceived innumerable Circles to pass through the two *Poles* of the *Earth*, and through each Place on its Surface, they will all be perpendicular to the Æquator, and they are called Secondaries of the Æquator: But in respect of the Places through which they pass, they are called *Meridians*; because when the *Sun* is seen in any Place, in the Plane of such Circle, it will be Mid-day to the Inhabitants of that Place. The Arch of one of these Secondaries intercepted between any Place and the Æquator, is called the *Latitude* of the Place, or its Distance from the Æquator, which may be likewise either *North* or *South*: And that Arch of the Æquator, that lies between the Intersection of the secondary passing through any Place, with the Æquator, and any other fixed Point in the Æquator, is called the *Longitude* of that Place.

The Meridians.

The Latitude of a Place.

Its Longitude

LECTURE

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Plate III front. p. 74.

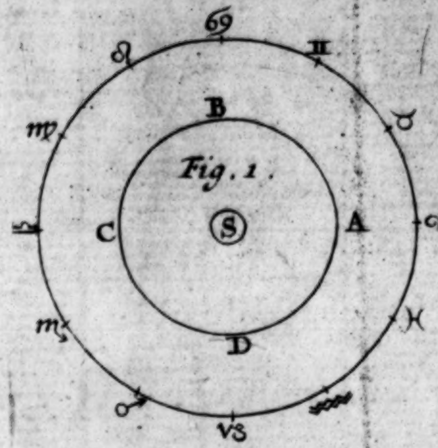

Fig. 1.

Fig. 2.

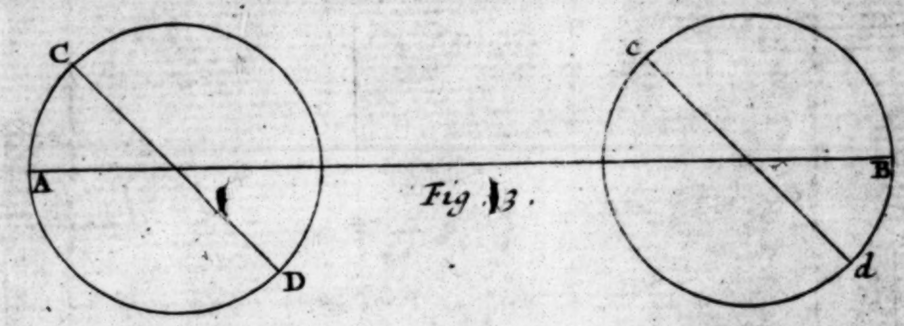

Fig. 3.

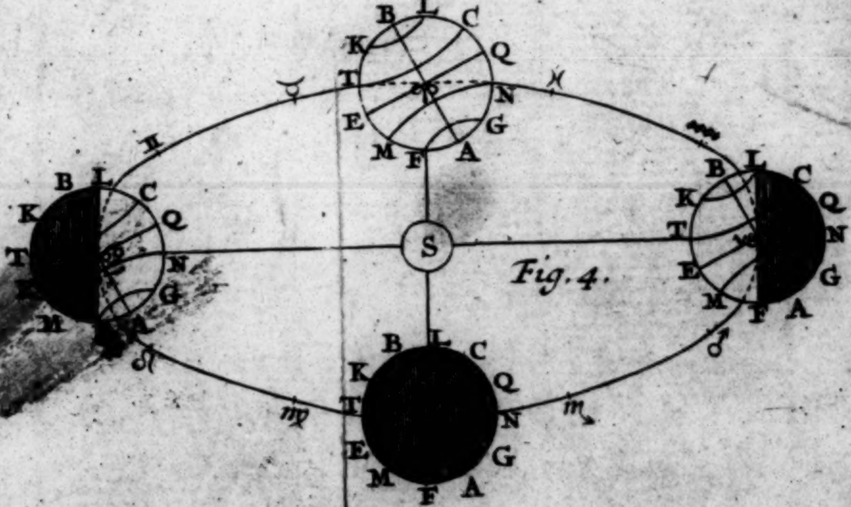

Fig. 4.

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LECTURE VIII.

Concerning several other Phænomena *or Appearances, which depend on the Motion of the* Earth.

SINCE the *Earth* turns so round the *Sun*, that its *Axis* always remains parallel to itself, it seems necessary that this *Axis*, at different Seasons of the Year should point to different *fixed Stars*, and that the *Star* or Point of the Heavens, which is directly over the *Pole* of the *Earth* in the Summer, should not be so directly over it in the Winter; but that the *Axis* should point to another *Star*, whose Distance from the former should be equal to the whole Diameter of the *Earth*'s Orbit.

The Axis *of the* Earth *ought to point to different* Stars *at different Times of the Year.*

FOR let ACBD be the Orbit of the *Earth*, in whose Center is the *Sun* S; A B the Diameter of the Orbit: When the *Earth* is at A, its *Axis* is directed to a *Star* E, which is directly over the *Pole* of the *Earth*: Now when the *Earth* comes to the opposite Point of the Orbit B, the *Axis* being in a Position parallel to its former Position, it will no longer point to the *Star* E, but to another *Star* F, which two *Stars* will be distant from each other the whole Length of the Diameter of the great Orbit. But the angular or observable Distance of the *Stars* is the Angle E B F, which is equal to the Angle A E B, by the 29th Prop. 1st Book of *Euclid*. But the Angle A E B is the Angle under which the Diameter of the great Orbit, or Orbit of the *Earth*, is seen from the *Star* E, which Angle E B F or

Plate IV. Fig. I.

AEB

Lecture VIII.

The Parallax *of the great Orbit.*

AEB is called the *Parallax of the great Orbit*; and if it could be observed, we might by it find the Distance of the *Star* E, from the *Earth*, in respect of the *Sun's* Distance from us. For in the Triangle EAB, we have the Angle E, which is equal to the Angle EBF, which we suppose we can observe; we have likewise the Angle E A B, which in the *Equinoctial Points* is always a right Angle; and in the *Solstices* it is equal to the Inclination of the *Earth's* *Axis* to the Plane of the Ecliptick, and is always equal to the visible Distance of the *Sun* from the *Pole*: Hence in this Triangle we have all the Angles: we have likewise the Side A B, and consequently, by *Trigonometry*, we can find the Side AE, or EB, the Distance of the *Star* E from the *Earth*.

This Parallax *scarcely to be observed.*

BUT the Truth is, that the Distance of the *Stars* is so great in respect of A B, and the Angle EBF is so very small, that there can be no Instruments made nice enough to observe it exactly; and they who have taken most Pains to find it out, could never observe it to be so great as one Minute: And since in the Observation of such small Angles, Errors are scarcely to be avoided, and such too as will in the Computation produce prodigious Differences in the Distances which depend upon them, we cannot safely trust such Observations: For if, with Mr. *Flamsteed*, we should suppose the Parallax, or the Angle EBF, to be 42 Seconds, and there be an Error committed in Observation, which makes the Angle 25 Seconds greater than it really is, (and no Man can be sure that he has not committed such an Error) the Distance of the *fixed Stars*, in that Case, will really be double of what our Observation makes it. But if the Observations happen to be less accurate, so that there may be a Minute or more between them and the Truth, (and most of our Observations are such) the Distance that arises from the Computations made upon such Observations, will be prodigiously wide of one another, and all of them very different from the Truth.

The Distance of the Stars *uncertain.*

HITHERTO

HITHERTO we have supposed the *Axis* of the *Earth* to have remained in an immutable Position, and to have continued in an exact Parallelism, and that the *Earth* had only two Motions, one Annual round the *Sun*, the other Diurnal round its *Axis*. But the *Astronomers*, from the Observations of many Years, have found that the *Axis* of the *Earth* has not exactly kept its Parallelism, but has deviated a little from that Position; so that though the Variation in the Space of two or three Years be scarcely sensible, yet in many Years, or in a Century or two, it is very observable: And therefore, while we were explaining the Appearances of one Year, we spoke nothing of this *Aberration*; for that could no Ways disturb the *Phænomena* that were then to be explained; yet in the Compass of several Years this Mutation or Change of the Position of the *Earth*'s *Axis* becomes very remarkable. So the Direction of the *Axis* has been sensibly changed, though its Inclination to the Plane of the Ecliptick has remained the same; and from hence we find, that the *Axis* of the *Earth* has another Motion, which is here to be explained.

Lecture VIII.

The Axis *of the* Earth *does not preserve an exact Parallelism.*

LET the Line DCH represent a Portion of the *Earth*'s Orbit, and let the Center of the *Earth* be C; from which erect C E perpendicular to the Plane of the Ecliptick, meeting with the concave Surface of the Heavens in E: This Line C E will be the *Axis* of the Ecliptick, and E the *Pole* of it. Let C P be the *Axis* of the *Earth* produced to the Heavens; P will be the *Pole* round which the Heavens have an apparent diurnal Revolution. Through the two Points E and P, draw a great Circle E P A, which passing through the *Poles* of both the Ecliptick and Æquator, will be perpendicular to both those Circles. Let it meet with the Ecliptick in A; the Arch P A will measure the Angle P C H, which is the Inclination of the *Axis* of the *Earth* to the Plane of the Ecliptick; that is, it will be $66\frac{1}{2}$ Degrees; and therefore the Arch E P, which is its Complement to a Quadrant,

Plate IV. Fig. 2.

The Axis *of the* E*cliptick.*

Lecture VIII.

The Pole of the World moves backward in a lesser Circle parallel to the Ecliptick.

drant, will be 23½ Degrees; which Arch will measure the Angle ECP, that the *Axis* of the Ecliptick and the Æquator make with one another. From the *Pole* E, describe through P a lesser Circle P F G, which will be parallel to the Ecliptick; and since the *Axis* of the *Earth* always keeps the same invariable Angle with the *Axis* of the Eliptick, it will always be directed to some Point in the Periphery P F G, and the *Pole* of the World must always be somewhere placed in it: So likewise, if the *Axis* of the *Earth* retained the same Direction without any Change, as often as the *Earth* came to the Point of its Oıbit C, the Pole of the Heavens would be constantly in the indivisible Point P: But we find, that the *Pole* of the World does constantly change its Place in the Periphery PF G; and the *Axis* of the *Earth*, which before pointed to P, after 72 Years will look to another Point Q, which is one Degree from P towards the *West*. And by this Means, the *Axis* of the *Earth*, or of the World, is carried in a *Conical Motion*, or describes the Surface of a *Cone*, whose Vertex is in the Center of the *Earth*, and its *Base* is the Circle P F G; and the *Pole* P will constantly move in the Periphery P F G, with a very slow and retrogade Motion, from the *East* to the *West*, and does not finish its Circulation in less than 25920 Years; after which Time the *Pole*, having left the *Star* at P, does again return thither. Hence it follows, that the *Star* which is now the *Polar*, and directly over the *Pole* of the *Earth*, after 12960 Years, which is half the Period of the *Polar* Revolution, will be 47 Degrees distant from the *Pole*, which will then be directed to G.

The Solstitial Colure.

THE Circle E P A being perpendicular to both the Ecliptick and the Æquator, will be the *Solstitial Colure*, and A will be the *Solstitial Point*, which Point of the Ecliptick is most distant from the Æquator: Now, after the *Axis* of the *Earth* produced comes into the Position C Q, if there be drawn through the Poles of the Ecliptick and the Æquator

the

the Circle E Q B, this Circle will then be perpendicular to both Æquator and Ecliptick; and therefore, when the *Axis* of the *Earth* is in the Position C Q, the Circle E Q B will be the *Solstitial Colure*, and the *Solstitial* Point will be B, where that Circle intersects the Ecliptick; and therefore the *Solstitial* Points will move backward equally with the *Poles*; the Motion of the *Pole* being in the Circle P Q G, which is parallel to the Ecliptick, the Arches P Q and A B will be like or similar; so that when P Q is an Arch of one Degree, A B will likewise be an Arch of one Degree.

Lecture VIII.

The Solstitial *Points move backward.*

Hence the *Solstitial* Points will always be receding from the *Stars* backward, so that if the *Solstitial* Point be this Day near the *Star* A, after 72 Years, it will be in B, one Degree removed towards the *West* of the *Star* A. And since the *Solstitial* Points move constantly backward, the Æquinoctial Points, which are always 90 Degrees distant from the *Solstices*, will also move constantly backward; and so likewise must all the other Points of the Ecliptick necessarily move back, equally with the *Solstices*, because they keep constantly the same Distances from them. Thus, since between the *Solstice*, and the Intersection of the Æquator and the Ecliptick, there are 90 Degrees, or a Quadrant of a Circle, when the *Solstice* has moved one Degree *Westward*; the Æquinoctial Intersection must likewise move one Degree *Westward*; otherwise they could not always keep the same Distance from one another. Therefore the Æquinoctial Points, and all the other Points of the Ecliptick, do move continually backward, or towards the *West*. And this Motion is said to be *in Antecedentia*, to the *Westward*, and contrary to the Order of the Signs: As the other Motion, whereby the *Earth* and all the Planets are carried round the *Sun* to the *Eastward*, is said to be *in Consequentia*, in Consequence, or according to the Order of the Signs, that is from ♈, ♉, ♊, *&c.* And this backward

The Æquinoctial Points move likewise backward.

The Motions in Antecedentia.

The Motions in Consequencia.

Lecture VIII.

ward Motion of the Æquinoctial Points is called the *Preceſſion* of the Æquinoxes, by which they are carried conſtantly back unto the preceding Signs or *Stars*, and fall more and more behind the ſucceeding *Stars*.

Preceſſion of the Æquinoxes.

SINCE the *fixed Stars* remain immoveable, and the common Interſection of the Equator and the Eccliptick conſtantly falls backward, it muſt neceſſarily happen, that the Diſtance of the *Stars* from the Æquinoctial Points be conſtantly changed, and the Interſections moving *Weſtward*, the *Stars* will ſeem to remove more and more *Eaſtward* in reſpect of the Æquinoctial Points: And therefore the Longitudes of the *Stars* which are computed from the firſt Point of ♈, or the vernal Interſection of the Æquator and Ecliptick, muſt conſtantly increaſe; and all the *Stars* will ſeem to have a Motion *Eaſtward*, not that they have really any ſuch Motion, but becauſe the Æquinoctial Point has a contrary Motion to the *Weſt*; ſo that the Diſtances of the *Stars* or their Longitudes from the firſt Point of ♈ reckoned *Eaſtward*, becomes conſtantly greater.

The Motion of the Æquinoctial Points backward makes the Stars *ſeem to move forward, or to the* Eaſt.

HENCE it is, that all the Conſtellations have changed their Places, and have deſerted the Stations they kept, when they were obſerved by the firſt *Aſtronomers*. Thus the Conſtellation of the *Ram*, which in *Hipparchus*'s Time was near the Vernal Interſection of the Æquator and Ecliptick, and gave its Name to that Portion of the Ecliptick, is now removed from that Interſection a whole Sign, or a twelfth Part towards the *Eaſt*, and is got into the Sign or Portion of the Ecliptic called ♉, or the *Bull*: Thus alſo the Conſtellation *Taurus*, or the *Bull*, does now reſide in *Gemini*, or the *Twins*; and the *Stars* which are called *Twins*, are at this Day advanced to ♋, or the *Crab*; the *Stars* in the *Crab* are got into the Place which was formerly poſſeſſed by the *Lion*, and the *Lion* has driven the *Virgin* a whole Sign forward; and ſo every Conſtel-

The Conſtellations have changed their Places.

Conſtellation has ſince the firſt Obſervation changed Place with the following. But here it is to be obſerved, that though the Conſtellations or Images have left their Places, yet the twelve Portions of the Ecliptick, which are called *Dodecatimoria*, retain ſtill the ſame Names which they had at firſt in the Time of *Hipparchus*: But to diſtinguiſh them from the Conſtellations, the Portions of the Ecliptick are called *Anaſtrous* Signs, or Signs without *Stars*; and the Conſtellations are called the *Starry* Signs.

Lecture VIII.

SOME antient *Aſtronomers* ſuppoſed the Interſections of the Æquator and Ecliptick to be immoveable; and becauſe they found that the *Stars* changed their Diſtances from theſe Interſections, they therefore imagined the Orb or Sphere in which the *fixed Stars* were placed, to have a ſlow Revolution about the *Poles* of the Ecliptick; ſo that all the *Stars* performed their Circulations in the Ecliptick, or its Parallels, in the Space of 25920 Years; after which Time the *Stars* would again return to their former Places. This Period of Time, which is five times greater than the Age of the World, they called the great Year; and imagined that when it was finiſhed, every Thing would begin again, and all Things happen and come up in the ſame Order they do now.

The great Year.

THE phyſical and efficient Cauſe of the Preceſſion of the Equinoxes, was unknown to all the *Aſtronomers* before Sir ISAAC NEWTON; none of them being able to gueſs from whence it did proceed. But Sir ISAAC NEWTON, having conſidered the Laws of Motion and Gravity, hath clearly demonſtrated, that it doth ariſe from the broad ſpheriodical Figure of the *Earth*: And that this broad ſperiodical Figure ariſes from the Rotation of the *Earth* round its *Axis*,

ALTHO' the *Earth* in its annual Motion does ſo go round the *Sun*, that it always performs its Period in equal Intervals of Time; yet its Motion in its Orbit is obſerved not to be equable and uniform,

The Motion of the Earth *in its Obit not equable.*

 but

Lecture VIII.

but in ſome Places it moves quicker, in other Places it ſlackens its Pace; and therefore the apparent Motion of the *Sun* in the Ecliptick cannot be regular and uniform; and he is not obſerved to go through the ſame Space of the Ecliptick every Day. In our Summer he is obſerved to go with a ſlower Motion, in our Winter he moves ſomewhat faſter; and the Difference of theſe Motions in Summer and Winter is ſuch, that his Place in the Ecliptick is ſometimes two Degrees above what it would be, if he had conſtantly kept the ſame Pace; and ſometimes it is two Degrees leſs: On which Account the *Sun* is obſerved to ſpend near eight Days more Time in the *Northern* Signs of the Ecliptick, than in the *Southern* Signs: ſo that from the Time of the *Sun*'s being in the *Vernal* Æquinoctie, till his coming into the *Autumnal*, there are 186½ Days; in which Time by his apparent Motion he is ſeen to deſcribe one half of the Ecliptick. But from the *Autumnal* Æquinoctie to the *Vernal*, there are only 178½ Days, in which Space of Time he finiſhes his Courſe through the other half of the Ecliptick, and viſits all the *Southern* Conſtellations. We are alſo aſſured by the Obſervations of *Aſtronomers*, that the apparent Diameter of the *Sun* in Winter, when the Motion of the *Sun* is quickeſt, is greater than the apparent Diameter in the Summer, when he ſlackens his Pace; and the Difference is ſo great, that when the *Sun* appears biggeſt, he is ſeen under an Angle of 32 Minutes 47 Seconds; but when he appears leaſt, he ſubtends an Angle only of 31 Minutes and 40 Seconds; and therefore the *Sun* muſt be farther from us in Summer than in Winter.

Our Summer is eight Days longer than Winter.

The apparent Diameter of the Sun *greater in Winter than Summer.*

SOME *Aſtronomers*, too pertinaciouſly keeping to circular Orbits, that they might give a ſatisfactory Account of theſe Appearances, ſuppoſed that the *Earth* did really move with an equable Motion in the Periphery of a Circle, and that if it were ſeen from the Center of that Circle, it would be obſerved to deſcrible equal Angles round it; but they ſuppoſed the *Sun* to be removed from that Center at ſome

some Distances. Let the Circle ABCD be the Orbit of the *Earth*, whose Center is E, and imagine the Center of the *Sun* not to be in E, but in S. Now when the *Earth* is in A, the *Sun* will be observed in the Point ♈; and when the *Earth* comes to B, the *Sun* will be observed in ♋: And again, the *Earth* being arrived at C, the *Sun* will appear in ♎; so that while the *Earth* describes the Arch A B C, which is more than a Semicircle, the *Sun* will appear to have gone through but one half of the Ecliptick; and the *Sun* will seem to have performed his Journey through other half, while the *Earth* is describing the other Portion of her Orbit ADC. Now since the Arch A B C is greater than the Arch ADC, it is easy to see, that the *Sun* must take more Time to describe, by its apparent Motion, that half of the Ecliptick ♈, ♋ ♎, than the other ♎, ♑, ♈. Moreover, when the *Earth* is in B, it is further distant from the *Sun* than when it is in D: And if its Motion were in itself perfectly equable, yet when it is seen from the *Sun*, which is not the Center of equable Motion, it would from thence appear to be unequal: In B it would appear to be slowest of all, and in D to be the quickest of all. But the apparent Motion of the *Sun* in the Ecliptick is constantly equal to the Motion of the *Earth* seen from the *Sun*; and therefore by this Supposition we can give an easy Account why in our Summer the *Sun* appears to have a slow Motion, and in the Winter a quicker; so that the unequal Motion of the *Sun* or *Earth* is not so in reality, but only Optical and apparent; arising only from this, that the *Sun* is not exactly in the Center of the *Earth*'s Orbit in E, but at some Distance from it at S: So they affirmed, that a Spectator in E would always observe, that the *Earth* had a most exact uniform Motion round that Center in its Orbit.

Lecture VIII. Plate IV. Fig. 3. *The Motion of the Earth in an Excentrick Circle.*

THIS *Hypothesis* appears at first Sight to be simple enough, and to answer well the Appearances we have related; and all the *Astronomers* before *Kepler* embraced it as a true *Hypothesis*. For they held it

Lecture VIII.

for an undoubted Truth, that all the Motions of the Heavens were exactly circular, and in themſelves equable: But after the great *Kepler* had more accurately ſurveyed theſe Motions, and relying upon the Obſervations of the moſt induſtrious *Tycho Brahe*, he then found that the circular *Hypotheſis* would by no Means anſwer to the true Motions of the *Planets*: And by a moſt certain and infallible Method of Reaſoning, he has ſhewn, that the Motions of the *Planets* are neither equable in themſelves, nor are their Orbits exact Circles. For by the Obſervations of *Tycho*, he has proved beyond all Diſpute, that the Figure of a planetary Orbit is an *Ellipſe*, which is deficient from a Circle, or of the Form of an Oval; and that the *Planets* Motion in this Ellipſe is really unequal, ſometimes quicker, and ſometimes ſlower; and that, according to its Diſtance from the *Sun*, the *Planet* ſlackens or quickens its Motion.

The true Motions of the Planets *are neither in Circles nor equable.*

The Elliptick Orbits of the Planets.

Plate IV. Fig. 4

The Deſcription of an Ellipſe.

Now the *Ellipſis* is a curved Line Figure, which the *Geometers* commonly ſhew by cutting a Cone or a Cylinder obliquely: But its Nature will be more clearly apprehended by Beginners from the following Deſcription: Imagine two ſmall round Sticks to be faſtened in any Plane or Paper, one in the Point H, the other in the Point G; and ſuppoſe a Thread doubled with the two Ends tied together, whoſe Length muſt be greater than the Diſtance of the Points G and H, which Thread put over the two round Sticks: And let there be a Pen put in the doubling of the Thread, which may keep it always ſtretched with the ſame Force. This Pen, going in this Manner round, will deſcribe by its Motion a curve Line, which is the *Ellipſe* we now ſpeak of. And if without changing the Length of the Thread, we ſhould bring the round Sticks a little cloſer together, we ſhall then have another Ellipſe of a different Kind from the former, and which will come nearer to a Circle: And by bringing them ſtill nearer, we ſhall always change the Form of our Ellipſe, and bring it nearer to a Circle, till the Sticks come to be joined together in one; and then the

Pen

Pen in the doubling of the Thread will describe an exact Circle. Either of the Points G or H is called the *Focus* or Nave of the Ellipsis; and if we bisect H G in C, the Point C is called its Center; the Line D K passing through each *Focus*, and at each End meeting with the Ellipse, is called its *Axis*: Hence it is evident, that if from any Point of the Ellipsis there be drawn to the two *Focus*'s, as for Example from B, two Lines B H and B G, these two Lines joined together will always be equal to the *Axis* of the Ellipsis, and likewise equal to the Length of the Thread, bating the Distance of the two *Focus*'s.

Lecture VIII.

The Focus *or* Umbilicus.

The Axis.

Now though this be the Form of the Orbit which the *Planets* describe, yet the Place of the *Sun* is not the Center of it, but he takes his Residence in one of the *Focus*'s: And the *Axis* of the Ellipse A P is called the Line of the *Apsides*; the Point A is termed the *higher Apsis*, and the *Aphelion*; the Point P is called the *lower Apsis*, and *Perihelion*: And S C the Distance between the *Sun* in the *Focus* and the Center, is called the *Excentricity*. If from the Center C, there be erected upon the *Axis* the Perpendicular C E, meeting with the Orbit in E, and there be drawn from the *Focus* the Line S E; this Line is called the *Mean Distance* of the *Planet* from the *Sun*, which is equal to half the *Axis*; it exceeding the shortest Distance by as much as the longest Distance exceeds it.

Plate IV. Fig. 5.

The higher Apsis *or* Aphelion.

The lower Apsis *or* Perihelion.

The Excentricity.

The Mean Distance.

In the planetary Orbits, the Forms of the Ellipses do not differ much from Circles; and in the Orbit of the *Earth*, the Excentricity S C is only 17 of such Parts as S E the mean Distance consists of 1000, which Excentricity is but half of that which the *Astronomers* that supposed circular Orbits, attributed to the Distance of the *Sun* from the Center.

The Motion of a *Planet* in the Periphery of an Ellipse, is not at all equable; yet it is regulated by a certain immutable Law, from which it never deviates

The Rule by which the Planets *Motions.*

Lecture VIII.

viates; which is, that a Line or Ray drawn from the Center of the *Sun* to the Center of the *Planet*, which is carried about with an angular Motion does so move, that it describes or sweeps an Elliptick *Area*, always proportional to the Time. Thus let the *Planet* be in A, from whence in a certain Time let it go to B; the Space or Area the Ray S A describes, is the Triline A S B: When afterwards the *Planet* comes to P, and from the Center of the *Sun* S, there be drawn the Line SD, so that the Elliptick Space or Area P S D, may be equal to the Area A S B; then in that Case, the *Planet* will move through the Arch P D, in the same Compass of Time that it did through the Arch A B, which Arches must be unequal, and nearly in a reciprocal Proportion to their Distances from the *Sun*; for because of the equal Area's, the Arch P D must be so much in Proportion greater than the Arch A B, as S A is greater than SP. This Law is sufficiently demonstrated by the most sagacious *Kepler*, in his Book which he intituled *Commentaries on the Motion of the Planet* Mars. And unto this his Invention, all the *Astronomers* do now give their Assent; for there is no other Rule to be found, which so well satisfies all the Appearances of the *Planets* Motions.

are regulated, is the equable Description of Elliptic Areas.

The Mean Anomaly.

An Arch of a Circle, or an Angle, or the Elliptick Area A S G, taken proportional to the Time in which the *Planet* descends from A to G, is called the *Mean Anomaly* of the *Planet*. But the Angle A S G, when the *Planet* comes from A to G, is called its *true Anomaly*. But when the Motion of the *Planet* is reckoned from the vernal Interfection of the Æquator and the Ecliptick, or from the Beginning of ♈, it is called its *Motion in Longitude*; which is either a mean Motion, such as the *Planet* would have, did it move uniformly in a Circle round the *Sun*, or else the true Motion wherewith the *Planet* describes its Orbit, and is reckoned by the Arch of the Ecliptick it is seen to describe; which

The true Anomaly.

The Motion in Longitude.

true

true Motion is ſometimes accelerated, and ſometimes retarded, according to the Diſtance of the *Planet* from the *Sun*, in the various Points of its Orbit.

Lecture VIII.

The Determination of a Planet's Place in its Orbit.

BY this Means, for any given Time after that the *Planet* has left its *Aphelion*, we find out its Place in its Orbit, *viz.* Let the Area of the Ellipſe be ſo divided by the Line SG, that the whole Elliptick Area may have the ſame Proportion to the Area ASG, as the whole periodical Time wherein the *Planet* deſcribes its Orbit, is to the Time given; and then G will be the Place of the *Planet* in its Orbit. The *Geometers* have given ſeveral Methods for dividing in this Manner the Area of an Ellipſe; ſome of which we will ſhew in its proper Place.

Why the Sun's Heat is greater, as he is further from us.

SINCE in our Summer we are further from the *Sun*, and when Winter comes on, we begin to approach him; ſome may wonder why the *Earth* grows warmer, while it is ſtill further removing from the *Sun*; and again in the Winter, why it ſhould be colder notwitſtanding its nearer Acceſs to him. But we muſt obſerve, that the Degrees of Heat and Cold do not altogether depend upon the Diſtances from the *Sun*; but there are other powerful and concurring Cauſes, which have certain Effects in this Matter: For, firſt of all, the direct Force of the Sun's Rays is much ſtronger, than when they are received obliquely: Now in the Winter the Rays fall upon the *Earth* very obliquely, and their Power is not only diminiſhed on the Account of their Obliqueneſs, but alſo becauſe the Light is not ſo denſe, there being much fewer Rays which can come to a certain Portion of the Surface to heat it. Moreover the *Sun* being low in the *Horizon* all the Winter, the Beams paſs through a much greater Quantity of Air, or are deeper immerſed in our Atmoſphere in the Winter, than they are in Summer, when the *Sun* approaches nearer to our Vextex, and the Force of the Rays is broke by the Reflections on ſo many Particles of Air: And the Difference is ſo very great, that when the *Sun* is in the *Horizon*, we can look upon him without hurting

Lecture VIII.

in our Eyes; but when he rises higher, there is no enduring his Sight without blinding us.

The Days longer than the Nights, increase the Heat.

But there is another very powerful Cause which produces the Variety of Seasons; which is, that the longer any hard and solid Body is exposed to the Fire, the hotter it grows. Now in the Summer for sixteen Hours we are continually in the *Sun*'s Heat, and we have only eight Hours in the Night to cool: The contrary of which happens in the Winter, and therefore it can be no Wonder, that there should be so great a Difference of Heat and Cold in these two Seasons.

Since the Power of the *Sun* is greatest when his Rays fall upon us most directly, and when the Days are longest; it would seem that the greatest Heat ought to be when the *Sun* enters the Tropick of ♋; for then the *Sun* comes nearest to our Vertex, and lieth longest upon us. But Experience shews us, that we have the greatest Heat after that the *Sun* has left the Tropick; and the Season becomes warmest about the End of *July*, in the *Dog-Days*, when the *Sun* has passed the Tropick, and is removed from it above a whole Sign.

That we may give the true Cause of this Effect, it is to be observed, that the Action of the *Sun*, by which all Bodies are heated, is not transient, as its Illumination is, but permanent: So that a Body which has been once heated by the *Sun*, retains its Heat for some Time after the *Sun* has gone off it: So that the heating Particles which flow from the *Sun*, and are absorbed by the heated Body, do for a certain Time remain within it, and do therein raise a Warmth or Heat. But afterwards, when these Particles fly off, or lose their Force, the Body begins to cool: And therefore, if the heating Particles, which are constantly received, be more than they which fly away, or lose their Force, the Heat of the Body must continually increase. And this is our present Case: After the *Sun* has entered the *Tropick*, the Number of Particles which heat our Atmosphere and *Earth* does constantly increase, there

Why the Heat is not greatest, when the Sun *is in the Summer* Tropick.

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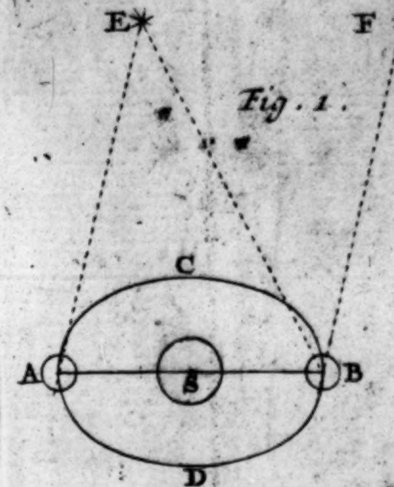

Fig. 1.

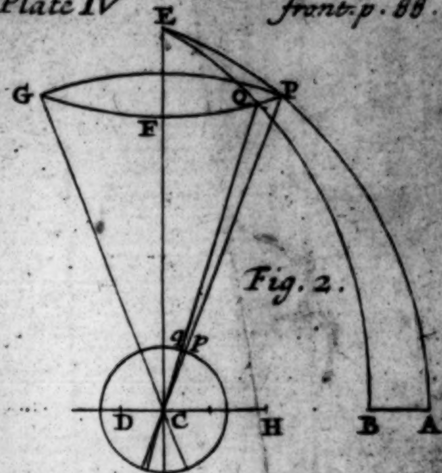

Fig. 2.

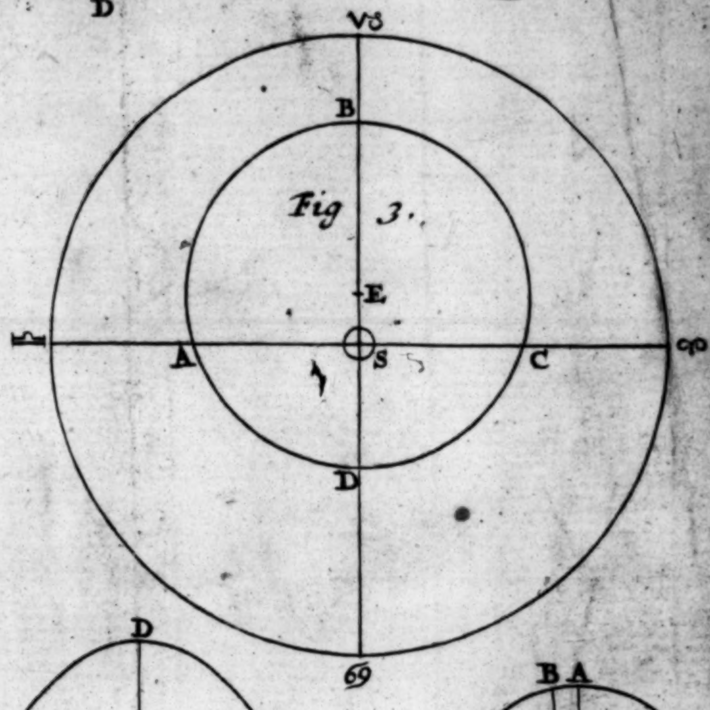

Fig 3.

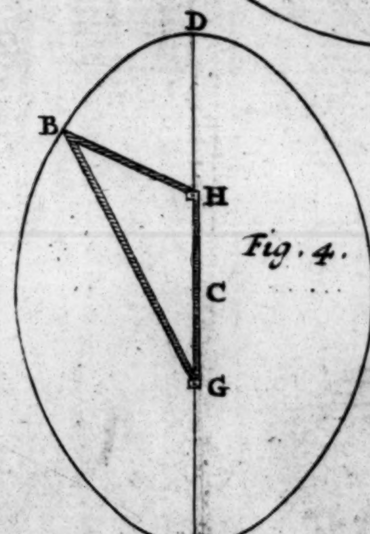

Fig. 4.

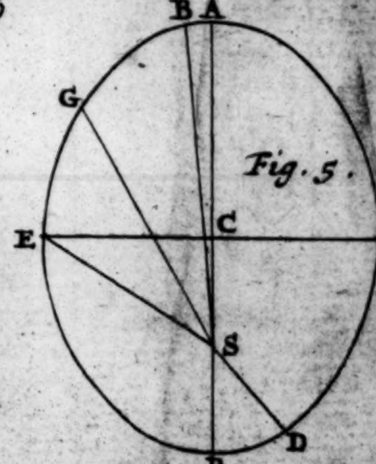

Fig. 5.

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entering more in the Day-time than what we loſe in the Night-time, and therefore our Heat muſt grow greater. Let us ſuppoſe, for Example's ſake, that there are a hundred heating Particles received in the Day-time in *Sun-ſhine*, and the Night being much ſhorter, there ſhould fly off only fifty of them, other fifty ſtill remaining there to excite Heat: The next Day, the *Sun* acting with almoſt the ſame Force, will impart another hundred Particles, of which no more than one half will fly away in the Night; ſo that on the Beginning of the third Day, the Number of Particles exciting Heat, will be increaſed by one Hundred: And thus, while there are more Particles that excite Heat received in the Day-time, than what fly away in the Night, the Heat will conſtantly grow ſtronger. But then as the Days decreaſe, and the Action of the *Sun* becomes weaker, there will at laſt be more Particles that fly away in the Night-time, than what we receive in the Day-time; by which Means the Heat of a Body will grow every Day leſs, and the *Earth* and Air will by Degrees cool.

Lecture VIII.

LECTURE

LECTURE IX.

Of the Moon, its Phases and Motion.

The Moon *is an Attendant on the* Earth.

OF all the Bodies in the Heavens, if you except the *Sun*, the *Moon* appears to be the most splendid and shining Globe, and does more particularly belong to our *Earth*, of which she is an inseparable Companion. And she does constantly abide so much in our Neighbourhood, that if she were looked at from the *Sun*, she could never be seen to depart from us by an Arch greater than ten Minutes. She therefore is tied to the *Earth* and waits upon her as an Attendant, going along with the *Earth* round the *Sun* in the Space of a Year; but in the mean Time she has a proper Orbit of her own, which she describes round the *Earth* in the Time of a Month.

It has various Shapes and Phases.

THE Primary *Planets* have the *Sun*, which they regard as a Center, for the Regulator of their Motions; and sometimes they approach us nearly, at other Times they move away to a great Distance from us. But the *Moon*, like an Earthly Body, is kept in our Neighbourhood by a natural Propension or Gravity towards us; by the Means of which it is constantly turned out of a rectilinear Course, and is obliged to perform its Revolution round about us, in the Space of 27 Days and seven Hours. The *Moon* puts on several Phases and Appearances, and is always changing its Figure; and with the Multitude of her Forms, she has frequently puzzled the Minds and Understandings of those *Philosophers*, who have most contemplated her: Sometimes she increases and grows

grows bigger, then again she wanes, and diminishes, as it were, in old Age; sometimes she is bended into Horns, and then again she appears like a half Circle; at other Times she looks gibbous or hump-backed, and immediately she assumes a full globular Face; and afterwards, by Degrees, she disappears and loses all her Lustre; sometimes she enlightens us the whole Night, at other Times she does not appear 'till late at Night: And, even in a total Eclipse, she is frequently visible, though with a very languid and pale Countenance: Sometimes she keeps in the *Southern* Region of the Heavens; at other Times, she rises high, and visits the *Northern* Hemisphere. All these Things were first found out by *Endymion* among the *Greeks*, who was the first among them who watched her Motions; and upon that Account, was supposed to have fallen in Love with her.

Lecture IX.

The Moon *is a Spherical Opake Body.*

THE *Moon*, like the *Earth*, is a dark, opake, and spherical Body; and only shines with the borrowed Light of the *Sun*: For it is the *Sun* who is the great Luminary in our System, and who always illustrates that Half of the *Moon*'s Body, which is turned towards him; whilst the other Half, which is opposite, is involved in Darkness: But the Face of the *Moon*, that can be seen by the Inhabitants of the *Earth*, is that which is turned toward the *Earth*: And therefore, according to the various Position of the *Moon*, in respect of the *Sun* and *Earth*, we do observe different Illuminations and Degrees of Illustration; at one Time a larger, at another a lesser Portion of the illuminated Surface is to be seen; sometimes there is no Part of it visible, and sometimes we observe the Whole, and see the *Moon* with her full Face. But for the better Understanding of this Matter, we will explain it by a Figure. Let S represent the *Sun*, T the *Earth*, R T S a Portion of the *Earth*'s Orbit, which it describes in its Annual Course round the *Sun*. Let A B C D E F G H be the Orbit of the

Plate V. Fig. 1, 2.

Lecture IX.

the *Moon*, in which she turns round the *Earth* in the Space of a Month, from the *West* towards the *East*.

The true Motion of the Moon *from* West *to* East.

THIS Motion of the *Moon* is evident to our Senses; for if the *Moon* be observed to arrive at the Miridian any Night with a *fixed Star*, the next Night she will be 52 Minutes later in coming to the Meridian, or in *Southing*, than the *Star*; she having receded from the *Star* about 13 Degrees towards the *East*. Join the Centers of the *Sun* and *Moon*, with the right Line S L; and through the Center of the *Moon*, imagine a Plane M L N to pass, to which the Line S L is perpendicular: The Section of that Plane, with the Surface of the *Moon*, will produce the Circle which bounds Light and Darkness in her, and seperates the inlightened Face, from the dark and obscure Side. In the same Manner, let the Centers of the *Earth* and *Moon* be joined by the right Line T L, which is perpendicular to a Plane P L O, passing through the Center of the *Moon*; that Plane will make, on the Surface of the *Moon*, the Circle which distinguisheth the visible Hemisphere, or that which is towards us, from the invisible, which is turned from us; which Circle may therefore be termed the *Circle of Vision*.

The Circle in the Moon *bounding Light and Darkness.*

The Circle of Vision.

The Phases of the Moon *explained*,

HENCE it is manifest, that whenever the *Moon* is in the Position A, in the Point of its Orbit opposite to the *Sun*, that then the Circle bounding Light and Darkness, and the Circle of Vision do coincide; and that all the illuminated Face of the *Moon* will be turned towards the *Earth*, and be visible by its Inhabitants: And then the *Moon* is said to be full, and she shines all Night long; and in respect to the *Sun* she is said to be in *Opposition*: For the *Sun* and *Moon* are seen in opposite Parts of the Heavens, the one rising when the other sets. When the *Moon* comes to B, the whole illuminated Disk M P N is not turned towards the *Earth*, there being a Part of it M P not to be seen by us; and

then

then the *Visible Illumination* will be deficient from a Circle, and the *Moon* will have a gibbous or humped Form, ſuch as is marked in B. The *Moon* arriving at C, where the Angle C T S is nearly right, there only one half of the illuminated Diſk is turned towards the *Earth*, and to be ſeen from thence; and then we obſerve a *Half-Moon* as in C, and ſhe is ſaid then to be *Biſected* or *Dichotomized*; that is cut in Halfs. In this Situation the *Sun* and *Moon* are a fourth Part of a Circle removed from each other; and the *Moon* is ſaid to be in a *Quadrate Aſpect*, or to be in her *Quadrature*. The *Moon* going forward to D, the illuminated Face MPN has but a ſmall Portion of itſelf turned towards the *Earth*, and the Side of the *Moon* turned towards the *Earth* is for the greateſt part in Darkneſs: And therefore of the ſpherical Figure of the *Moon* which appears to us to be plain, that ſmall Part which ſhines upon us, will ſeem to be bended into narrow Points or Angles, and will look like what we call Horns; for there the Circle bounding Light and Darkneſs with the Circle of Viſion, doth form two ſmall Angles at their Interſections, and the *Phaſis* ſeen from the *Earth* will appear as in D. The *Moon* at laſt coming to E, will ſhew no Part of its illuminated Face to the *Earth*, but all the dark Side of the *Moon* will be turned towards it; and then the *Moon* diſappears, and ſhe is ſaid to be in *Conjunction* with the *Sun*, the *Sun* and ſhe being in the ſame Point of the Ecliptick. This Poſition we call *New Moon*. When the *Moon* advances further to F, ſhe again aſſumes a horned or crooked Figure; and as before the *New Moon* the Horns were turned *Weſtward*, ſo now, after the Time of *New Moon*, they change their Poſitions and look *Eaſtward*. When the *Moon* has proceeded to G, and is again in a *Quadrate Aſpect* with the *Sun*, ſhe will appear biſected, and like a *Half-Moon*. In H ſhe will be bigger, but will ſtill be deficient from a whole Circle, and be ſeen gibbous: But in

Lecture IX.

The Gibbous Figure.

Half Moon, *or the* Moon *Dichotomized.*

The Quadrature.

New Moon *or the* Conjunction.

Lecture IX.

The Elongation *of the* Moon *from the* Sun.

in A she will again appear circular, and in her full Splendor.

THE Arch EL, or the Angle STL, contained under Lines drawn from the Centers of the *Sun* and *Moon* to the Center of the *Earth*, is called the *Elongation* of the *Moon* from the *Sun*. And the Arch MO, which is that Portion of the illuminated Circle MON, which is turned towards the *Earth*, and which is the Measure of the Angle that the Circle bounding Light and Darkness, and the Circle of Vision, make with one another, is every-where nearly similar to the Arch of Elongation EL; or, which is the same Thing, the Angle STL is nearly equal to the Angle MLO, which I thus demonstrate: Produce SL at Pleasure unto X, and the Angles TLP and MLS will be equal, they being both Right Angles: But the Angles OLS and PLX are also equal, because they are vertical to each other; therefore taking away those equal Angles, the Angle MLO will remain equal to the Angle TLX; but the Angle TLX is the external Angle of the Triangle STL, and is therefore equal to both the inward and opposite Angles STL and TSL, by the 32d Proposition of Book I. of *Euclid*. But the Angle TSL is exceeding small, and next to nothing; for, when biggest in the Quadratures, it does scarce exceed ten Minutes of a Degree; the Distance of the *Moon* from the *Earth*, in comparison of that of the *Sun*, being so small, that the Angle which it subtends at the *Sun* vanishes. And therefore the Angle STL by itself, is nearly equal to the Angle MLO; whence the Arch MO will be similar or like to the Arch EL.

THE Semicircle OMP, since its Plane passes through the Eye, will be projected into a Right Line, or appear like a Right Line on the Disk of the *Moon*; but the Circle bounding Light and Darkness in the *Moon*, since it is seen obliquely from the *Earth*, will be projected into an Ellipse, in which Form it will appear. Hence having the Elon-

Elongation of the *Moon* from the *Sun*, it will be an easy Matter to shew its *Phasis*, or how it happens at that Time. Let the Circle COBP represent the Disk of the *Moon*, which is turned towards the *Earth*; and let OP be the Line in which the Semicircle OMP is projected, which suppose to be cut by the Diameter BC, at Right Angles; and making LP the Radius, take LF equal to the Cosine of the Elongation of the *Moon* from the *Sun*: And then upon BC, as the great *Axis*, and LF the lesser *Axis*, describe the Semi-Ellipse BFC. This Ellipse will cut off from the Disk of the *Moon* the Portion BFCP of the illuminated Face, which is visible to us from the *Earth*.

Lecture IX.

A Delineation of the Phasis of the Moon, *for any* Elongation. Plate V. Fig. 3, 4, 5.

By making LP the Radius, LF becomes the Cosine of the Elongation of the *Moon* from the *Sun*; PF, in that Case, must be the versed Sine of the said Elongation; and BFC (the Line which divides the illuminated and dark Parts of the Disk) will be an Ellipse, whose greater *Axis* is the Diameter of the Disk BE, and half the lesser *Axis* is the Semidiameter of the same Disk, diminished by the versed Sine of the Elongation. Suppose now that OBPC were the Disk of the *Moon* turned towards the *Earth*, and BFC the Semi-Ellipse dividing Light and Shadow: Draw any Line GHN parallel to the lesser *Axis*, and which meets with the greater *Axis* in M, by the Nature of the Circle and the Ellipse, LP will be to LF, as GM is to MH; and by Division of the Ratio, LP is to PF, as GM is to HG; and doubling of the Antecedents, PO will be to PF, as GN is to GH. The same Thing may be shewn of any other Line, which is parallel to the lesser *Axis*; and therefore by the 12th Prop. Book V. of *Euclid*, as PO is to PF, so will all the Lines GN be to all the Lines GH. But all the Lines GN compose or make up the whole *Lunar* Disk, it consisting of an infinite Number of Parallelograms, whose Heights are the Lines GN, and whose Bases are

The Quantity of Illustration.

Lecture IX. are indifinitely little: So likewise all the Lines GH make up that Part of the Disk which is illuminated.

And therefore, as PO is to PF, that is, as the Diameter of a Circle is to the versed Sine of the *Moon*'s Elongation from the *Sun*; so is the whole Disk of the *Moon*, to that Part of it which is illuminated by the *Sun*. And hence the Illustration of the *Moon*, at any Time, is to its greatest Illustration, which is at *Full Moon*, as the versed Sine of the Elongation is to the Diameter of a Circle.

The Earth *illuminates the* Moon *by a Reflex Light.*

As the *Moon* by reflected Light from the *Sun* illuminates the *Earth*, so the *Earth* does more than repay her Kindness, in enlightening the Surface of the *Moon*, by the *Sun*'s reflex Light, which she diffuses more abundantly upon the *Moon*, than the *Moon* does upon us: For the Surface of the *Earth* is above fifteen Times greater than that of the *Moon*; and therefore, if both Bodies have the same Power of reflecting in Proportion to their Bigness, the *Earth* would send back fifteen Times more Light to the *Moon* than it receives from it. For the *Earth* appears fifteen Times bigger to the Inhabitants of the *Moon*, than the *Moon* does to us. In *New Moons* the illustrated Side of the *Earth* is fully turned towards the *Moon*, and will, therefore, at that Time, illuminate the dark Side of the *Moon*; and then the *Lunarians* will have a *Full Earth*, as we, in a similar Position have a *Full Moon*. And from thence arises that dim Light which is observed in the *Old* and *New Moons*, whereby, besides the bright and shining Horns, we can perceive the rest of her Body behind them, though but dark and obscure. Now, when the *Moon* comes to be in *Opposition* to the *Sun*, the *Earth* seen from the *Moon*, will appear in *Conjunction* with him, and its dark Side will be turned towards the *Moon*, in which Position the *Earth* will disappear; after the same Manner as the *Moon* does disappear to us in the Time of *New Moons*, or in her *Conjunction* with the

the *Sun*. After this the *Earth* will appear to the Inhabitants of the *Moon* in a horned Form. In a Word, the *Earth* will ſhew all the ſame Appearances to the Inhabitants of the *Moon*, as the *Moon* does to us.

Lecture IX.

ALTHOUGH the *Moon* circulating round the *Earth* deſcribes its Orbit in the Space of twenty-ſeven Days and ſeven Hours, which Space of Time is called a periodical Month; yet the Time the *Moon* takes to go from one Conjunction to the next, which is a ſynodical Month or a *Lunation*, is greater than the periodical. For while the *Moon* in its proper Orbit finiſhes its Courſe, the *Earth*, with this her Companion and its Orbit are going on their Way round the *Sun*, and are advanced almoſt a whole Sign towards the *Eaſt*; ſo that the Point of the Orbit, which in the former Poſition was placed in a right Line joining the Centers of the *Earth* and *Sun*, is now more *Weſterly* than the *Sun*: And therefore, when the *Moon* has again arrived to that Point, it will not yet be ſeen in Conjunction with the *Sun*.

The periodical Month, and the ſynodical.

FOR let A B repreſent a Portion of the Orbit of the *Earth*, and when the *Earth* is in T, ſuppoſe the *Moon* in L, in Conjunction with the *Sun* in S: While the *Moon* leaves the Point L, and proceeds in deſcribing its Orbit L A C D; the *Earth* in the mean Time, by its Motion round the *Sun*, is carried through the Arch T *t*; and when it is come to *t*, the Orbit of the *Moon* is in the Poſirion *l a c d*, and the Point of the Orbit L will be now in the Line *t l*, which is parallel to the former Line T L. Hence it is plain, that when the *Moon* has come to *l*, and deſcribed its whole Orbit, it is not then arrived at a Conjunction with the *Sun*; but it muſt ſtill go further, and move through the Arch *l* M, before it can get between the *Earth* and the *Sun*: And ſince the *Moon* finiſhes her Courſe in the Space of twenty-ſeven Days, in that Time the *Earth* will have completed an Arch of twenty-ſeven Degrees in the Ecliptick; now the Arch *l* M and T *t* are alike or ſimilar, becauſe the Lines

Plate V. Fig. 6.

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Lecture IX.

Lines L T and *l t*, being parallel, the Angles *l t* M and L S M are equal. But indeed it is required, that the *Moon* ſhould deſcribe a greater Arch than *l* M, before it gets between us and the *Sun*, becauſe the *Earth* is ſtill moving in the mean Time: And therefore the whole *Lunation*, or Time from *New Moon* to *New Moon*, is not finiſhed but in the Space of 29 Days and a half; and the *Moon* does every Day recede from the *Sun* about twelve Degrees and ſome odd Minutes, which is called the diurnal Motion of the *Moon* from the *Sun*.

The diurnal Motion of the Moon *from the* Sun.

If the Plane of the *Moon*'s Orbit coincided with the Plane of the Ecliptick, that is, if the *Earth* and *Moon* moved both in the ſame Plane, the Way of the *Moon* in the Heavens ſeen from the *Earth*, would be exactly the ſame with the Circle the *Sun* is ſeen to deſcribe; only the *Sun* would be obſerved to deſcribe that Circle in the Space of a Year, which the *Moon* does in a Month. Now, in reality, the Plane in which lies the *Moon*'s Orbit, is not coincident with the Plane of the Ecliptick; but theſe two Planes cut one another in a right Line, which paſſes through the Center of the *Earth*; and they are inclined to one another in an Angle of about five Degrees.

Plate V. Fig. 7.

The Moon *does not move in the Ecliptick.*

Let A B be a Portion of the *Earth*'s Orbit, T the *Earth*, and let the Circle C E D F repreſent the Orbit of the *Moon*, in which is the Center of the *Earth*; with the ſame Center T, in the Plane of the Ecliptick, let there be deſcribed another Circle CGDH, whoſe Semidiameter may be equal to the Semidiameter of the *Moon*'s Orbit; theſe two Circles being in different Planes, and having the ſame Center T, will interſect each other in a Line D C, which paſſes through the Center of the *Earth*; and C E D, one half of the Orbit of the *Moon*, will riſe above the Plane of the Circle CGH, towards the *North*. The other half of the Orbit D F C will be depreſſed below it, towards the *South*. The right Line D C, wherein the two Circles cut one another,

another is called the Line of the *Nodes*; and the Points of the Angle C and D are called the *Nodes*. And the *Node* C, where the *Moon* ascends *Northward* above the Plane of the Ecliptick, is called the *ascending Node*, and the *Head of the Dragon*, and is thus marked ☊. The other D, from whence the *Moon* descends to the *South*, is named the *descending Node*, and the *Tail of the Dragon*, which by the *Astronomers* is marked in this Manner ☋. If the Line of the *Nodes* were immoveable; that is, if it had no other Motion, than that whereby it is carried round the *Sun*, it would always look to the same Point of the Ecliptick, that is, it would always keep parallel to itself, as we shewed the *Axis* of the *Earth* ought to do: But we find by Observation, that this Line of the *Nodes* does constantly change its Place, and shifts its Situation from *East* to *West*, contrary to the Order of the Signs; and by a retrograde Motion finishes its Circulation in the Compass of alost nineteen Years: After which Time either of the *Nodes*, having receded from any Point of the Ecliptick, returns to the same again. And when the *Moon* is in the *Node*, she is also seen in the Ecliptick.

Lecture IX.

The Line of Nodes.

The ascending Node, *or the* Dragon's *Head.*

The descending Node, *or the* Dragon's *Tail.*

The retrograde Motion of the Nodes.

HENCE it is evident, that the *Moon* can never be observed precisely in the Ecliptick, but twice in every Period, that is, when she enters the *Nodes*; when she is in any other Place of her Orbit, she deviates from it, and is sometimes nearer, sometimes further removed from the Ecliptick, according as she happens to be nearer, or further off from the *Nodes*: But she is at her greatest Distance from the *Nodes*, when she is in the Points of her Orbit E or F, which are the middle Points between the *Nodes*; and these Points are called the *Limits*. The Distance of the *Moon* from the Ecliptick is called her *Latitude*, which is measured by an Arch of a Circle drawn through the *Moon*, perpendicular to the Ecliptick; the Arch of this Circle, intercepted between the *Moon* and the Ecliptick, measures the *Moon*'s

The Limits.

The Moon's Latitude.

Lecture X.

Latitude, or her Distance from the Ecliptick: And therefore such Circles, perpendicular to the Ecliptick, are called Circles of *Latitude*; the *Latitude* of the *Moon*, when it is at the biggest, as in E or F, does never exceed five Degrees, and about eighteen Minutes, which *Latitude* is the Measure of the Angles at the *Nodes*.

LECTURE X.

Of the Inequalities in the Lunar *Motions. The Face of the* Moon, *her Mountains and Vallies.*

The Orbit of the Moon *an Ellipse.*

OBSERVATIONS have discovered to us that the Distance of the *Moon* from the *Earth* does constantly change; sometimes the *Moon* comes nearer to us, sometimes goes further from us; the Reason of which is, because the *Moon* does not move in a circular Orbit, which has the *Earth* for its Center: But the real Orbit of the *Moon* is of an Elliptick Form, such as is represented in the Figure A B P D, one of whose *Focus's* is always the Center of the *Earth*; A P is the greater *Axis* of the Ellipse, and the Line of the *Apsides*; T C is the Excentricity; the Point A, which is the highest *Apsis*, is called the *Apogeon* of the Moon; the lowest *Apsis*, which is the Point P, is called the *Perigeon*, in which the *Moon* comes nearest the *Earth*. And if the Orbit of the *Moon* had no other Motion besides that wherewith it is carried round

Plate VI. Fig. 1.

The Moon's Apogeon, *and* Perigeon.

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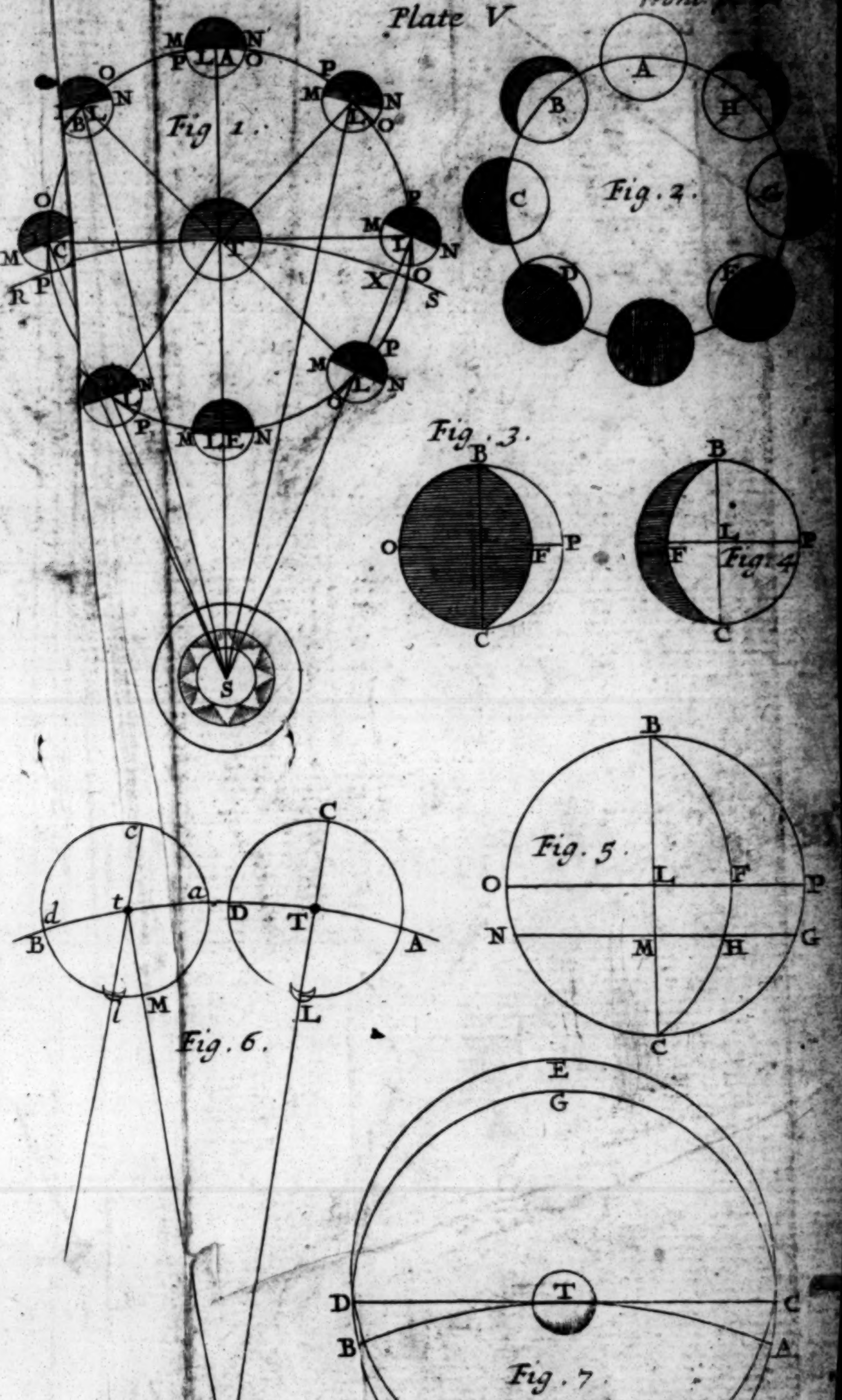
Plate V
front. p. [illegible]
Fig 1.
Fig. 2.
Fig. 3.
Fig. 4
Fig. 5
Fig. 6.
Fig. 7
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round the *Sun*, it would always retain a Position parallel to itself, and would always point the same Way, and be observed in the same Point of the Ecliptick; and whenever the *Moon* came to that Point, it would constantly be at the same Distance from us: But this Line of the *Apsides* is likewise observed to be moveable, and to have an angular Motion round the *Earth* from the *West* towards the *East*, according to the Order of Signs; so that it does not return to the same Situation, till after the Space of almost nine Years.

Lecture X.

THE Motions of the *Moon*, and that of her Orbit, do not observe the same Inequalities. For, *First*, When the *Earth* is in her *Aphelion*, at the greatest Distance from the *Sun*, the *Moon* being so likewise, the *Moon* does somewhat quicken her Pace, and performs her Circulation in less Time. On the contrary, when the *Earth* approaches nighest to the *Sun* in the *Perihelion*, the *Moon* is likewise nearer, and then she slackens her Motion: Upon which Account it is, that the *Moon* revolves about the *Earth* in shorter Time, when the *Earth* is in her *Aphelion*, than when she is in her *Perihelion*; so that the periodical Months are not all equal.

The Inequalities of the Motions of the Moon.

Secondly, When the *Moon* is in the *Syzygia*, that is, in the Line which joins the Centers of the *Earth* and *Sun*, (which may be either in her Opposition or Conjunction) all other Things being alike, she has a swifter Motion round the *Earth*: But in the Quadratures she she goes slower.

Syzygia.

Thirdly, According to the different Distance of the *Moon* from the *Syzygia*, that is, from Opposition or Conjunction, she changes her Motion; and in the first Quarter of her Motion; that is, from Conjunction to her first Quadrature, she loses something of her Swiftness: In her second Quarter, from the Quadrature to her Opposition, she increases in Velocity: In her third Quarter, from Opposition to the last Quadrature, she again loses of her Motion; and from that Quadrature to the Conjunction, she again

Lecture X. again recovers her Swiftneſs. This Inequality in the *Moon*'s Motions was firſt diſcovered by the noble *Tycho*, who called it the *Moon*'s *Variation*.

The Variation. *Fourthly*, The *Moon* moves in an Ellipſe, whoſe *Focus* is in the Center of the *Earth*, round about which ſhe deſcribes *Area*'s proportional to the Times, as the primary *Planets* do round the *Sun*; whence the Motion of the *Moon* muſt be quickeſt in the *Perigeon*, and ſloweſt in the *Apogeon*.

The Orbit of the Moon, *and its Excentricity, changeable.* *Fifthly*, the very Orbit of the *Moon* is changeable, and does not always keep the ſame Figure; but its Excentricity does now and then grow greater, and now and then it diminiſhes: And it is greateſt, when the Line of the *Apſides* is coincident with the *Syzygia*, or is in the Line which joins the Centers of the *Sun* and *Earth*: And the Excentricity is the leaſt, when the Line of the *Apſides* cuts the other at right Angles. The Difference between the greateſt and leaſt Excentricity is ſo conſiderable, that it exceeds the half of the leaſt Excentricity.

The Apogeon *has an unequal Motion.* *Sixthly*, The very *Apogeon* of the *Moon* has an unequal Motion, and ſometimes moves forward, and ſometimes backward; when it is coincident with the *Syzygial* Line, its Motion is forward; but when it cuts that Line at right Angles, its Motion is backward, and its Progreſs and Regreſs are no ways equal. But when the *Moon* is in her Quadratures with the *Sun*, the *Apogeon* goes but ſlowly forward, or even may ſtand ſtill, or go backward. But when the *Moon* comes to be oppoſite or conjoined to the *Sun*, the *Apogeon* has a quick Motion forward.

Seventhly, The Motion of the *Nodes* is not at all uniform; for when the Line of the *Nodes* coincides with the Line of the *Syzygia*, then they ſtand ſtill without any Motion; but when they cut that Line at right Angles, they go backward, or from *Eaſt* to *Weſt*, with a conſiderably quick Motion. The moſt ſagacious Sir ISAAC NEWTON was the firſt, and the only Man, who has diſcovered the true Cauſes

of

of all these Inequalities; and has demonstrated, that they all arise, according to the Laws of Mechanism, from the *Theory of Gravitation* of Matter to Matter. It is very surprising, that the *Moon*, which of all the heavenly Bodies is nearest to us, should be of such difficult Access; and that it should be so hard to find out her Ways, and the Causes of all her Irregularities.

Lecture X.

The Moon *moves uniformly about her* Axis.

THE only equal Motion of the *Moon* is that wherewith she turns round her *Axis* in the same Time that she moves round us in her Orbit; from whence it comes to pass, that she always keeps the same Face towards us: But this very Equability in Rotation is the Cause of an apparent Inequality; that the *Moon* appears to librate about its *Axis* sometimes from the *East* to the *West*, and now and then from the *West* to the *East*; and that some Parts in the *Western* Limb or Margin of the *Moon* recede from the *Center* of the Disk, and sometimes they move towards it. Some of these Parts, which were before visible, set and hide themselves in the invisible Side of the *Moon*, and afterwards become again conspicuous. Such a Motion in the *Moon* is called her *Libration*, and it arises from the unequal Motion of the *Moon* in the Perimeter of her Orbit: For if the *Moon* moved in a Circle, whose Center coincided with the Center of the *Earth*, and turned round its *Axis* in the precise Time of its Period round the *Earth*, in that Case the Plane of the same *Lunar* Meridian would always pass through the *Earth*; and the same Face of the *Moon* would be constantly and exactly turned towards us. But since the real Motion of the *Moon* is in an Ellipse, in whose *Focus* is the *Earth*, and the Motion of the *Moon* about her *Axis* is equable; or, which is the same Thing, every Meridian of the *Moon* by this Rotation describes Angles proportional to the Times, the Plane of no one Meridian will constantly pass through the *Earth*.

The Moon's Libration.

Lecture X. Plate VI. Fig. 2.

For let ALP be the Orbit of the *Moon*, in whose *Focus* is the *Earth* in T; and when the *Moon* is in A, its Meridian MN produced, will paſs thro' the *Earth*: And if the *Moon* only revolved in her Orbit, without any Motion round an *Axis*, the ſame Meridian MN would always keep a Poſition parallel to itſelf; ſo that when the *Moon* comes to L, the Meridian MN would be in the Poſition PQ, which is parallel to MN; but on the Account of the *equable Rotation*, the Meridian MN changes its Situation, and deſcribes Angles proportional to the Times; ſo that in the periodical Time of the *Moon*'s Revolution ronnd the *Earth*, it deſcribes four right Angles; and therefore in L it will have the Poſition mLn; ſuch that the Angle QLn may have the ſame Proportion to a right Angle, as the Time the *Moon* takes to deſcribe the Arch AL has to a fourth Part of the periodical Time. But the Time the *Moon* takes to deſcribe the Arch AL is to the fourth Part of the periodical Time, as the *Area* ATL is to the *Area* ACL; that is, to $\frac{1}{4}$ Part of the *Area* of the Ellipſe: Therefore the Angle QLn will be to a right Angle in the ſame Proportion. But the *Area* ATL is greater than the *Area* ACL, or than the fourth Part of the *Area* of the Ellipſe: The Angle therefore QLn will be bigger than a Right, or bigger than QLC; but QLC is bigger than QLT; wherefore QLN will be much bigger than QLT. The Meridian therefore MN, whoſe Plane paſſed through the *Earth*, when the *Moon* was in A, now the *Moon* is arrived at L, does not look towards the *Earth*: And therefore the Hemiſphere of the *Moon* which is towards the *Earth*, the *Moon* being at L, is not the ſame with that which was towards the *Earth* when the *Moon* was in A; and thoſe Parts of the *Moon*'s Surface beyond Q will come under Obſervation, which before, when the *Moon* was in A, were not to be ſeen, being in the Side of the *Moon* quite oppoſite to us. But as ſoon as the *Moon* arrives at her *Pe-*

rigeon P, then the Meridian MN has deſcribed in its Rotation a Semicircle; and then again its Plane paſſes through the *Earth*, and the former Point N will be directly towards us, and be in the Center of the Diſk. Hence it is evident, that this *Libration* of the *Moon* is reſtored twice in each Period of the *Moon*, that is, when ſhe comes to her *Apogeon* and *Perigeon*.

The Surface of the Moon *rugged and mountainous.*

If the Surface of the *Moon* were ſmooth and poliſhed like a Looking-glaſs, it would not then reflect Light upon all Sides, and every Way; but it would ſhew us only in ſome Poſitions the Image of the *Sun*, no bigger than a Point, but with an immenſe Luſtre. But as in all our *Earthly* Bodies, ſo in the *Moon*, its Surface is very rough and uneven; upon which Account it diffuſes the Light by reflecting it to all Sides, without producing any Image of the *Sun*, as poliſhed Glaſſes do.

A Demonſtration that there are Mountains in the Moon.

But the Surface of the *Moon* is not only rough and uneven, but there are upon it moſt prodigious high Mountains, and deep Vallies, which cover the whole Face of the *Moon*: This we thus prove. If there were no Parts in the *Moon* higher than the reſt, no prominent Points, then a Right Line in the *Dichotomy* or Quadrature, and an Elliptick Line in all the other *Phaſes*, would terminate the light and dark Parts of the Diſk: but when the *Moon* is viewed with a Teleſcope, we find that there is no regular Line, which ſeparates Light and Darkneſs in the *Moon*'s Surface; but the Confines of theſe Parts appear, as it were, toothed, and cut with innumerable Notches and Breaks; and even in the dark Part near the Borders of the lucid Surface, there are ſeen ſome ſmall Places enlightened by the *Sun*'s Beams: And upon the fourth Day after *New Moon*, there may be perceived ſome ſhining Points, like Rocks or ſmall Iſlands, within the dark Body of the *Moon*; but not far from the Confines of Light and Darkneſs, there are obſerved other little Spaces, which join to the enlightened Surface, but run out into

Lecture X. into the dark Side; which by Degrees change their Figure, till at laſt they come wholly within the illuſtrated Face, and have no dark Parts round them. Afterwards we obſerve many more ſhining Spaces to ariſe by Degrees, and to appear within the dark Side of the *Moon*, which, before they drew near to the Confines of Light and Darkneſs, were inviſible; being without any Light, but wholly immerſed in the Shadow. The contrary is obſerved in the decreaſing *Phaſes*, where the lucid Spaces which joined the illuminated Surface, by Degrees recede from it; and after they are ſeperated quite from the Confines of Light and Darkneſs, remain for ſome time viſible, till at laſt they alſo diſappear: Now it is impoſſible that this ſhould be, unleſs theſe ſhining Points were higher than the reſt of the Surface, ſo that the Light of the *Sun* may reach them.

In the Moon *large Cavities and Pits.*

THESE ſhining Points ſituated in the *Moon's* Surface, without the Confines of the illuminated Surface, are the Tops of very high Mountains, which riſing far above the other Parts of the Surface, are ſooner reached by the *Sun's* Beams, and remain longer in the Light, than the reſt of the Parts do which are lower. Beſides theſe, we likewiſe obſerve, even in the illuminated Face of the *Moon*, many dark and obſcure Spots, which ſeem to be only Caverns, or large Cavities; on which the *Sun* ſhining very obliquely, and touching only their upper Edge with his Light, the deeper Places remain without Light: But as the *Sun* riſes higher upon them, they receive more Light, and the Shadow or dark Parts grow ſmaller and ſhorter, till the *Sun* comes at laſt to ſhine directly upon them, and then the whole Cavity will be illuſtrated, and the Parts which were obſcure before will then look as bright as the Tops of the Mountains. From theſe conſtant Obſervations, it is plain to a Demonſtration, that the *Moon's* Face is covered with Mountains in ſome Places, and that in others it is cut with deep Pits and Caverns.

THE *Lunar* Mountains are much higher in Proportion to the Body of the *Moon*, than any Mountain upon our Globe; for the *Geometers* can take the Height of them, as easily as they can find the Measure of a Mountain upon our *Earth*. The Way of finding the Height of a *Lunar* Mountain is this: Let E G D be the Hemisphere of the *Moon* illuminated by the *Sun*, and E C D the Diameter of the Circle, bounding Light and Shadow, A the Top of a Hill, within the dark Part, when it first begins to be illuminated. Observe with a Telescope the Proportion of the Right Line A E, or the Distance of the Point A, from the lucid Surface, to the Diameter of the *Moon* E D; and because in this Case the Ray of Light E S touches the Globe of the *Moon*, A E C will be a right Angle, by the 16th *Prop.* Book Third, of *Euclid:* And therefore having in the Triangle A E C, the two Sides A E and E C, we can find out the third Side A C, from which subducting B C, or E C, there will remain A B, the Height of the Mountain. *Ricciolus* affirms, that upon the fourth Day after *New Moon*, he has observed the Top of the Hill called *St Katherine* to be illuminated, and that it was distant from the Confines of the lucid Surface, about a sixteenth Part of the *Moon*'s Diameter, or an eighth Part of her Semidiameter. And therefore if C E be 8, A E will be 1; and the Square of A C will be equal to the Squares of C E and EA, by *Prop.* 47, Book First, of *Euclid*. Now the Square of C E being 64, and the Square of A E being 1, the Square of A C will be 65, whose square Roor is 8,062 which expresses the Length of A C: From whence deducting $BC=8$, there will remain $AB=0{,}062$. So that C B or CE is therefore to AB, as 8 is to 0,062; that is, as 8000 to 62: and therefore, since the Semidiameter of the *Moon* is 1182 Miles, if we make the Proportion as 8000 to 62, so 1182 is to 9: we shall have 9 Miles for the Height of that Mountain, which

Lecture X.

The Lunar Mountains higher than the Mountains of the Earth.

Plate VI. Fig. 3.

A Method of measuring them.

Lecture X.

which is therefore three times higher than the Tops of our highest Hills on *Earth*.

Great Varieties to be observed in the Face of the Moon.

WHOEVER shall contemplate the Face of the *Moon* with a Telescope, will discern it distinguished with an admirable Variety of Spots; some Parts have a most bright Lustre, and some *Philosophers* have imagined them to be Rocks of Diamonds; others have compared them to Pearls, or some precious Stones: But they seem to be the solid Parts of high Mountains, which are endued with a Quality whereby they strongly reflect the Light. There are again other Places and Parts of the *Moon*'s Face, and they are not a few nor small, which look dark and of a duskish Colour, which the *Philosophers* have fansied to the Seas, Lakes, and Fens: But yet we find, that they cannot be Seas, nor any thing of a liquid Substance; for when they are looked at with a good Telescope, we find they consist of an Infinity of Caverns and empty Pits, whose Shadows fall within them; which can never be in a Sea or liquid Body. These black Spots therefore cannot possibly be Seas: But they consist of some darker and sad-coloured Matter, which does not reflect the Light so strongly, as the solid and shining Mountains do. But even within these dark Spots, we observe some Bodies of a brighter Light, wherewith they outshine the rest.

There are no Seas.

No Clouds.

THERE seem to be no Clouds nor Vapours in the *Moon*, from whence Rain may be generated: For such Clouds would sometimes cover the Face of the *Moon*, and hide some of its Regions from our Sight, which we never observe them to do: But in the *Moon* there is a constant Serenity, without any dark Weather; and when there are no Clouds in our Air, the *Moon* constantly appears with the same Lusture.

No Atmosphere.

It is probable likewise, that the *Moon* has no Atmosphere to surround it: For the *Planets* and *Stars*, which sometimes are seen very near its Limb, have not their Light refracted, as it is when it passes through our Atmosphere.

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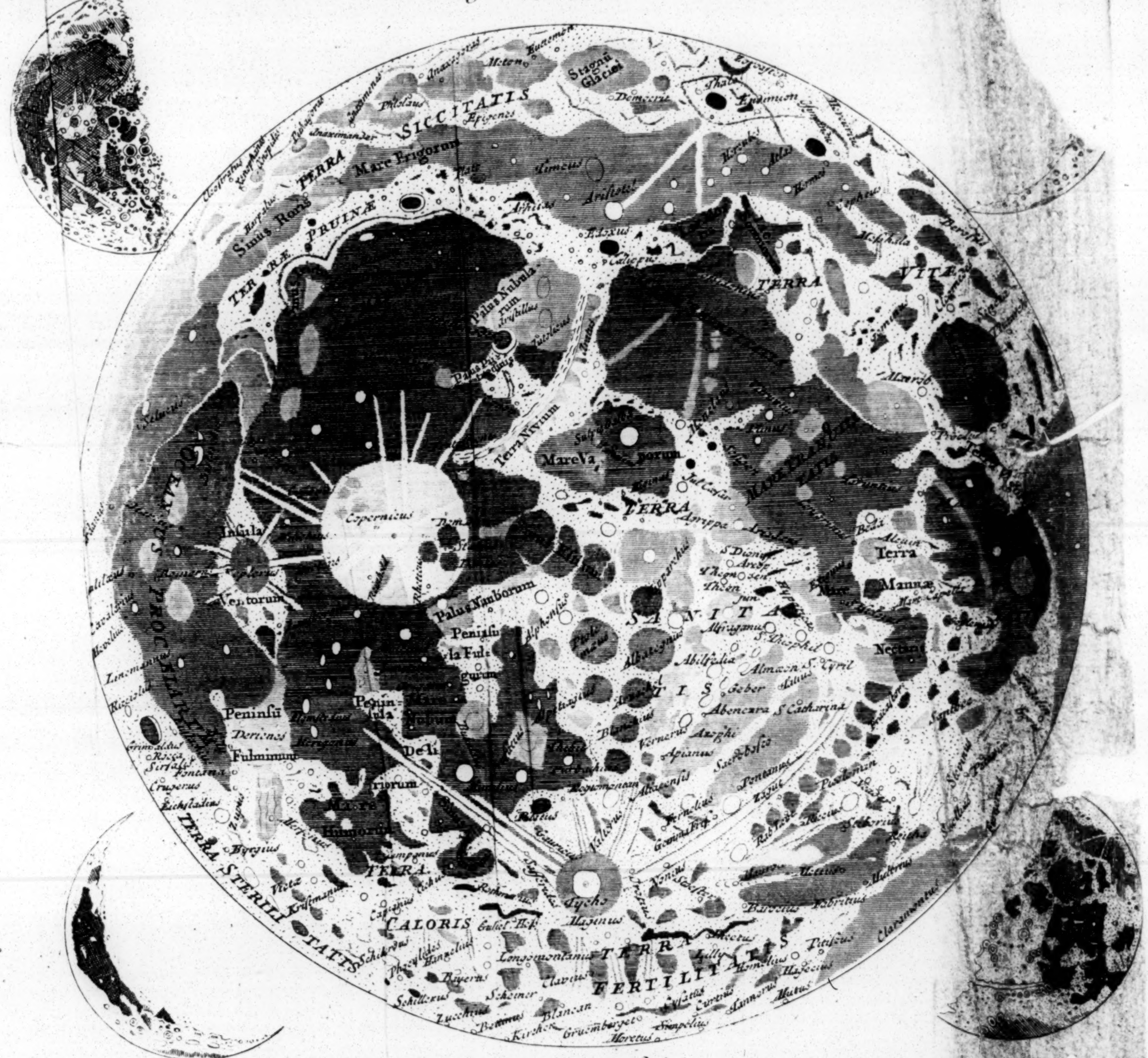

I. Senex sculp^t.

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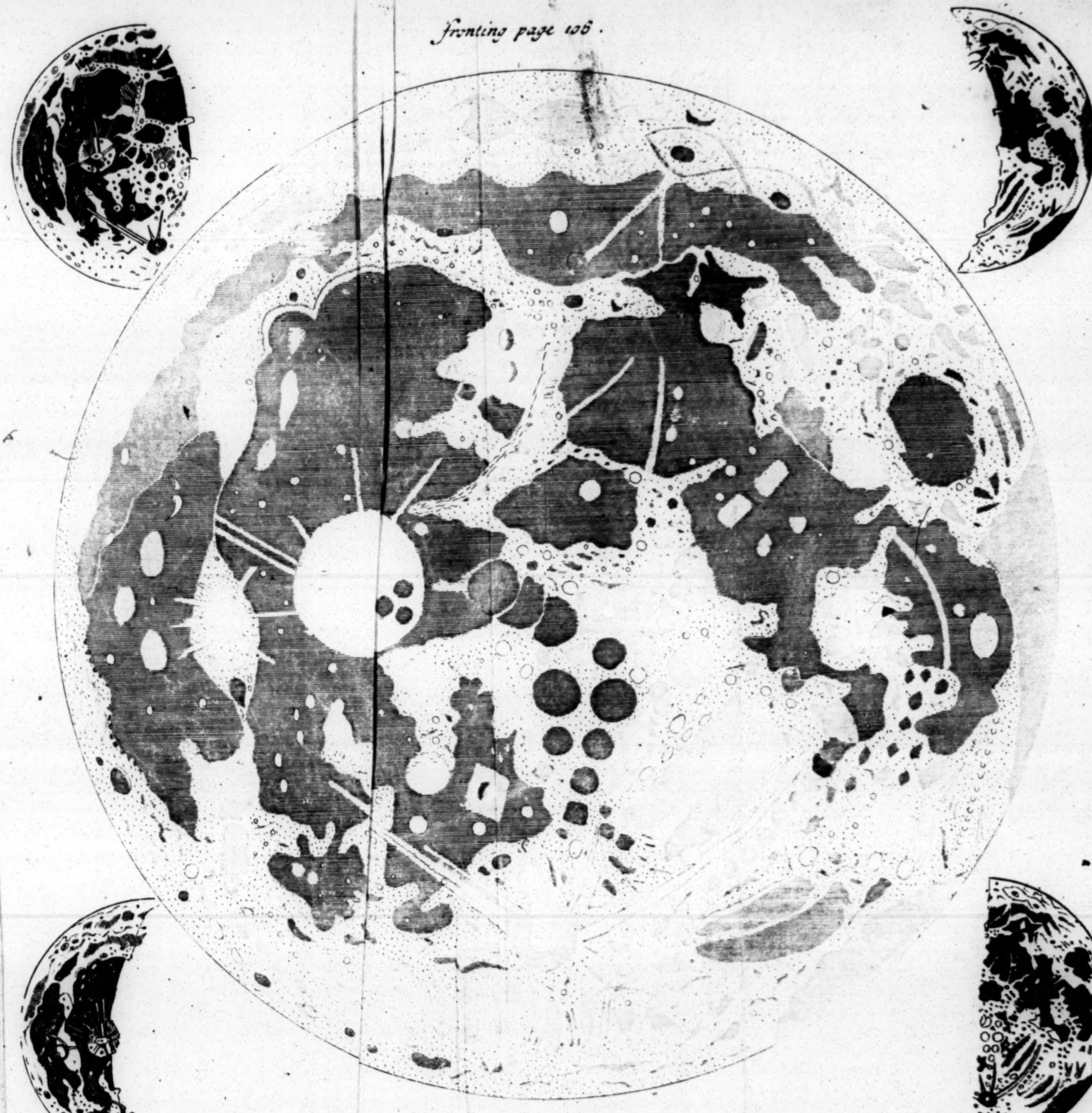


I. Senex sculpt.

THE *Astronomers* have drawn the Face of the *Moon*, according as it is seen with the best Telescopes; for which we are obliged to the accurate Labours of these famous Selenographers *Florentius*, *Langrenus*, *John Hevelius* of *Dantzig*, *Grimaldus* and *Ricciolus*, *Italians*; who have taken particular Care to note all the shining Parts of the *Moon*'s Face and for the better distinguishing them, they have given to each Part a proper Name. *Langrenus* and *Ricciolus* have divided the *Lunar* Regions among the *Philosophers* and *Astronomers*, and other eminent Men: But *Hevelius*, fearing lest the *Philosophers* should quarrel about the Division of the Lands, has spoiled them of this their Propeıty, and gives the Parts of the *Moon* those *Geographical* Names, that belong to the different Islands, Countries and Seas of our *Earth*, without any Regard to Situation or Figure.

The Selenographers *or* Astronomers *who have described the* Moon's *Surface.*

LECTURE XI.

Of the Obscurations or Eclipses *of the* Sun *and* Moon.

THERE is nothing in *Astronomy*, which shews the great Sagacity of human Understanding, and its deep Penetration more, than a clear Explication of the sudden Disappearings of the *Sun* and *Moon*, that is, of their *Eclipses*; and the accurate Predictions when they are to come to pass, which the *Astronomers* can now foretel almost to a Minute. Tho' this be the nicest and most subtle Speculation of our Science, yet it is certain and undoubtable, than which nothing

Lecture XI. nothing can be more ſublime, or worthy of our Contemplation.

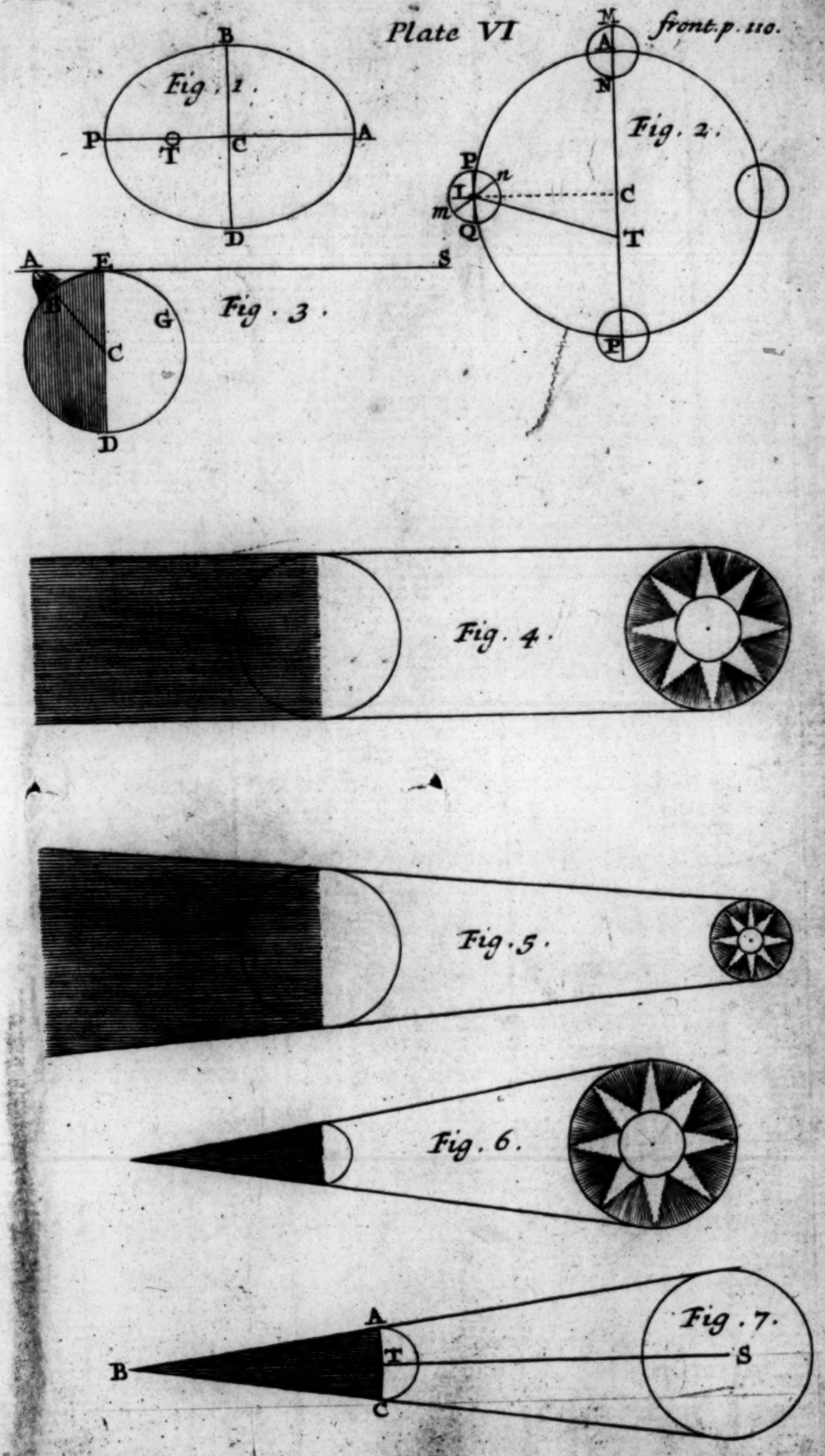
An Eclipſe, what. THE Word *Eclipſe* is derived from the *Greek* ἐκλείπω, which ſignifies to faint, or to ſwoon away: So ſick and dying Perſons, when a ſwooning Fit, and a Death-like Faintneſs comes over them, were ſaid by the *Greeks* to fall into an Eclipſe: After the ſame Manner the *Moon*, when ſhe ſhines with a full Face, if ſhe falls into the Shadow of the *Earth*, does loſe the enlivening Beams of the *Sun*'s Light, and grows pale, as if ſhe were about to die. And the *Sun* again when the *Moon* interpoſes her Body, and deprives us of his Heat and Light; though in himſelf he retains his Luſtre, yet to us he ſeems to vaniſh and grow dark. At ſuch Times the *Sun* and *Moon* are ſaid to ſuffer, and fall into an Eclipſe. The Eclipſes muſt be here explained: And, that we may begin from the firſt Principles:

A Shadow, what. IT is to be obſerved, that all opake and dark Bodies, when they are expoſed to the direct Light of the *Sun*, caſt a Shadow behind them, that is oppoſite to the Line the *Sun* is in. This Shadow is nothing but the Loſs or Privation of Light, in the Space oppoſite to the *Sun*, by reaſon the *Sun*'s Rays are intercepted by the opake Body. Now ſince the *Earth* is an opake Body, it muſt likewiſe caſt a Shadow towards the Space oppoſite to the *Sun*; in which Space if the *Moon* ſhould come, it muſt neceſſarily be darkened, and loſe the Light which it had before from the *Sun*. And becauſe the Figure of the *Earth* is Spherical, the Figure of the Shadow would be Cylindrical, if the *Earth* and *Sun* were of equal Bigneſs, or if the *Earth* were bigger than the *Sun*, the Shadow would have the Figure of a Cone, which had loſt a Piece at his Top or Vertex; and the farther it were extended, would grow thicker and thicker. (Plate VI. Fig. 4, 5, 6.)

The Figure of the Shadow. AND in both theſe Caſes, the Shadow would run out into infinite Space, without ever having an End: And then it would involve ſometimes the other *Planets*, *Mars*, *Jupiter*, and *Saturn*, within

Plate VI

front. p. 110.



in it, when they come to be opposite to the *Sun*, and enter within that Space; But this is never observed, for then these *Planets* would be eclipsed: And therefore the *Sun* must necessarily be greater than the *Earth*, whose Shadow must consequently be of a conical Figure, and end in a Point.

Lecture XI.

The Sun *bigger than the* Earth.

But the *Moon*, since its Diameter is contained about three times in the Diameter of the Shadow, and the Diameter of the Shadow is less than that of the *Earth*, must needs be much less than our *Earth*.

Let S represent the *Sun*, T the *Earth*, and the Cone A B C the Shadow. It is evident, there can be no Line drawn from the *Sun*, to any Point of the Space A B C, which does not fall upon the *Earth*: And therefore, since the *Earth* is an opake Body, it will not suffer any Rays to pass through. or to illustrate the Space A B C. Now if the *Moon*, when she is opposite to the *Sun*, should come into this Space, she must then be involved in Darkness; and would then suffer an Eclipse in the very Time of *Full Moon*.

Plate VI. Fig. 7.

The *Moon* likewise, upon the same Account, must have a Shadow of a conical Figure opposite to the *Sun*; and if this Shadow should fall upon the *Earth*, which can never happen, but when the *Moon* is in Conjunction with the *Sun*, the Inhabitants of the *Earth*, on whom the Shadow falls, will be involved in Darkness; and the *Sun* will seem to them to be in an Eclipse, so long as the Shadow covers them; but because the *Moon* is much less than the *Earth*, its Shadow can never cover the whole *Earth*, but only a small Part of it, such as B C: And within that Space only, where the Shadow comes, there will be total Darkness; and the rest of the circumjacent Places will be illustrated with some of the *Sun*'s Beams, and their Inhabitants will only see a Part of the *Sun*'s Disk obscured; which will be greater or less, according as they are nearer or further removed from the Shadow:

When there can happen an Eclipse of the Moon.

Plate VII. Fig. 1.

Lecture XI.

In some Places of the Earth *an Eclipse may be total, in others partial, and in others none at all. When an Eclipse of the* Sun *happens.*

Shadow: Particularly, they who live about P, will see half the *Sun* eclipsed; but whosoever lives between M and N, will see at the same time all the *Sun*'s Body, and perceive no Eclipse.

Hence it is manifest that there can be no Eclipse of the *Moon* but in *Full Moons*, when she is opposite to the *Sun*; as the Shadow always is. Nor can there be any Eclipse of the *Sun* but in the *New Moons*, when she is in Conjunction with the *Sun*, for then only she can cast her Shadow on the *Earth*. Since therefore in every Month there is one *Full Moon*, and one *New Moon*; it may be asked how it comes that the *Sun* and *Moon* do not suffer Eclipses every Month. And indeed if the *Moon* did always move in the Plane of the Ecliptick, since the *Axis* of the Shadow is always in the same Plane, the *Moon* would then every *Full Moon* pass through the Body of the Shadow, and there would be a total Eclipse of the *Moon*. So likewise in every *New Moon*, if she were not then too far off us, she should cast her Shadow on the *Earth*, and produce an Eclipse of the *Sun*, in some or other of the Regions of the *Earth*. But the Case is otherwise; for we have shewed, that the Plane of the *Moon*'s Orbit does not coincide with the Plane of the Ecliptick, but that it cuts it in a Line which passes through the Center of the *Earth*: And therefore the *Moon* is never in the Plane of the Ecliptick, but when it is in this Line, which is the Intersection of the two Planes, that is, when it enters the *Nodes*. And therefore, when it happens, that the *Moon* at *Full* shall likewise be in one of the *Nodes*; then the *Axis* of the Shadow will pass through the Center of the *Moon*, and then she will be in a total and central Eclipse. Let the Circle MN represent the transverse Section of the Shadow at the Distance of the *Moon*; and the Line CD a Portion of the Orbit of the *Moon*, which the *Moon* describes in the Time of *Full Moon*; which, because it is but a small Portion, may be well enough represented by

Why there happen not Eclipses of the Sun *and* Moon *every Month.*

Plate VII. Fig. 2. *Total and central Eclipses of the* Moon.

by a right Line: Let the right Line B G A be in the Plane of the Ecliptick, and let F be the Position of the *Moon*'s Center, when she first touches the Shadow; E the Position of the same Center, when she first leaves it; G the same Center of the *Moon*, when the *Axis* of the Shadow passes through it: It is evident, that such an Eclipse will be Central and Total; and there will always be such Eclipses, when the Center of the *Moon*, and *Axis* of the Shadow, meet in the *Nodes*. Hence the Duration or Time that an Eclipse can last, may be as long as the *Moon* is passing through an Arch, that is, equal to E F, or four Diameters of the *Moon*; that is, about two Degrees, which Space the *Moon* generally moves thro' in the Space of four Hours.

Lecture XI.

Because of the largeness of the Diameter of the Shadow in Comparison of that of the *Moon*, there may be total Eclipses, which are not central, where the *Node* does not coincide with the *Axis*; and may even lie without the Shadow, as the Figure sufficiently shews. The *Node* may likewise be at such a Distance from the Shadow, that there may be only a Part of the *Moon*'s Body that can enter it; and then we shall have a partial Eclipse of the *Moon*, as is manifest by the Figures; and these partial Eclipses will be greater or less, according as the Distance of the *Node* from the Shadow is less or greater. But when it happens that the *Node*, in the Time of *Full Moon*, is further removed from the *Axis* of the Shadow than twelve Degrees, the *Moon* then will have so much Latitude, or its Distance from the Ecliptick will be so great, that it cannot be obscured by the Shadow.

Plate VII. Fig. 3.

Partial Eclipses. Plate VII. Fig. 4, 5.

As the Shadow of the *Earth* cast upon the *Moon* produces an Eclipse of the *Moon*, so, if the Shadow of the *Moon* should fall upon the *Earth*, it will cause an Eclipse of the *Earth*, at least on that Part of the *Earth* on which the Shadow falls. For the *Moon* being much less than the *Earth*, cannot with its Shadow involve the whole Disk of the *Earth*,

Eclipses of the Earth.

but only a very ſmall Part of it; and ſo all the Eclipſes of the *Earth* will be Partial, and not Total; and ſuch Eclipſes only will produce a Darkneſs upon thoſe Places where the Shadow falls; and the Inhabitants within this Shadow will only ſee the *Sun* totally darkened, and therefore they will call them Eclipſes of the *Sun:* But this is improperly attributed to the *Sun*, who all the Time retains his Light without the leaſt Diminution; and it is only thoſe Inhabitants of the *Earth* that are under the Shadow, that are truly eclipſed, and involved in Darkneſs.

Lines drawn from the Center of the Sun *to any Point of the* Earth, *may be reckoned as parallel.*

THAT we may deſcend more particularly to explain the Phænomena or Appearances of Eclipſes; it will be requiſite to ſhew the Method of meaſuring the Dimenſions of the conical Shadows of both *Earth* and *Moon:* For which Purpoſe we will firſt lay down the following *Poſtulatum:* If from the Center of the *Sun* there be drawn right Lines to every Point of the *Earth*, or to as many as you pleaſe, theſe Lines may all of them be eſteemed as parallel. For parallel Lines are ſuch as do not meet, till they are produced to an infinite Diſtance; and therefore ſuch Lines as do not meet but at a Diſtance immenſely great, in Compariſon of the Diſtance of the Lines from one another, are nearly, or, as we may ſay, phyſically parallel; that is to ſay, they will have the ſame Effect in Nature, and the phyſical Obſervations that are to be made from them, will be the ſame, as if the Lines were abſolutely parallel. Now the Diſtance of the *Earth* from the *Sun* is ſo great, that the Diameter of the *Earth*, compared with it, is but as a Point, as is now acknowledged by all Mathematicians; for this Diameter, ſeen from the *Sun*, does appear under an unperceptible Angle, or which is ſo ſmall, that the Eye cannot obſerve it, and the *Earth* appears only like a Point: and therefore in Compariſon of the great Diſtance of the *Sun*, it vaniſhes; and conſequently Lines drawn from the Center of the *Sun* to different Parts of the *Earth*, will be at leaſt phyſically parallel. Moreover it is known

known in Geometry, that if a right Line falls upon two other right Lines, so as to make the two internal Angles on the same Side equal to two right Angles, that these two Lines on which it falls are parallel, by 29 *Prop.* Book First, of *Euclid* Let therefore the Line A B be the Diameter or Semidiameter of the *Earth*, and C the Center of the *Sun*; drawing A C and B C, the Angles A, B and C of the Triangle A B C are equal to two right Angles: Now the Angle C at the *Sun* vanishes, and is next to nothing; for the *Earth*, seen from the *Sun*, looks like a Point; and therefore the Angles at A and B must make by themselves two right Angles very nearly, and therefore the right Lines A C, B C, are nearly parallel. It is upon the same Account, that if there be taken two Threads with Plumbets to make them hang perpendicularly, the Directions of those Threads are by all Artificers esteemed as parallel, though their Directions will meet at the Center of the *Earth*, to which all heavy Bodies have a *Tendency* or *Propension*. Lecture XI. Plate VII. Fig. 6.

WHAT we have said in this Case concerning the *Earth*, is also true of the *Moon*; for its Diameter has a much less Proportion to the Distance of the *Sun*, than that of the *Earth* has to it. And not only Lines drawn from the *Sun* to any Points of the *Earth* and *Moon*, are to be reputed parallel; but if there be two Lines drawn, one from the Center of the *Sun* to the *Earth*, the other from thence to the *Moon*, these may be likewise taken as Parallels; for they will not sensibly differ from a Parallelism, especially in the Time of Eclipses: The Difference of these Lines from real Parallels is so small, that it will make no sensible Error in the Calculation of Eclipses.

WE likewise premise the following *Lemma*, which is easily demonstrated:

IF *two right Lines* A E, B F, *touch a Circle, and there be drawn from the Points of Contact to the Center the Lines* A D, B D; *the Angle at the Cen-* Plate VII. Fig. 7. A Lemma.

Lecture XI.

ter contained under these Lines, will be equal to the Angle that the Tangents make with one another. For in the four-sided Figure G A B D, all the Angles make four Rights: But the Angles A and B make two Rights, by the 18 *Prop.* Book Third, of *Euclid:* Wherefore the Angles A G B and D, are equal to two Rights. But by the 13th *Prop.* Book First, of *Euclid*, the Angles; AGB and EGB are equal to two right Angles; and therefore the Angles D and E G B are equal, since the Angle A G B makes two right Angles with either of them.

The Dimension of the Angle of the Conical Shadow. Plate VII. Fig. 8.

LET the Circle ABK represent the Globe of the *Earth*, AM the Line which joins the Centers of the *Sun* and *Earth*; to which let the Diameter CB be perpendicular: If from B there be drawn to the Center of the *Sun* the Line E F, this Line will be parallel to the Line CM, as has been shewn: Make the Angle BCD equal to the apparent Semidiameter of the *Sun*; that is, equal to the Angle under which the Semidiameter of the *Sun* is seen from the *Earth*, and then through D draw the Tangent D G. By the *Lemma* above demonstrated, the Angle G E F will be equal to the Angle B C D, or to the apparent Semidiameter of the *Sun*; and therefore, since the Line B F produced goes to the Center of the *Sun*, the Line G E D must touch its Circumference, and it will also touch the *Earth*, and being produced, will meet with the *Axis* of the Shadow C H in H, so that the Angle D H C will be half the Angle of the conical Shadow. Now because EF is parallel to M H, the Angles D H C and GEF are equal, by the 29th *Prop.* Book First, of *Euclid*. But G E F is equal, as has been shewed, to the apparent Semidiameter of the *Sun*; wherefore the whole conical Angle K H D is equal to the apparent Diameter of the *Sun*.

These Angles in all Conical Shadows are equal.

THE same Thing is to be demonstrated of the *Moon*, and universally, the *Sun*'s apparent Diameter remaining the same in all Spheres, which are not bigger than the *Earth*; the Angles of the conical

Figures

Figures which include the Shadows are all equal, and all their Shadows will be similar Figures. This may likewise be demonstrated in this Manner. Lecture XI.

LET AGF be the *Sun*, DHE the *Earth*, SC a Line joining the Centers of the *Sun* and *Earth*, AD a right Line which touches both Bodies; and let the Lines AD, SC produced, meet in M; the Angle AMS will be half the Angle of the shadowed Cone. Now in the Triangle SDM, the outward Angle ADS is equal to both the inward and opposite Angles, by *Prop.* 32. Book First, of *Euclid*; that is, the Angles DMS and DSM are equal to the Angle ADS; but the Angle DSM is nothing, or next to nothing, being the Angle under which the Simidiameter of the *Earth* appears as seen from the *Sun*; and the Angle ADS is the apparent Semidiameter of the *Sun*; therefore the Angle DMS, or the Semiangle of the Cone, is equal to the apparent Semidiameter of the *Sun*. Plate VII. Fig. 9.

Lecture XII.

LECTURE XII.

Of the Penumbra *and its Cone; the Height of the Shadow, and the apparent Diameters of the Shadows.*

Penumbra *what.*

BESIDES the Shadow which is deprived of all the *Sun*'s Light, there is a certain Space which is but a partial Shadow, and is called a *Penumbra*; for though all the *Sun*'s Body does not illuminate it, there are, for all that, Rays coming from ſome Part of the *Sun* which do enter it, and render it lucid, the reſt of the *Sun*'s Beams being intercepted by the opake Body of the *Earth*; and the Parts of this *Penumbra* will have different Degrees of illumination, according as they are nearer or further removed from the Shadow. The Space of the *Penumbra* is to be determined in this Manner:

Plate VIII. Fig. 1.

LET the Circle A E F G repreſent the *Sun*, H E D any opake Sphere, for Example, the *Moon*, it being her *Penumbra* that we are at preſent concerned with; S C the Line which joins the Centers of both Spheres. Draw the Line F D O, touching the left Side of the *Sun*, and the right Side of the *Moon*; and the Line AHP, which touches the right Side of the *Sun*, and the left of the *Moon*: Let theſe two Lines cut the Line S C in I. The Point I remaining immoveable, either of the right Lines I D O, or I H P, being extended indefinitely, let them

2.

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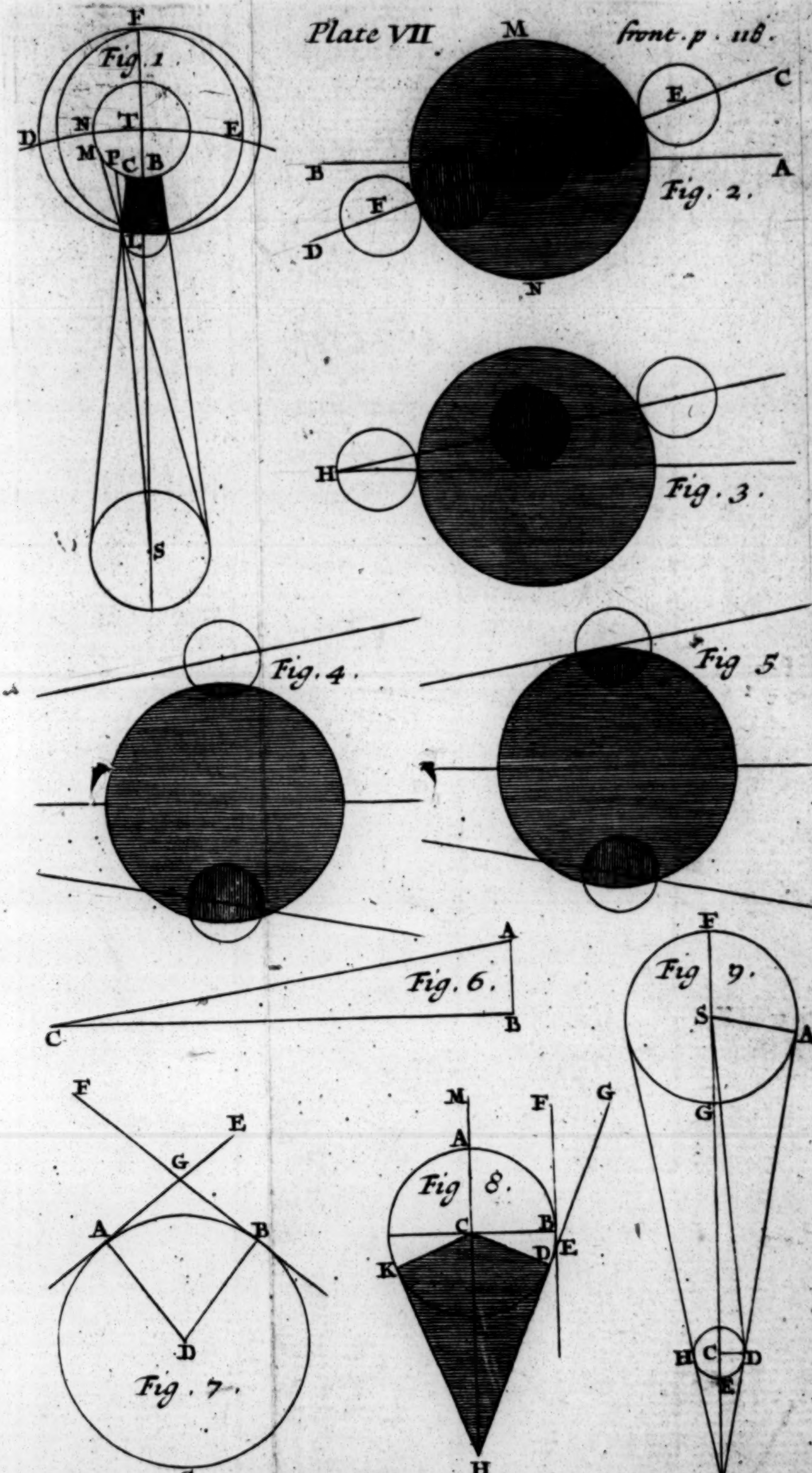
Plate VII front. p. 118.



them be turned round the *Axis* IM, with a conical Motion, ſo that they may always touch the Globe of the *Moon*; there will by that means be generated an indefinite conical Surface, including both the Shadow and the circumambient Space ODM PHM; into which Space ſome Rays of the *Sun* are hindered from entering by the opake Body of the *Moon*: And this is the Space which we call the *Penumbra*, which is darker in X or Y, which are nearer the Borders of the Shadows than in V and N, which are nearer the conical Surface. For the Places X and Y are illuſtrated with a ſmaller Portion of the *Sun*'s Diſk, than the other Places further diſtant from the *Axis* of the Cone. Now, if the *Earth* come within this Space, a certain Portion of its Surface at S may be included in the total Darkneſs, and the Inhabitants of that Region will ſee a total Eclipſe of the *Sun*; but thoſe who live without this Shadow; but are ſtill within the *Penumbral* Space, as about Q and X, will have no total Darkneſs, ſome Part of the *Sun*'s Diſk being ſtill viſible, while the reſt is hid by the *Moon*, For let us draw from Q the Line QD, touching the Globe of the *Moon*; which being produced to the *Sun*, the Point Q being immoveable, if the Line QD indefinitely extended be moved by a conical Motion round the *Moon*, the conical Surface it deſcribes, will cut off a Portion of the *Sun*'s Diſk, which is covered by the *Moon*.

Lecture XII.

WE find the Dimenſions of the Cone of the *Penumbra* in this Manner: Let the Circle HDL repreſent the opake Sphere of the *Moon*, SC the Line joining its Center with the Center of the *Sun*; and let CB, the Semidiameter of the *Moon*, be perpendicular to CS, and BF parallel to it touching the *Moon* in B. Make the Angle BCD equal to the apparent Semidiameter of the *Sun*, and through D draw DG a Tangent to the *Moon*. And by the *Lemma* premiſed, the Angle FEG will be equal to the Angle BCD, or to the apparent Semidiameter

Plate VIII. Fig. 2. *The Dimenſions of the conical* Penumbra.

Lecture XII. diameter of the *Sun*; and therefore, since the Line EF goes to the Center of the *Sun*, the Line EDG must touch the inferior Limb of the *Sun*: But it also touches the *Moon*; and therefore the Point I of this Line being immoveable, if it be carried by a conical Motion round the *Moon*, it will generate the Surface which includes the *Penumbra*. And because of the Parallels EF, CS, the alternate Angles FEI and EIC will be equal; but the Angle EIC is the Semiangle of the Cone, and FEI is the apparent Semidiameter of the *Sun*; and therefore half the Angle of the *Penumbral* Cone is always equal to the apparent Semidiameter of the *Sun*. The Cone therefore of the total Shadow, and that Part of the *Penumbral* Cone which lies between the *Sun* and the *Moon*, are equal and similar Figures, for they have their vertical Angles and Bases equal.

Plate VIII. Fig. 3. THE Height of the Shadow of the *Earth* is thus determined: Let CT be the Semidiameter of the *Earth*, TM the Height of the Cone or Shadow: If TM be the Radius, CT will be the Sine of the Angle TMC, which is half the Angle of the Cone. And this Angle is equal to the apparent Semidiameter of the *Sun*, as has been shewed, and in the mean Distance of the *Sun*, is about 16 Minutes. Let therefore the Sine of 16 Minutes be to the Radius, as the Semidiameter of the *Earth* to a fourth; and we shall find TM equal to 214, 8 Semidiameters of the *Earth*: But when the *Sun* is at his greatest Distance, half the Angle of the Cone is fifteen Minutes, and 50 Seconds; and the Height of the Shadow becomes 217 Semidiameters of the *Earth*: And since the Diameter of the *Earth* is to the Diameter of the *Moon*, as 100 is to 28: the Altitude of the *Earth*'s Shadow will be to the Altitude of the *Moon*'s in the same Proportion; for the conical Shadows are similar Figures; and therefore the Height of the *Moon*'s Shadow will be 59,36 Semidiameters of the *Earth*. Hence, if the Distance of

The Height of the Earth's Shadow.

The Height of the Moon's Shadow.

of the *Moon* from the *Earth* be greater than her mean Distance, which is about 60 Semidiameters of the *Earth*, the Shadow of the *Moon* cannot reach the *Earth*; in which Case there may be a central Eclipse of the *Sun*, but not a total one: But round the *Moon* there will appear Part of the *Sun*'s Body, in the Form of a luminous Circle, which, like a bright shining Ring of Gold, will embrace the Body of the *Moon*. It also follows, that if in the Time of the Eclipse, the Anomaly of the *Moon* be less than three Signs, or bigger than nine Signs, there can no where be a total Eclipse of the *Sun*; for in all these Degrees of Anomaly, the Distance of the *Moon* is greater than her mean Distance.

Lecture XII.

The Portion of the Earth's *Surface that may be involved in the Shadow.*

To find how much of the *Earth*'s Surface can be involved in the *Moon*'s Shadow in the Time of an Eclipse, when it directly falls upon it: Let us suppose the Distance of the *Sun* to be the greatest that can be, in which Case the Height of the conical Shadow of the *Moon*, is about 60 Semidiameters of the *Earth*: Let us likewise suppose the Distance of the *Moon* from us, to be the least that can be, that the Earth may receive the most of the Shadow. This least Distance of the *Moon* from the *Earth* is about 56 Semidiameters of the *Earth*: Now let L represent the *Moon*, and T the Center of the *Earth*; L T the Distance of the *Moon* from the *Earth*, which is equal to 56 Semidiameters; and since L M is 60, T M must be four Semidiameters, and T B will be to T M as 1 to 4. But as T B is to T M, so is the Sine of the Angle T M B, which is 15′ 50″, to the Sine of the Angle T B M; by which Means we find the Angle T B M to be 63 Minutes and 10 Seconds. But the Angle A T B is equal to both T M B and T B M, by 32 *Prop.* Book 1st, of *Euclid*; and therefore the Angle A T B is 79 Minutes, and such is the Arch A B; the double of which is the Arch B C, equal to 158 Minutes, or to 180 *English* Miles. We have here supposed the *Axis* of the Shade to pass through the

Plate VIII. Fig. 4.

Center

Lecture XII.

of the Shade to pass through the Center of the *Earth*; which it does when the Centers of the *Sun*, *Earth*, and *Moon*, are in the same Right Line precisely: But when the three Centers do not lie in the same Right Line, the conical Shade is cut obliquely, and the Figure of it upon the *Earth* is Oval, whose Diameter is easily to be determined by the Distance of the *Moon* from the *Sun* seen from the Center of the *Earth*.

How much of the Surface may be included in the Penumbra. Plate. VIII. Fig. 5.

If we inquire how much of the Surface of the *Earth* can be involved in the *Penumbra*, it may be found by this Means: Let us suppose the Apparent Diameter of the *Sun* to be the greatest, which is when the *Earth* is in her *Perihelion*, and is about 16′ 23″. Let A B D be the *Earth*, L the *Moon*, and A M B be half the Angle of the Cone, which is likewise 16′ 23″: By which we shall find the Height of it L M, equal to 58 and a half Semidiameters of the *Earth*. Let the *Moon* be in her *Apogeon*, and therefore at her greatest Distance from the *Earth*, which is 64 Semidiameters of the *Earth*; and T M is equal to both T L and L M, that is to 122½ Semidiameters. But by a Trigonometrical Theorem, T B is to T M, as the Sine of the Angle T M B is to the Sine of the Angle M B N or M B T. But T B is to T M as 1 is to 122½, and the Angle T M B is 16 Minutes 23 Seconds; and therefore we shall find out the Angle M B N to be 35 Degrees and 42 Minutes, which Angle is equal to both the Angles T M B and M T B. If therefore from the Angle M B N 35 Degrees 42 Minutes, we subtract the Angle T M B, there will remain the Angle M T B 35 Degrees 25 Minutes, which is the Measure of the Arch A B; whose double 70 Degrees and 50 Minutes, is equal to the Arch C A B, which makes about 4900 *English* Miles.

If the conical Shadow of the *Earth* at the Distance of the *Moon*, be cut with a Plane perpendicular to its *Axis*, the Section will be a Circle, which

which is called the Shadow, whose Apparent Diameter seen from the Center of the *Earth* is thus determined: Let T be the Center of the *Earth*, C M T half the Angle of the Cone, F L H the Section whose Diameter is F H. Having the Angle of the Cone, we by what Means can find its Altitude T M; but we have also T L, the Distance of the *Moon* from the *Earth*: and therefore we can find M L: But we have also the Angle F M L; therefore we can find out F G half the Diameter of the Shadow at the Distance of the *Moon*: And therefore in the rectangled Triangle F T G, having the two Sides F G and T G, we can find by Trigonometry the Angle F T G, the apparent Semidiameter of the Shadow seen from the Center of the *Earth*. It may likewise be thus found: Having F T the Distance of the *Moon* from the *Earth*, and C T the Semidiameter of the *Earth*; in the Triangle C F T, we may find out the Angle C F T, which is equal to the two Angles F M T and F T M: If therefore we subtract the Angle F M T, which is half the Angle of the Cone, from the Angle C F T, we shall find the Angle F T M or F T L: The Angle C F T is the apparent Semidiameter of the *Earth*, seen from the *Moon*, and is called the *Horizontal Parallax* of the *Moon*, the Reason of which we shall shew, when we come to treat about *Parallaxes*. These apparent Semidiameters of the *Earth*, or *Horizontal Parallaxes* of the *Moon*, constantly vary as the Distance of the *Moon* does, and they are found ready calculated in all *Astronomical* Tables.

Lecture XII.

The apparent Diameter of the Shadow of the Earth *at the Distance of the* Moon *determined.*

Plate VIII. Fig. 3.

Another Way of finding the same.

The Horizontal Parallax *of the* Moon.

LET ☊ M be a Portion of a Line in the Plane of the Ecliptick ☊ L a Portion of the Lunar Orbit which the *Moon* moves through, about the Time of *Full Moon*; which because it is small, may be represented by a Right Line. Let the Circle F M O represent the Shadow of the *Earth* at the Distance of the *Moon*, and its Center G; G L is the Latitude of the *Moon* in the Time of *Full Moon*, which is

Plate VIII. Fig. 6.

Eclipse of the Moon.

Lecture XII. is almoſt equal to the ſhorteſt Diſtance of the *Moon* from the Plane of the Ecliptick. It is manifeſt that if G L, the Latitude of the *Moon*, be greater than the Sum of the Semidiameters of the *Moon* and Shadow; then the *Moon* will no Part of it enter into the Shadow: But if the Latitude of the *Moon* be juſt equal to theſe two Semidiameters, then the Limb of the *Moon* will juſt touch the Shadow, but not enter it: But if the Latitude of the *Moon* be leſs than this Sum, but greater than their Difference, there will be a partial Eclipſe; and if the Latitude of the *Moon* be leſs than the Difference of the Semidiameters of the *Moon* and Shadow, the Eclipſe will be total: And by this Means we can find out the Ecliptick Terms or *Limits*, which are ſuch, that if the Diſtances of the *Moon* from the *Node* be leſs than they are, in the Time of *Full Moon*, there will be an Eclipſe; if greater, there can be no Eclipſe. Let ☊ S repreſent a Portion of a Line in the Plane of the Ecliptick, parallel to the *Earth*'s Orbit; ☊ L, a Portion of the *Moon*'s Orbit; S L, the Latitude of the *Moon*, when at *Full*: And let us ſuppoſe the Latitude to be ſuch, that the Margin of the *Moon* may juſt touch the Shadow, and let the *Node* be at ☊. The Angle L ☊ S is the Inclination of the Orbit of the *Moon* to the Plane of the Ecliptick, which is five Degrees; and L S the Latitude of the *Moon*, when its Limb touches the Shadow, equal to 66 Minutes; and therefore in the rectangular Triangle ☊ L S, having L S and the Angle L ☊ S, we can find ☊ S, the Diſtance of the *Node* from the Point of the Ecliptick oppoſite to the *Sun*, which is 754 Minutes, or 12 Degrees 34 Minutes. And if in the Time of *Full Moon*, the *Moon*'s Place in the Ecliptick be further diſtant from the *Node*, than 12 Degrees 34 Minutes, there can then be no Eclipſe of the *Moon*.

Plate IX. Fig. 1, 2.

The Limits *of Eclipſes.*

Plate IX. Fig. 3.

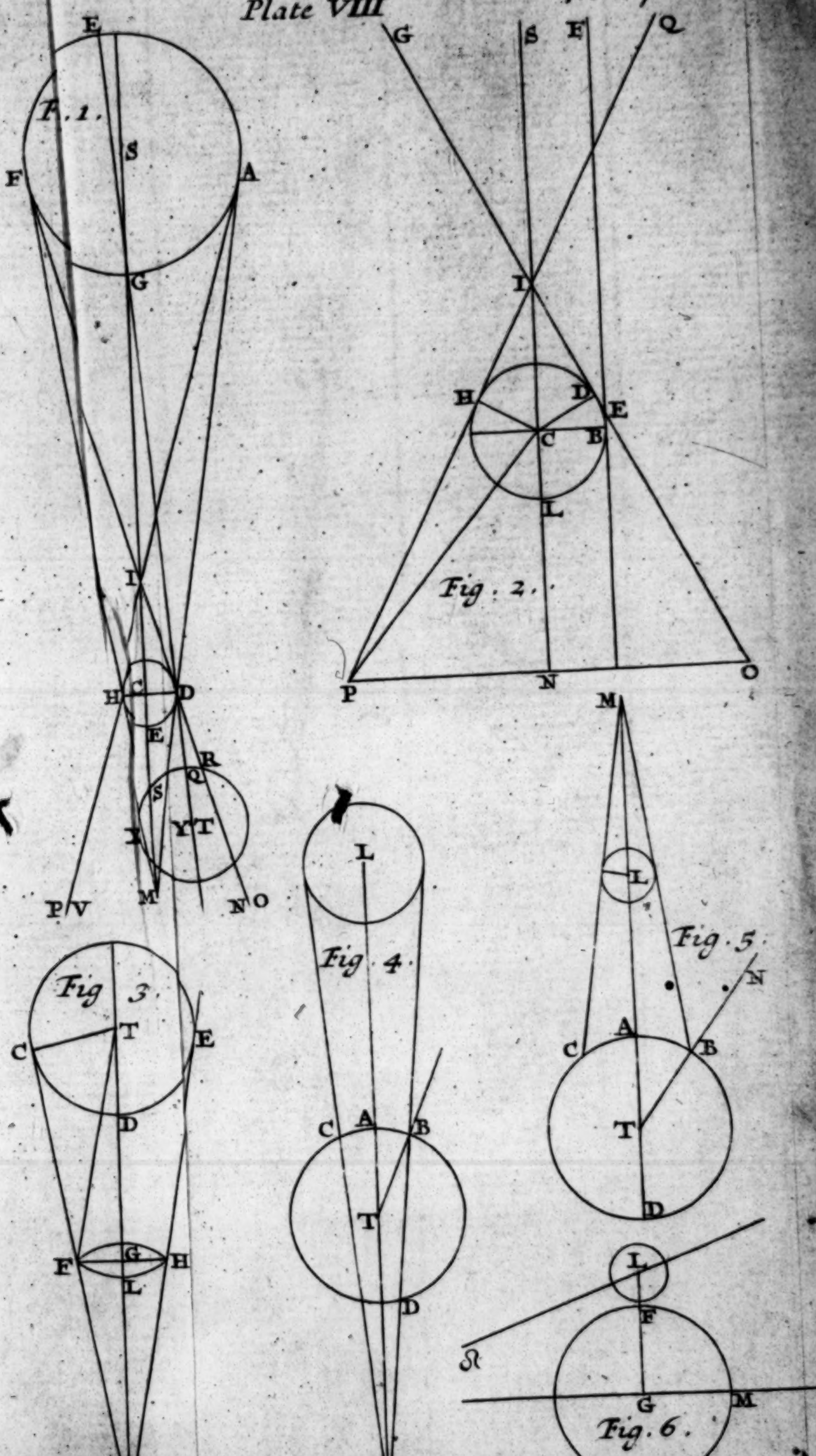
The Limits *determined.*

Let L be the Center of the *Moon*, whoſe conical Shadow is D M E. Imagine this Shadow cut

10

Plate VIII

front. p. 104.



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at the Distance of the *Earth*, with a Plane perpendicular to its *Axis*; the Section will be a Circle, whose Semidiameter T P is called the Semidiameter of the *Moon's* Shade: Now the Angle under which this Semidiameter appears, seen out of the *Moon*, is equal to the Difference of the Semidiameters of the *Sun* and *Moon* seen from the *Earth*: For the Angle L P D is the apparent Semidiameter of the *Moon* seen from the *Earth*, which is equal to the two internal Angles P M L and P L M; and therefore, if we subtract from the Angle L P D, which is the apparent Semidiameter of the *Moon*, the Angle P M L, which is equal to the apparent Semidiameter of the *Sun*, there will remain the Angle P L T, the apparent Semidiameter of the Shadow seen from the *Moon*.

Lecture XII. Plate IX. Fig. 4. *The Diameter of the* Moon's *Shadow at the* Earth, *when seen from the* Moon.

AGAIN: Let L be the Center of the *Moon*, A M B the *Penumbral* Cone of the *Moon*, extended as far as the *Earth*, and its *Axis* M T; if this Cone at the Distance of the *Earth* be cut by a Plane transversely, the Section will be a Circle whose Semidiameter is A T, and is called the *Semidiameter of the Penumbra*. And the Angle under which it appears from the *Moon*, is the Angle T L A, which is the external Angle of the Triangle L M A, and is equal to both the inward Angles L A M and L M A. But the Angle L M A is half the Angle of the Cone, which is the same with the apparent Semidiameter of the *Sun*; and M A L or C A L is equal to the apparent Semidiameter of the *Moon*, is from the *Earth*; and therefore the Apparent Semidiameter of the *Penumbra* seen from the *Moon* seen equal to the Sum of the Semidiameters of both the *Sun* and *Moon*.

Plate IX. Fig. 5.

The Apparent Semidiameter of the Penumbra.

IF the *Sun* had no apparent Motion arising from the real Motion of the *Earth*, the Way of the *Moon* from the *Sun* would be the same with her real Way in her Orbit: But because, while the *Moon* is proceeding in her Orbit, the *Sun* also seems to move in the Ecliptick, the Way the *Moon* moves towards

The Way of the Moon *from the* Sun.

Lecture XII. Plate IX. Fig. 6.

or from the *Sun*, will be different from that which ſhe has in her Orbit; and its Inclination to the Ecliptick will be greater, than the Inclination of the Orbit to it. Let ☊ A be a Portion of the *Moon*'s Orbit produced to the Ecliptick: And ſuppoſe the *Sun* and *Moon* in Conjunction in the *Node*: Now, then, while the *Moon* in her Orbit deſcribes the Space ☊ L, the *Sun* by his apparent Motion will deſcribe the Space ☊ S in the Ecliptick, and S L will be the Way of the *Moon* from the *Sun*. Now if two Bodies be both moved the ſame Way, but one faſter than the other, their relative Motion, whereby the one recedes from the other, is the ſame as if the floweſt Body ſtood ſtill, and the other moved on with the Difference of Velocities, as we have demonſtrated in our Phyſical Lectures. Thro' the Place of the *Moon* L, draw B L parallel to the Ecliptick, to which let ☊ B be perpendicular. Now while the *Moon* in her Orbit deſcribes the Space ☊ L, its Motion according to the Ecliptick is equal to the Space B L; take L *l* equal to ☊ S, and draw ☊ *l*, it will be parallel to S L; and the Motion of the *Moon* from the *Sun* will be the ſame as if the *Sun* had remained in the *Node*, and the *Moon*, according to the Ecliptick, had been carried with the Velocity B *l*, which is the Difference of the Velocities of the *Sun* and *Moon*, according to the Ecliptick. Becauſe the Angles B L ☊ and B *l* ☊ are but ſmall, the Angle B L ☊ will be to the Angle B *l* ☊, as B *l* is to B L; that is, as the Difference of the Motions of the *Sun* and *Moon*, according to the Ecliptick, is to the Motion of the *Moon*, in the Ecliptick, ſo will the Angle which the Orbit of the *Moon* makes with the Ecliptick, be to the Angle B *l* ☊, which is equal to the Angle L S E, or the Inclination of the Way of the *Moon* from the *Sun* to the Ecliptick: And by this Means we can find out the Angle, which a Circle of Latitude, drawn through any Point of the Ecliptick, makes with the Way of the *Moon* from the

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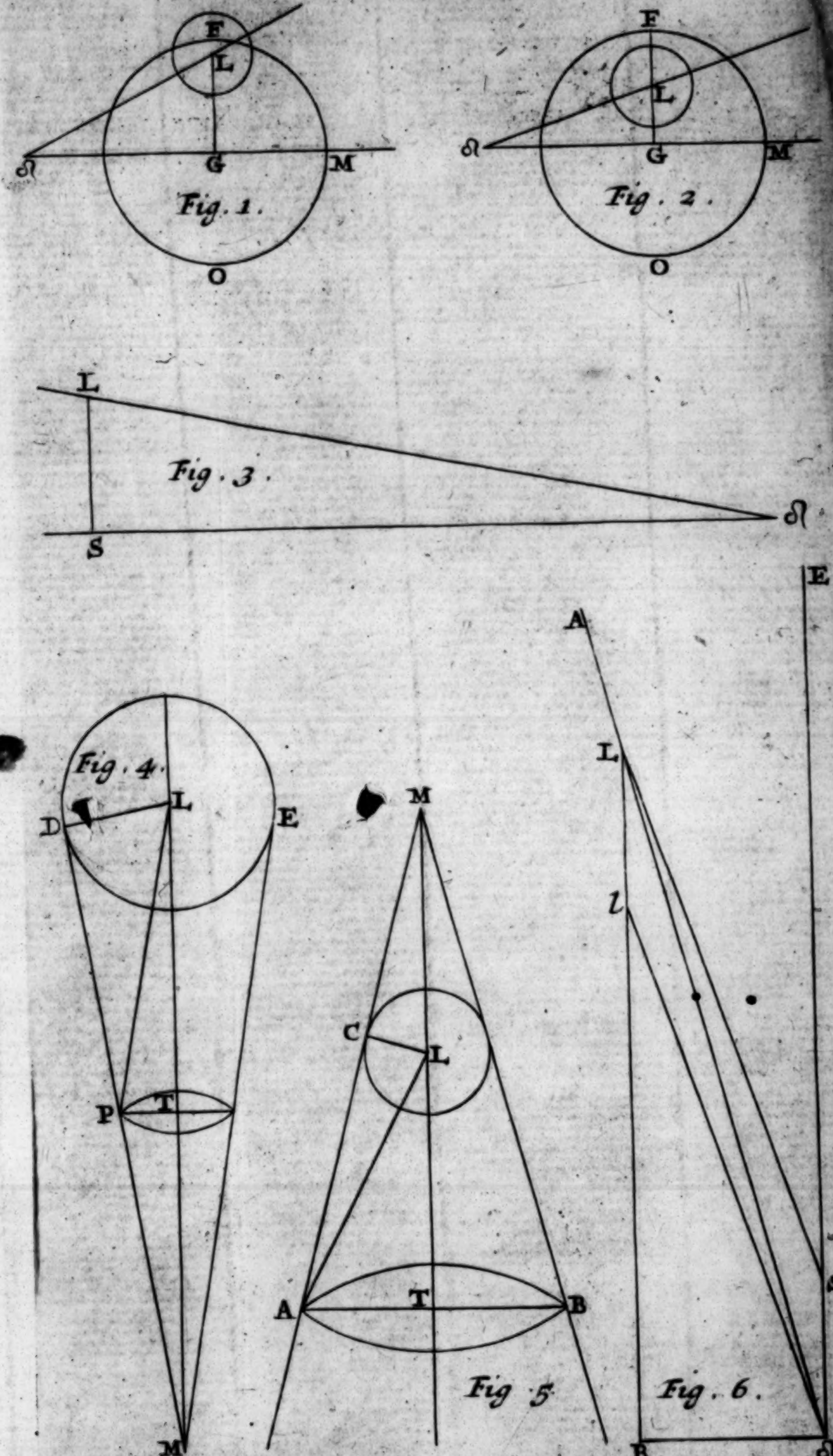
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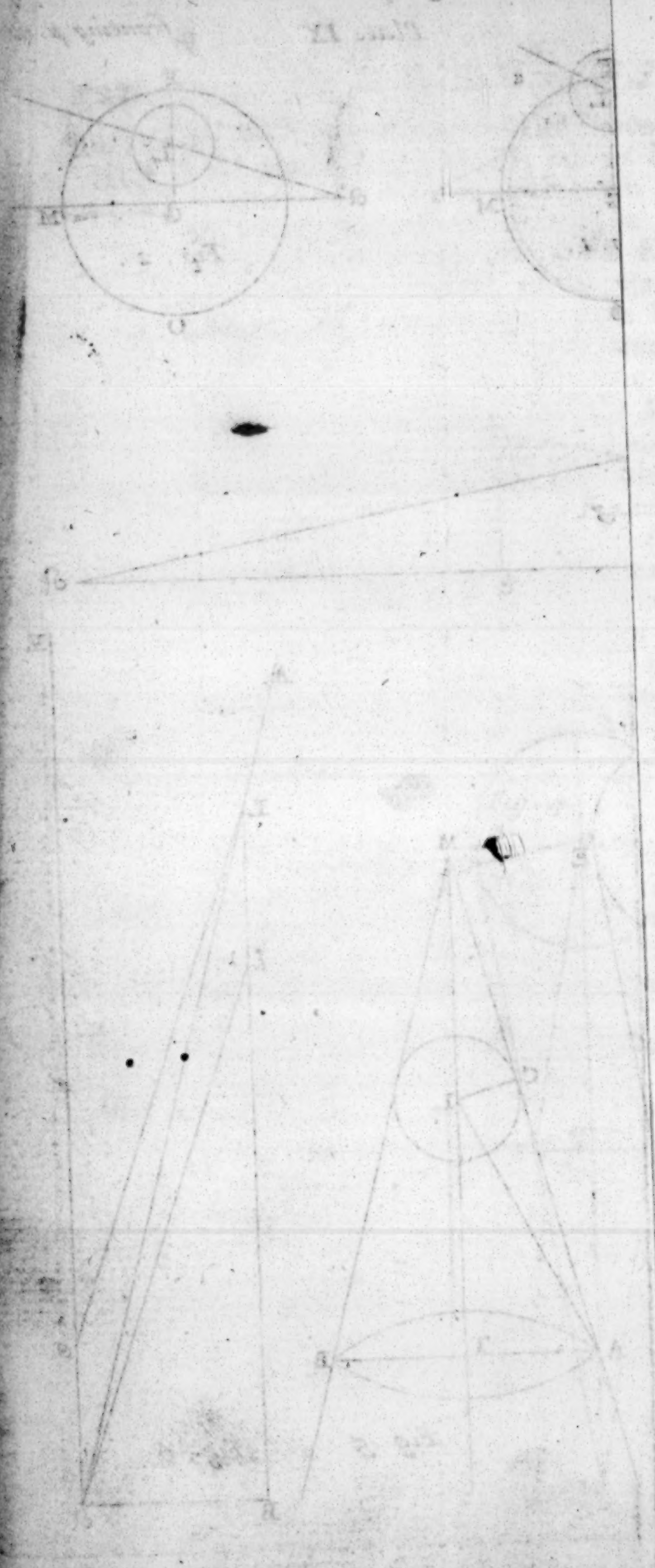


the *Sun*. For in the rectangular sperical Triangle, which the Ecliptick, the Circle of Latitude, and the Way of the *Moon* from the *Sun* do form, we have one Angle, which is the Inclination of the Way of the *Moon* from the *Sun* to the Ecliptick, and its Base, which is the Distance of the Circle-Latitude from the *Node*; and therefore we can find the other acute Angle. Lecture XIII.

LECTURE XIII.

Of the Projection of the Moon's *Shadow on the Disk of the* Earth.

IF a Right Line be projected on a Plane that is parallel to it, by letting fall from all its Points Perpendiculars on the Plane, the Projection or the Place where all Perpendiculars meet with the Plane will be a Right Line, parallel and equal to the former Line which was projected. For the Perpendiculars that fall from the Extremities of the Right Line, are parallel and equal; and therefore the Lines which join them will be parallel and equal.

HENCE, if two Right Lines touching one another, be parallel to any Plane, the Projections of these two Lines upon that Plane, will contain an Angle, equal to the Angle the Lines themselves make together; this is plain by *Prop*. 10. Book XI. of *Euclid*. Hence all Plane Figures projected on a Plane parallel to themselves have for

Lecture XIII.

for their Projections, Figures exactly similar and equal to themselves.

Plate X. Fig. 1.

But if a Line be inclined to any Plane, its Projection upon that Plane, made by letting fall from it Perpendiculars to the Plane, will be to the Line itself, as the Cosine of the Inclination of the Line is to the Radius. For let A B be a Line inclined to the Plane, and let D E represent the Plane: Letting fall from the Points A and B, the Perpendiculars A *a*, B *b*; *a b* will be the Projection of the Line A B; to which if we draw through B the parallel Line B C, meeting with the Perpendicular A *a* in C, this Line B C is equal to *a b*: But B C is to A B, as the Sine of the Angle C A B, or the Cosine of the Angle A B C to the Radius; that is, as the Cosine of the Angle of Inclination is to the Radius, so is *a b* to A B. Hence it follows, that every Figure, whose Plane is perpendicular to the Plane of the Projection, is projected in a right Line. For the Perpendiculars from every Point of the Figure, will all fall upon the common Intersection of the Plane of the Figure, with the Plane of the Projection. Such a Projection of Lines and Figures is called an *Orthographical* Projection.

An Orthographical *Projection, what?*

If we imagine a Plane to pass through the Center of the *Earth*, so that the Line which joins the Centers of the *Sun* and *Earth*, may be perpendicular to this Plane, it will make on the Surface of the *Earth* a Circle, which will separate the illuminated Hemisphere of the *Earth* from the dark. This Circle we before called the Circle bounding Light and Darkness, but we will now call it the illuminated Disk; which Disk is directly seen by a Spectator placed at the Distance of the *Moon*, in the Right Line which joins the Centers of the *Sun* and *Earth*. Upon this Circle the *Earth*'s Equator, its Parallels, *Poles*, and all the other Circles which we imagined, are to be supposed, projected *Orthographically*. For all Lines drawn from the Center

The Disk the Earth.

The Orthographical *Projection on the Disk.*

Center of the *Sun* to every ſingle Point of the Diſk, are to be accounted parallel; and therefore ſince that Line which is drawn to the Center of the Diſk is perpendicular to it, all the reſt will be perpendicular to it; and therefore all Lines drawn from the Center of the *Sun*, and paſſing through every Point of any Circle upon the *Earth*'s Surface, when they are produced, will be perpendicular to the Plane of the Diſk. Moreover a Spectator in the *Moon* will ſee all Countries, Cities and Towns to move upon the Diſk, which Motion is occaſioned by the Rotation of the *Earth* round its *Axis*, and every Point will have its Way on the Diſk; for by the diurnal Gyration all Places deſcribe either the Æquator, or one of its Parallels; and if the *Sun* be in the Plane of the Æquinoctial, or rather, if the Plane of the Æquinoctial paſſes through the *Sun*, the Æquinoctial and all its Parallels are in that Caſe projected into right Lines; for they will all be perpendicular to the Diſk, or the Plane of the Projection. But in other Poſitions the Projections of theſe Circles will be Ellipſes, which ere the Ways that all the Places of the *Earth* are ſeen to move in on the Diſk: Now if through the *Pole* and the *Sun* there be a great Circle drawn which cuts the *Earth*, and this Circle be projected on the Diſk, we ſhall have an univerſal Meridian, to which when any Place is obſerved to come, the Inhabitants of that Place will have Mid-day. And when any Place is firſt ſeen to touch the *Weſtern* Limb, or Edge of the Diſk, the Inhabitants of that Place will then ſee the *Sun* ariſing upon them. But a Spectator at the *Moon* will ſee the Place to riſe and come upon the Diſk, and will ſee it move towards the *Eaſt*: And as ſoon as it has paſſed the univerſal Meridian, the Place then being gone to the *Eaſtward*, the *Sun* ſeen out of the *Earth* from the Place will appear to move *Weſtward*. But when the Place comes to the *Eaſtern* Edge of the Diſk,

Lecture XIII.

The univerſal Meridian.

Lecture XIII. our Spectator in the *Moon* will obſerve the Place to ſet in the Diſk, and hide itſelf in the dark Side; but the Inhabitants of that Place upon the *Earth's* Surface will ſee the *Sun* ſet in the *Weſt*, and withdraw himſelf out of Sight.

The Bigneſs of the Diſk determined. THE Bigneſs of the Diſk is to be eſtimated by the Angle under which the *Earth* is ſeen from the *Moon*, and is of the ſame Quantity with the *Horizontal Parallax* of the *Moon*. And if from the *Moon* there be let fall a Perpendicular upon the Plane of the Ecliptick which meaſures the Diſtance of the *Moon* from the Ecliptick, this Line being parallel to the Plane of the Diſk, will be projected into a Line equal and parallel to itſelf; and the Angle under which the Projection of this Line appears from the *Moon*, will be equal to the Angle under which the ſame Line is ſeen from the *Earth*; for equal right Lines at equal Diſtances, being ſeen directly, appear under equal Angles. *The Projection of the* Moon's *Way on the Diſk.* The Way of the *Moon* from the *Sun*, if ſuch a ſmall Portion of it be taken as is turned towards the Diſk in the Time of an Eclipſe, may be eſteemed as a right Line, and will be projected upon the Diſk into a right Line, which is equal to itſelf; and its Projection with the Projection of the Circle of Latitude will contain the ſame Angle, that theſe two Lines make in the Heavens: The Spectator in the *Moon* will ſee this Line to be deſcribed upon the Plane of the Diſk, by the Center of the Shadow and of the *Penumbra* which coincide.

Plate X. Fig. 2, 3, 4, 5. LET now the Circle DGK repreſent the Diſk of the *Earth*, whoſe Semidiameter let us ſuppoſe to be divided into as many Parts as the *Horizontal Parallax* of the *Moon* contains Minutes and Seconds. Let the Line NT repreſent the Diſtance of the *Moon* from the Ecliptick in the Time of *New Moon*, projected on the Plane of the Diſk; which alſo muſt conſiſt of as many Parts as the Latitude of the *Moon* contains Minutes; and let likewiſe K ☊ be a Portion of the Ecliptick, ☊ *l*

a Portion

a Portion of the Way of the *Moon* from the *Sun*, both projected on the Plane of the Disk. From the Center T, let fall on the Way of the Shadow a Perpendicular T V; this Line will measure the least Distance that is between the Centers of the Disk and Shadow. At the Center V describe a small Circle, whose Semidiameter may be equal to the Difference of the apparent Semidiameters of the *Moon* and *Sun*; this Circle will represent the Breadth of the Shadow: For we have shewed, that this Shadow, seen from the *Moon*, at the Distance of the *Earth*, was equal to the Excess whereby the apparent Diameter of the *Moon* exceeds that of the *Sun*. Again, let there be described another Circle H M, at the Center V, whose Semidiameter V H bears the same Proportion to the Semidiameter of the Disk, as the Sum of the Semidiameters of both *Sun* and *Moon* bears to the apparent Semidiameter of the *Earth* seen from the *Moon*; or, which is the same, to the *Horizontal Parallax* of the *Moon*: This Circle will represent the *Penumbra*, projected at its shortest Distance from the Center of the Disk: For we have shewed, that the apparent Semidiameter of the *Penumbra*, seen from the *Moon*, was equal to the Semidiameters of both *Sun* and *Moon*. And therefore, if this Circle does not touch the Disk there can be no *Eclipse*; that is, if the Distance V T be greater than the Sum of the Semidiameters of the Disk and the *Penumbra*; or, which is the same, greater than the Sum of the Semidiameters of the *Sun* and *Moon*, and of the *Horizontal Parallax* of the *Moon*, there can be no *Eclipse*: But if the Distance V T be equal to this Sum, the *Penumbra* will touch the Disk, without obscuring the *Sun*, or any Part thereof: But if V T be less than this Sum, that is, if it be less than V M and T R, the *Penumbra* will cover some Part of the Disk; and those that lie within the Segment R Z M Y, will see at least a partial Eclipse of the *Sun*. But if the Distance V T be less than the Difference of the Semidiameter of the Disk, and the Semidiameter of the Shadow, then the Shadow

Lecture XIII.

The Proportion of the Moon's *Latitude.*

The Shadow projected.

The Penumbra *projected.*

When there can be no Eclipse of the Earth.

When there must be an Eclipse.

Lecture XIII.

dow will cover some Part of the Disk, and make a total *Eclipse* of the *Sun* in all those Places it passes over. These total *Eclipses* are always but for a small Portion of Time, because the Diameter of the total Shade is but small; the apparent Diameter of the *Moon* never exceeding the apparent Diameter of the *Sun*, but by a very small Matter, which is seldom equal to two Minutes; which Space the Center of the Shade passes over on the Disk in the Space of four Minutes of Time: But yet the Stay that it may make on any Place, may be longer than this; upon the Account of the Motion of the Place which follows the Shade.

Total Eclipses of the Sun.

The Ecliptick Limits.

HENCE we may know the *Ecliptick Limits*, or the Distance of the *Moon* from the *Nodes*, at the Time of *New Moon*; so that an Eclipse of the *Sun* may be possible: For let the Circle R O G represent the Disk of the *Earth*; ☊ T K and ☊ N, the Projections of a Portion of the Ecliptick, and of the Way of the *Moon* from the *Sun* upon the Plane of the Disk. And let T V be the least Distance of the Center of the Disk and Shadow; which suppose equal to the Sum of the Semidiameters of the Disk and *Penumbra*: Then in the rectangular Triangle ☊ T V, we have the Side T V; which, when it is biggest, is about $94\frac{1}{2}$ Minutes. We have likewise an Angle at ☊, which, when it is least, is 5 Degrees and 30 Minutes: From whence we shall find ☊ T equal to 986 Minutes; or to 16 Degrees and 26 Minutes. And since in this Case the *Penumbra* only touches the Disk, it is plain, that there can be no Eclipse, unless the *New Moon* be nearer to the *Node* than 16 Degrees and 26 Minutes.

Plate X. Fig. 6.

LET the Circle R K G, as before, represent the Disk of the *Earth*; ☊ T K a Portion of the Ecliptick projected on the Plane of the Disk; ☊ *l* the Way of the *Moon*'s Shadow on the Disk; T N the Latitude of the *Moon*, and T V the shortest Distance of the Centers of the Shadow and Disk. Let the Circle O P Q be the *Penumbra*, moving from

from D by V to l; in the Middle of which is a small Circle representing the Shadow: And let us suppose we know the Time of the Conjunction; that is, when the Center of the Shadow is in N, which we find by Astronomical Tables; by that Means we can find the Time when the Center of the Shade is in V, that is, the Time of the Middle of the general Eclipse. For in the rectangled Triangle T V N, we have T N the Latitude of the *Moon*, and the Angle T N V, which the Circle of Latitude makes with the Way of the *Moon* from the *Sun*; and therefore we can find V N and T V. But by the Horary Motion of the *Moon* from the *Sun*, we can find out the Time the Shadow will pass through the Space N V; and this Time either added or subtracted from the Time of the Conjunction, will give the Time of the Middle of the Eclipse. Moreover in the Triangle D V T, right-angled at V, we have the Side D T, which is the Sum of the Semidiameters of the Disk and *Penumbra*, and the Side T V the shortest Distance of the Shade from the Center of the Disk; therefore we can find out the Side D V, and by it the Time when the Shade enters the Disk; and from that we shall have the Semiduration, or half the Time the Eclipse continues upon the Disk. After the same Manner we shall find the Time when the Shade leaves the Disk, or the Time of the End of the Eclipse.

Lecture XIII.

The Time of the Middle of the Eclipse.

The Semiduration.

Having the Place of the *Sun* in the Ecliptick for any Moment of Time, we can thereby find the Place on the Surface of the *Earth*, upon which the *Sun* at that Time is vertical: For the Latitude of the Place is equal to the Declination of the *Sun* at that Time; and its Longitude is found by turning the Time from the Meridian into Degrees and Minutes of the Æquator, allowing for every Hour 15 Degrees, and for every Minute of an Hour 15 Minutes of a Degree. For Example. The Longitude of a Place, in whose *Vertex* the *Sun* is, when at *Oxford* we reckon 9 and an half in the Morning, is

To find the Place to which the Sun *is vertical.*

Lecture XIII. known by subtracting 9 and a half from 12, and then there will remain 2 Hours and thirty Minutes; which multiplied by 15, make 37 Degrees and 30 Minutes. And therefore that Place will be 37 Degrees and 30 Minutes to the *East* of *Oxford*.

Plate X. Fig. 7.

LET the Circle FRK represent the Disk, as before; FTK a Portion of the Ecliptick projected on the Disk; on which from the Center erect the Perpendicular TR: This Line will be the Projection of the *Axis* of the Ecliptick, and the Point R of its Pole. Let the Point P be the Projection of the *Pole* of the *Earth*: Through T and the Pole P let us imagine a Circle to pass, and to be projected on the Disk. This Circle will represent the universal Meridian; and the Elevation of the Pole above the Plane of the Disk, will always be equal to the Declination of the *Sun*. For the Arch of the Meridian between the *Sun* and *Periphery* of the Disk, is a Quadrant of a Circle; and the Arch of the Miridian between the Æquator and the Pole, is likewise a Quadrant: Wherefore taking away from equal Arches the common Arch TP, there will remain PS, the Elevation of the Pole above the Disk, equal to the Distance of the *Sun* from the Æquator.

The Elevation of the Earth's *Pole above the Disk.*

To find the Position of the Meridian which passes thro' the Sun.

It is here to be observed, that when the *Sun* is in the Signs ♑ ♒ ♓ ♈ ♉ ♊, or rather when the *Earth* is in the opposite Signs, the Point S, wherein the universal Meridian cuts the Limb of the Disk, will fall towards the Right Hand of the Pole of the Ecliptick. But when the *Sun* is in the other six Signs, the Point S falls upon the other Side of the Pole of the Ecliptick, contrary to what happens when the Projection is supposed to be made in a Plane parallel to the Disk at the Orbit of the *Moon*.

To find the Angle RTS, or the Arch of the Disk intercepted between the Pole of the Ecliptick and the Point S: In the right angled spherical Triangle RPS, we have the Arch RP, the Distance of the Poles of the Ecliptick and Æquator $23\frac{1}{2}$ Degrees; also the Side PS, which is equal to the Declination

clination of the *Sun:* Wherefore, by *Trigonometry*, we may find the Side R S, or the Measure of the Angle R T S. Lecture XIII.

To find the Place upon the *Earth*'s Surface Q, where the *Sun* rising begins to be in an Eclipse in its upper Limb, and where the Shadow enters the Disk: Draw through the Pole the Meridian P Q, to the Point Q, where the *Penumbra* first touches the Disk and first, in the right-lined Triangle D T V, having DT and T V, we may know the Angle DTV; to which if we add or subtract a given Angle V T P, which is the Sum or Difference of two known Angles V T N and NTB, we shall have the Angle Q T P. And then in the spherical Triangle on the Surface of the *Earth* S P Q, which is right-angled at S, we have S P equal to the Declination of the *Sun*; and the Arch S Q, which is the Measure of the known Angle S T Q; from whence we may find the Arch P Q, the Complement of the Latitude of the Place Q, and the Angle S P Q, whose Complement to two Rights is the Angle Q P T, contained between the Meridian of the Place Q, and the Meridian of that Place to which the *Sun* is then vertical; and is the Measure of the Distance of the Meridian of the Place Q, from that of the Place which has the *Sun* in the Meridian at that Time. But we know the Place which has the *Sun* at that Time in its Meridian; wherefore we know likewise the Meridian or Longitude of the Place Q: But its Co-Latitude P Q was before found out. Having therefore both the Longitude and Latitude of that Place, the Place itself must be known.

To find the Place of the Earth, which is first touch'd by the Penumbra.

By the same Method we can find out the Place of the *Earth*, which is first involved in the total Shadow. And by a like Process we may find out the Place M on the Surface of the *Earth*, which is under the total Shadow at any Point of Times either before or after the Middle of the Eclipse. For the Time being known, we may find by the Horary Motion of the *Moon* from the *Sun*, the Line M V and the

Lecture XIII.

the Point M, where the Center of the Shadow lies; and in the right-lined Triangle M V T, having M V and V T, we may find M T and the Angle M T V; to which if you add or ſubtract the known Angle V T P, we ſhall have the Angle M T P. But M T is the Sine of an Arch of the vertical Circle which paſſes through M, and the Point of the *Earth*'s Surface directly under the *Sun*, the Semidiameter of the Disk being made the Radius. And therefore, if we ſay, As the Semidiameter of the Disk is to M T, ſo is the Radius to the Sine of an Arch; the Arch found out by this Proportion will be the Diſtance of the *Sun* from the *Vertex* M: And therefore in the ſpherical Triangle on the Surface of the *Earth* M P T, we have P T the Diſtance of the *Sun* from the Pole; and M T the Diſtance of the *Sun* from the *Vertex*, and the Angle M T P. Hence we can find M P, which is the Complemet of the Latitude of the Place; and the Angle M P T, which ſhews the Difference of the Meridians of the Place M, and of that Place to which the *Sun* at that Time is vertical; which Place by the Time is known, and therefore we can find the Place M. And by this Method we may find ſeveral Places over which the Shade does paſs; and if they be joined by Lines, we ſhall have the Way of the Shade upon the Surface of the *Earth*.

The Determination of the Place involved by the Shadow, for any given Time.

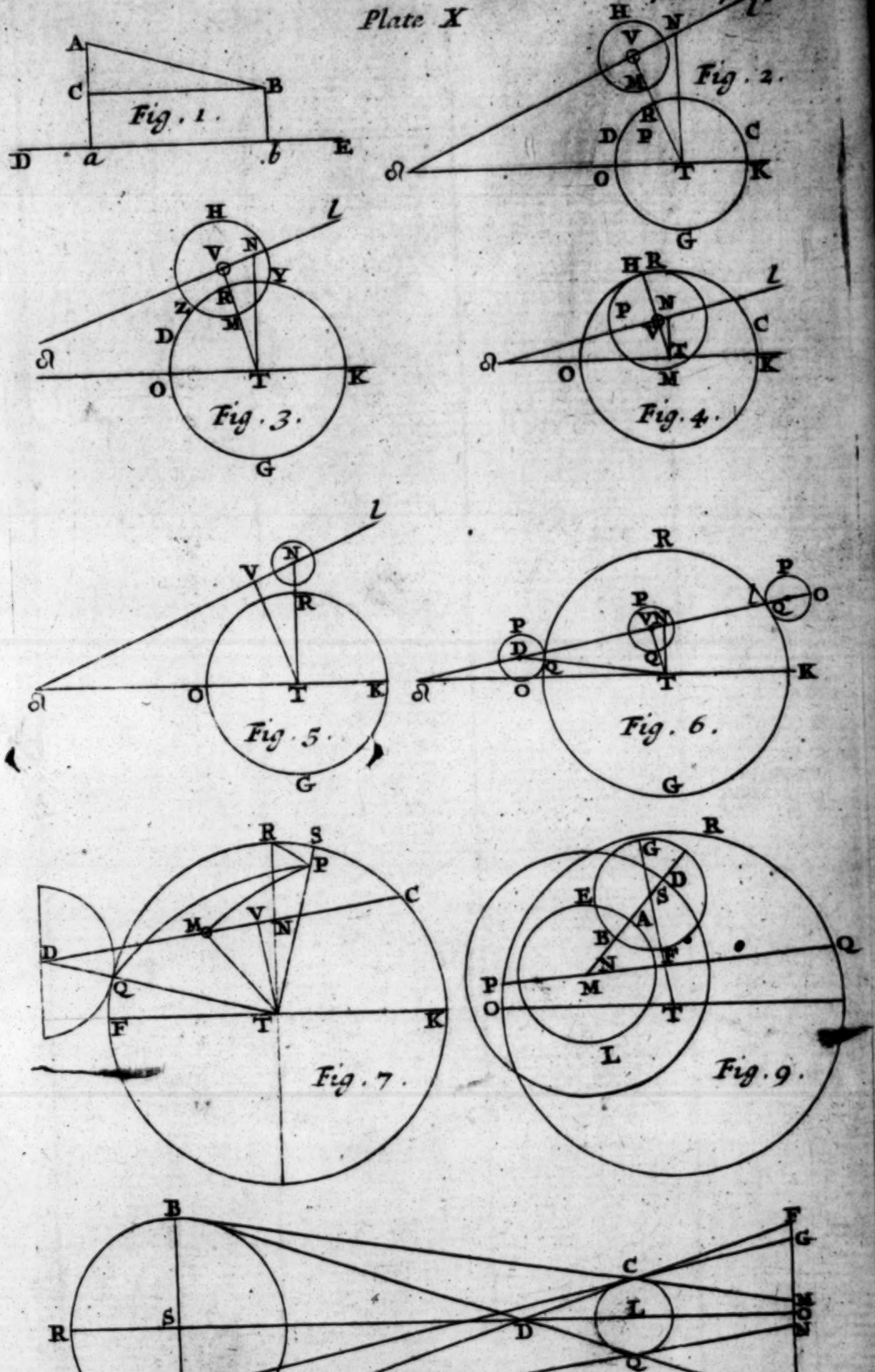
THE Portion of the Diameter of the *Sun* obſcured by the *Moon*, is known by the Situation of the *Spectator* within the *Penumbra*; or by his Diſtance from the Center of the Shade. For let ASB be the Diameter of the *Sun*, parallel to the Diameter of the *Penumbra* E F; draw the Line M C B, touching the ſuperior Edges of both *Sun* and *Moon*; and let F C A alſo touch the inferior Margin of the *Sun*: Then the Angle A C B is equal to the apparent Diameter of the *Sun*; and the Triangles ACB and M C F are ſimilar. Suppoſe then a *Spectator* within the *Penumbra* at G; draw the Line G C P, touching the Globe of the *Moon*, which nearly paſſes through the former

Plate X. Fig. 8.

The Portion of the ſolar Diſk obſcured by the Eclipſe.

front. p. 138.

Plate X



mer Point C, and cuts off A P, that Part of the *Sun*'s Diameter, which is obscured by the *Moon*, to the *Spectator* in G. But the right Line G P, since it very nearly passes through the *Vertex* of the Triangles M C F and A C B, will divide the Bases A B and M F in a like Proportion. And therefore A P is to A B, as F G to M F; that is, the obscured Portion of the *Sun*'s Diameter is to the whole Diameter, as the Distance of the Place from the Edge of the *Penumbra*, which is F G, is to F M the Semidiameter of the *Penumbra* diminished by the Semidiameter of the total Shadow.

The Quantity of the Eclipse estimated by Digits.

THE *Astronomers* commonly divide the Diameters of both Solar and Lunar Disks into twelve equal Parts, which they call *Digits*; and by them they measure the Quantity of the Obscuration. And they say, the Eclipse is of so many *Digits*, as the obscured Portions consists of such Parts.

Having the Position of the Place on the Disk, to find the Phasis *of the Eclipse.*

IF we know the Position of any Place upon the Disk for any Point of Time, and it be desired to find the *Phasis* of the Eclipse for that Time, as it is seen from the Place, it is to be found in this Manner: Let S be the Position of the Place on the Disk; find out for that Point of Time, the Place of the Center of the Shade in its Path, which let it be M. At the Center M, with a Semidiameter, equal to the Semidiameter of the *Moon*, describe the Circle A F L: Also at the Center S, with a Semidiameter S B, equal to that of the *Sun*, describe the Circle E F G, which the Circle A F L, cuts in E and F; then E B F A will be that Part of the *Sun*, which is covered by the *Moon* from a *Spectator* in S. For produce the Semidiameter of the *Moon* M A through S, that A D may be equal to the Semidiameter of the *Sun*, which is equal to B S; and then M D will be equal to the Sum of the Semidiameters of both *Sun* and *Moon*; and therefore it will be equal to the Semidiameter of the *Penumbra*: And the Distance of the Place from the Edge of the *Penumbra* is S D: And because B S is equal to A D, A B will be equal to

Plate XI. Fig. 9.

Lecture XIII.

to S D. Take A N equal to the Semidiameter of the *Sun*, and then M N will be equal to the Difference of the Semidiameters of the *Sun* and *Moon*, which is equal to the Semidiameter of the total Shadow. But we shewed, *pag.* 136, that S D was to D N, as A B the Part of the *Sun*'s Diameter obscured was to the *Sun*'s Diameter. But A B is equal to S D, and D N is equal to D A and A N, which are equal to two Semidiameters of the *Sun*, or to one Diameter of the *Sun*. Therefore A B is to the Diameter of the *Sun*, as S D, the Distance of the Place from the Edge of the *Penumbra*, is to D N, which is the Semidiameter of the *Penumbra* diminished by the Semidiameter of the total Shadow. And therefore it is plain, that A B must represent the Portion of the *Sun*'s Diameter, which is then seen to be obscured.

HENCE also is determined the Position of the *Cuspides*, Points or Horns of the Eclipse: For by drawing the vertical Circle T S G, the Arches G E, G F, shew the Distance of the Points from the supreme or highest Point of the *Sun*'s Limb.

The Velocity of the Shadow on the Disk.

IF the Velocity wherewith the Shadow goes over the Disk, be inquired for, it must be observed, that the Way of the *Moon* from the *Sun* is projected on the Disk, in a Line parallel and equal to itself; which Line is described by the Motion of the Shadow: And therefore the Velocity of the Center of the Shade is equal to the Motion of the *Moon* from the *Sun*. Now the Motion of the *Moon* from the *Sun* is about $30\frac{1}{2}$ Minutes in an Hour; though it is sometimes more, and sometimes less. And therefore the Space, which the Center of the Shade moves thro' in an Hour, is about $30\frac{1}{2}$ Minutes of the Lunar Orbit. Now the Semidiameter of the Lunar Orbit is about 60 Semidiameters of the *Earth*; and therefore one Minute of the Orbit of the *Moon* is as much as 60 on a great Circle of the *Earth*, or as much as one Degree, that is, 69 *English* Miles. And therefore $30\frac{1}{2}$ Minutes are equivolent to 2104 *English* Miles; which Space the Shade moves through on the

Disk

Disk in one Hour. But though this be the Velocity of the Shadow on the Disk, yet the Velocity whereby the Shade recedes from any given Place on the *Earth*'s Surface, is less than it; by reason that while the Shadow moves from the *East* to the *West*, all the Places of the *Earth* are likewise carried by the Rotation of the *Earth* the same Way: And therefore, following the Motion of the Shadow with a slower Pace, they diminish the Velocity whereby the Shade moves from them. Lecture XIV.

LECTURE XIV.

A new Method of computing Eclipses of the Sun, *as they are to be observed from any given Place on the* Earth's *Surface.*

HITHERTO we have explained all the *Phases* and Appearances of a general Eclipse, such as a *Spectator* at the *Moon* would observe; and we have shewed the Methods whereby the Beginning, Middle, and End of an universal Eclipse may be determined. But this Beginning and End can be observed by only a few, who lie near the Edge of the Disk, where the Shadow enters. In other Places lying towards the Middle of the Disk, there will be no Eclipses seen, till some Time after, when the Margin or Edge of the *Penumbra* comes to touch them. And the End will be when the opposite Part of the *Penumbra* leaves them. And therefore, according to the different Situation of Places, the Quantity of the visible Eclipses will be different, as likewise their Beginning, End, and Duration.

The Beginning of an universal Eclipse can be observed by very few.

And the Beginnings in particular Places are very different.

THEREFORE to compute the particular *Phases* of an Eclipse, as it is to be observed from a certain Place,

Lecture XIV. Place, we must here explain a new Method, whereby, without the troublesome and tedious Calculations of *Parallaxes*, which all the *Astronomers* have made use of before, the particular *Phases* may be shewn.

A new Method to compute the Phases *for a particular Place.* Plate XI. Fig. 1.

LET therefore the Semicircle A E B be half the Disk of the *Earth*, in whose Circumference is the Point E, which answers to the Pole of the Ecliptick. And let the Pole of the *Earth*, or the Æquator, be projected into P. Because all the Places of the *Earth*, being carried by the diurnal Rotation, describe Circles, which are the Æquator or its Parallels, and all the Parallels, except in the *Æquinoxes*, are inclined to the Plane of the Disk, the Parallel which any Place describes will be projected into an Ellipse, which will be the Way or Path in which that Place will be seen to move by a *Spectator* at the *Moon*. Let therefore F XII D be the Ellipse in which the Parallel of any Place is projected on the Disk: And let the Points in which the Horary Circles cut the Parallels, be likewise projected: And let these Points be VI, VII, VIII, IX, X, XI, XII, I, II, III, IV, V, VI, So at Six in the Morning the Place on the *Earth*'s Surface will be seen on the Disk at VI; at the seventh Hour it will be at VII, at Eight of the Clock it will at VIII, and at Nine it will be in the Point IX, and so on.

All Parallels projected into an Ellipse.

LET C T be a Portion of the Path of the Shadow received on the Plane of the Disk: And suppose the Center of the Shade at Two of the Clock to be in the Point 2; at Three of the Clock let the Place of the Shadow be 3; and at Four let it be in the Point 4, *&c.* At Two of the Clock the Seat of the *Spectator* on the Disk is II; and therefore his Distance from the Center of the Shadow is 2 II: But if we measure this Distance according to the Path of the Shadow, we must let fall from the Place a Perpendicular to the Path, which let be II L, and the Distance thus estimated will be 2 L, and the Point L will be the Position of the Place reduced to the Path of the Shadow. At Three of the

Position of a Place reduced to the Path of the Shadow.

the Clock the Shade is at 3, and the Place at III, and their Distance is 3 III, which is less than the former Distance at Two. At the fourth Hour the Shadow is in 4, and the Place in IV. All which Time the Shadow advances nearer to the Place, till at last the Edge of the *Penumbra* touches it, and then the Eclipse will begin at that Place. At the fifth Hour, when the Center of the Shadow is at 5, and the Place is in V, it will be there more within the *Penumbra* than it was before, and the Shadow will be near the Place: But at Six, the Shadow being at 6, and the Place at VI, the Shadow will have got to the *East* of the Place, which is then in the Point of the Disk at VI; and therefore the Center of the Shadow has passed by the Place, and the Time of the shortest Distance between the Center of the Shadow and the Place, will be between the Hours of 5 and 6. After this Time the Distance of the Shadow and Place will constantly grow greater, and at last the *Western* Edge of the *Penumbra* will leave the Place, and there will be an End of the Eclipse upon that Place. But by the following Method the Beginning, Middle, End and Duration of the Eclipse will be more accurately determined. Upon which Account we must first premise the two following Problems:

Lecture XIV.

PROBLEM I.

To find upon the Disk of the Earth *the Position of a Place for any given Point of Time.*

Let the Semicircle A E B represent half the Disk, A B a Portion of the Ecliptick projected on the Disk. its *Axis* projected S E, meeting with the *Periphery* of the Disk in E, which will be the Pole of the Ecliptick on the *Earth*'s Surface. Let S P be the Line on which the *Axis* of the *Earth* is projected, and P the Projection of the Pole. As the *Radius* is to the *Sine* of the Latitude of the Place, so let S P be to S H, and the Point H will be the Projection of the Center

Plate XI. Fig. 2.

Lecture XIV.

Center of the Parallel. Through H draw H G equal to the Semidiameter of the Parallel, or to the *Sine* of the Distance of the Place from the Pole, which must be perpendicular to P S; this will be half the greatest *Axe* of the Ellipse into which the Parallel is projected. As the *Radius* is to the *Sine* of the Pole's Elevation above the Plane of the Disk, so let G H be to H L, and H L will be the lesser half *Axis* of the Ellipse. In G H take H Q, which has the same Proportion to G H, that the *Sine* of the Angle, which the Horary Circle and the Meridian make together, has to the *Radius*; and draw Q R perpendicular to G H: Also make the Proportion, as the *Radius* to the Cosine of the Angle which the Horary Circle and the Meridian make together, so the lesser *Semi-axe* H L to a fourth, to be laid on the Line Q R from Q to R; and then the Point R will be the Position of the Place on the Disk required at the Moment of the Time given.

The same may be done by the Help of the Horary Circle.

Plate XI. Fig. 3.

Let A O B be half the Disk, P the Pole, S P the *Universal Meridian*, meeting with the Disk in G; and let F P O be the Horary Circle for the Moment of Time given. In the right-angled Triangle P G O, we have P G the Elevation of the Pole above the Disk, and the Angle G P O, which the Horary Circle makes with the Meridian; whence we shall find out the Angle G O P, the Inclination of the Horary Circle to the Plane of the Disk, and the Arches P O and G O; and therefore we have the Point O, where the Horary Circle cuts the Disk. Draw S O, which will be the common Section of the Horary Circle with the Plane of the Disk. Let P F be the Complement of the Latitude of the Place. And S O being the *Radius*, take S Q, equal to the *Sine* of an Arch, whose Complement is the Sum of two Arches that are known, *viz.* F P and P O: and let D be the Cosine of the same Arch whose *Sine* is S Q. At Q upon O S erect the Perpendicular,

Q R,

QR, ſo that D may have the ſame Proportion to QR, that the *Radius* has to the *Coſine* of the Inclination of the Horary Circle to the Plane of the Disk; and R will be the Point which was deſired, and ſhews the Poſition of the Place on the Disk for the Time given. Lecture XIV.

The ſame Thing may be likewiſe found out this Way: Calculate, (by the *Problems* demonſtrated in the *Spherical Doctrine*, and in the *Problems* to be found where the Uſe of the Globes is taught) for the Moment of Time given, the *Sun*'s Altitude, and the Angle between the Vertical and Hour-Circle at his Center, and make the Angle RSP equal thereto; and take SR equal to the *Sine* of the Complement of the Altitude, or the *Sine* of his Diſtance from the *Vertex*, SE being the *Radius*, and then R will be the Point required. By the ſame Methods, for all other different Moments of Time, we can find out other Poſitions of the Place on the Disk. The Demonſtrations of all theſe Practices are eaſily deduced from the Laws of *Orthographical Projection*. Plate XI. Fig. 2.

PROBLEM II.

To find in an Eclipſe the Poſition of the Center of the Shadow on the Diſk, for any given Time.

Let AEB, as before, repreſent the *Semi-diſk*, SE the *Axis* of the Ecliptick, CT the Path of the Shadow, while it paſſes over the Diſk; and let it cut the *Axis* of the Ecliptick in N. Now when the Center of the Shadow is in N, then is the true Conjunction of the *Sun* and *Moon*, whoſe Time is known by Aſtronomical Tables. We have likewiſe by the ſame Tables the Horary Motion of the *Moon* from the *Sun*. Say, As the Horizontal *Parallax* of the *Moon* is to the Horary Motion of the *Moon* from the *Sun*, ſo is the Semidiameter of the Diſk to a fourth Line M: This will be the Space which the Shadow moves through upon the Disk in an Hour. Then Plate XI. Fig. 4.

ſay,

Lecture XIV. say, As one Hour is to the Time between the Conjunction and the Time for which the Position of the Shadow is sought; so is the Line M to a fourth; this Line will shew the Distance of the Shadow in its proper Path, from the Point of Conjunction N, and consequently the Place of the Shadow for the Time given. Suppose the Hour which immediately precedes the Time of Conjunction, to be any Hour you please; for Example, Let it be the Fourth Hour; say, As one Hour is to the Time between the Fourth Hour and the Time of the Conjunction, so let the Line M be to another, which is N 4; and the Point 4 will be the Position of the Shadow at Four of the Clock. Take likewise 4 3, 3 2, 4 5, 5 6, equal to M; and the Points 2, 3, 4, 5, 6, will shew the Place of the Shadow at the Hours 2, 3, 4, 5 and 6.

THESE Things being premised, let A E B be half the Disk, as before, C T the Path of the Shadow upon the Plane of the Disk, which the *Axis* of the Ecliptick cuts in N; and when the Shadow comes to N, then is the Time of the true Conjunction. Let, for Example, the 2d be the Hour which immediately precedes the Time of the true Conjunction, and then mark in the Path of the Shadow its Places at the Hours 1, 2, 3, 4, 5; and likewise at the same Time mark the Situation of the Place on the Disk at the same Hours; let them be I, II, III, IV, V. At One of the Clock the Distance of the Place and Shadow is 1 I; this, by applying to a Scale of equal Parts, is to be measured, and taken in Numbers; and from thence deduct the Semidiameter of the *Penumbra* measured by the same Scale, and we have the Distance of the Place from the Edge of *Penumbra*.

Plate XI. Fig. 4.

The Calculation of the Beginning of the Eclipse.

AT Two of the Clock, after the same Manner, take the Distance of the Edge of the *Penumbra* from the Place which is then in II; the Difference of these Distances, since the Edge of the *Penumbra* is in both Cases more *Westerly* than the Place, is the

the Appropinquation or relative Motion of the Place to the Shadow in one Hour. Say then, as the Appropinquation of the Margin of the *Penumbra* to the Place in one Hour is to the Diſtance of the ſaid Margin from the Place at Two of the Clock, ſo one Hour is to the Diſtance of Time from Two till the Beginning of the Eclipſe; which Time, added to the ſecond Hour, ſhews the Time when the Eclipſe begins.

The Calculation of the Time of greateſt Obſcuration.

FROM the Poſition of the Place at II, to the Path of the Shadow, let fall the Perpendicular II *a*: And becauſe the Center of the Shadow is at 2, the Diſtance of it from the Place reduced to the Path, is 2 *a*. Alſo at the third Hour the Poſition of the Place being III, let fall from thence a Perpendicular III *b* on the Path; the Diſtance of the Shadow from the Place reduced is 3 *b*; the Difference of theſe two Diſtances is the Acceſs of the Shadow to the Place reduced, in the Time of one Hour. Meaſure this Difference with a Scale, and by the Rule of Proportion ſay, As the Acceſs of the Shadow to the Place reduced in one Hour, is to the Diſtance of the Shadow and Place reduced at Three of the Clock, ſo is one Hour, or 60 Minutes, to a fourth Time; which Time added to the third Hour, gives the Time of the Middle of the Eclipſe, or of the greateſt Obſcuration.

The Calculation of the End of the Eclipſe.

AT Four of the Clock the Center of the Shadow is in 4, and the Place in IV; and becauſe 4 IV is leſs than the Semidiameter of the *Penumbra*, ſubtract it from the Semidiameter, and there will remain the Diſtance of the Place from the Edge of the *Penumbra*. Again, at Five of the Clock the Shadow is in 5, and the Place in V; and their Diſtance is 5 V, which is greater than the Semidiameter of the *Penumbra*; and therefore the *Weſtern* Edge of the *Penumbra* is now more advanced towards the *Eaſt* than the Place is; and therefore before that Time the *Penumbra* has quitted the Place, and the Eclipſe is at an End. From the Diſtance 5 V, ſubduct the Semidiameter of the *Penumbra*, and there will be left the Diſtance between the Place and the *Weſtern* Side of the

Lecture XIV.

the *Penumbra*: And because, in the former Case, at Four of the Clock, the Edge was *Westward* of the Place, and it has now got to the *East* of it, the relative Motion of the *Penumbra* and Place must be estimated by the Sum of these two Lines or Distances. Say then, As the Sum of these two Distances is to the Distance of the Edge of the *Penumbra* from the Place at the fourth Hour, so is one Hour to a fourth Time; which Time, added to Four, gives the Time when the *Penumbra* leaves the Place, or it will shew the End of the Eclipse.

THE Motion of the Shadow in its Path is equable, at least all the Time of an Eclipse it may be esteemed equable. But the Motion of a Place upon the Disk is no ways equable, but towards the Edge of the Disk it is slower; when it comes towards the Middle it goes through larger Spaces in equal Time. Moreover, our Calculus supposes, that the relative Motion of the *Moon* and Shadow are equable; and the Middle of the Eclipse, or greatest Obscuration, to be where the Line which joins the Place, and the Center of the *Penumbra*, is perpendicular to the Path of the Shadow; neither of which is precisely true, and therefore there will arise some small Error; but it may be corrected in this Manner: At the Time of the Beginning of the Eclipse, find out the Place of the Shadow, and likewise, for the same Time, the Situation of the Place upon the Disk. At the Center of the Shadow, with a Distance equal to the Semidiameter of the *Penumbra*, describe a Circle: If this Circle passes through the Point the Place is in, then is the Beginning of the Eclipse rightly determined; but if this Circle does not pass through the Place, note the Distance of the Place and the *Periphery*, and take the relative Motion of the Place and Margin of the *Penumbra* for half an Hour; and work again by the Rule of Proportion, as before, and we shall then have the true Time of the Beginning of the Eclipse. And by the same Method we may correct the Error that may arise in computing the End of the Eclipse. And by this means we may have the

Correction of an Error which may arise.

Beginning

Beginning and End of Eclipſes as accurately as by the common Method, which is by a troubleſome Calculation of the *Parallaxes*; where they likewiſe ſuppoſe, that the viſible Motion of the *Moon* is equable for a certain Time; which, nevertheleſs, is as unequable as the Motion of a Place upon the Diſk is.

Lecture XIV.

If about the Time of the Middle of the Eclipſe, at the Center of the Shadow, a Circle be deſcribed, whoſe *Radius* is equal to the Semidiameter of the *Moon*; and if likewiſe we deſcribe another Circle, whoſe Center is the Place of the *Spectator*, and whoſe *Radius* is the Semidiameter of the *Sun*; the Interſections of theſe two Circles will ſhew the *Phaſes* at the greateſt Obſcuration.

The Quantity of the greateſt Obſcuration.

If there be ſome who are not pleaſed with this mechanical Way of meaſuring Lines and Diſtances by a Scale, they may compute all the Lines by *Trigonometry* in the following Manner: As before, let AEB be the Diſk of the *Earth*, P the Projection of the *Pole*, CNT the Way of the Shadow, the Point 2 its Poſition at Two of the Clock; and, for the ſame Time, let II be the Situation of the Place of the *Spectator*. Let SE be the *Axis* of the Ecliptick, which cuts the Path of the Shadow in N; SN will be the Latitude of the *Moon* at the Time of the *Conjunction*. From the Center of the Shadow and the Place draw the Line 2 S, II S, to the Center of the Diſk, and join 2 II: Then in the right-lined Triangle 2 N S we have N S the Latitude of the *Moon*, and 2 N its Diſtance from *Conjunction* at Two of the Clock. We have likewiſe the Angle 2 N S, which is the Inclination of the Path to the Circle of Latitude: Wherefore we can find out 2 S, and the Angle 2 S N. Again, in the ſpherical Triangle P S II, we have the Side P S the Complement of the *Sun*'s Declination, and P II the Complement of the Latitude, together with the Angle S P II, which is known by the Time; by which we can find the Arch S II, which is the Diſtance of the *Sun* from the *Vertex*; and the *Sine* of this Arch is juſt

The Computation by Trigonometry.

Plate XI. Fig. 5.

 equal

Lecture XIV.

equal to the Diſtance SII, SE being made the *Radius:* We may alſo find the Angle PSII; to which if we add, or take away the known Angle PSE, we ſhall have the Angle NSII. But the Angle 2SN was found out before; wherefore we have the whole Angle 2SII. Laſtly, in the right-lined Triangle 2SII, we have the two Sides 2S and IIS, and the Angle contained between the two Sides; and therefore by plain Trigonometry we may find the Side 2II, which was to be found out. Proceeding by this Method, there is no need for inquiring into the Poſitions of Place and Shadow on the Diſk; for they are to be found out by a *Calculus* without Protraction.

To find the Longitude of Places by Obſervations of Solar Eclipſes. Plate XI. Fig. 2.

THE Longitudes of Places on the Surface of the *Earth* may be found out by Obſervations of Eclipſes of the *Sun*, as well as by thoſe of the *Moon*, *viz.* if we obſerve in that Place whoſe Longitude is wanted, the Moment of Time when the Eclipſe begins or ends. Let that, for Example, be at Five of the Clock; and at the Center V, with a Diſtance equal to the Semidiameter of the *Penumbra*, deſcribe an Arch of a Circle cutting the Path of the Shadow in *d*, that Point *d* will ſhew the Place of the Center of the Shadow at that Time. Meaſure the Diſtance N *d* with a Scale, which being given, together with the Motion of the *Moon* from the *Sun*, we ſhall find the Time in the Place of the Obſervation, when the true *Conjunction* is celebrated. In like Manner, by an Obſervation in any other Place, we may find when the ſame *Conjunction* is celebrated according to the Time computed from the Meridian of that Place; and the Difference of thoſe Times being turned into Degrees and Minutes of the Æquator, will ſhew the Difference of Longitude between thoſe two Places.

IN Practice it is convenient to make the Semidiameter of the Diſk ten Inches, that it may be divided by a Scale into 1000 Parts, which is done by the Help of a Diagonal Scale; for this Number is the *Tabular Radius:* And let SN the Latitude of

13

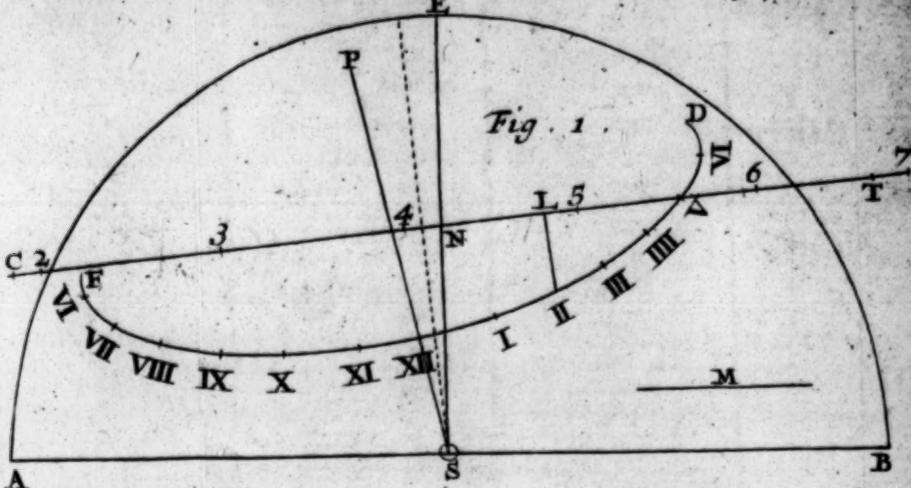


Fig. 1.
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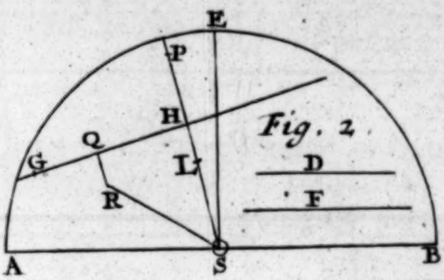
Fig. 2.
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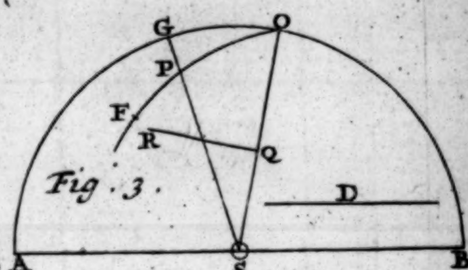
Fig. 3.
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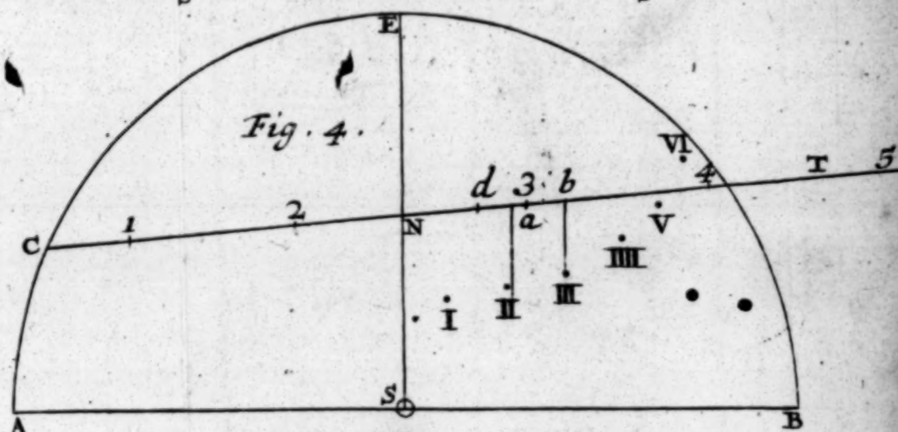
Fig. 4.
E
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4
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2
N
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V
IIII
II
III
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S
A
B

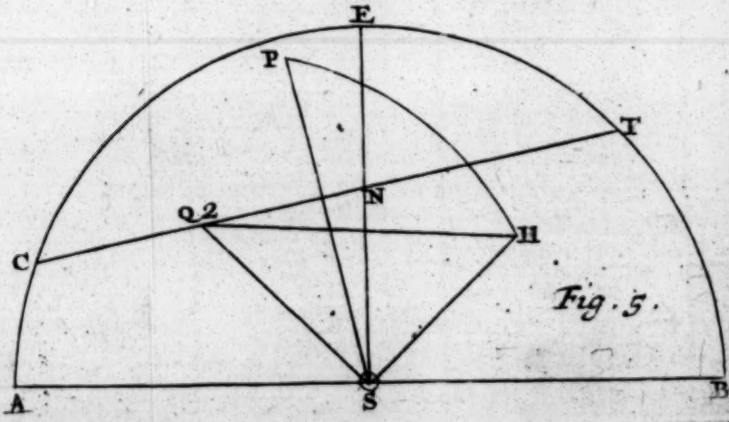
Fig. 5.
E
P
T
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Q 2
H
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B

of the *Moon*, and all the Lines whose Dimensions are necessary to be known, be expressed by the same Parts. For if we say, As the Horizontal *Parallax* of the *Moon*, which is expressed in Astronomical Tables in Minutes and Seconds, is to the *Moon*'s Latitude, so is 1000 to a fourth Number; and then we take the Line S N out of the Scale, whose Dimension is expressed by this fourth; this Line will represent the Latitude of the *Moon*. And in like manner we are to operate to find out the Length of the Way the Shadow advances in an Hour in its Path. And now we have shewed a new Way, by which the Times and *Phases* of an Eclipse are to be defined, as they are to be seen from a particular Place, which does not require a frequent or repeated Calculation of the *Parallax*, to have the visible Place of the *Moon* in the Heavens, both as to Longitude or Latitude; which Method is received by most *Astronomers*. But our Method is much easier, and, as I think, no less accurate: For in the common Method the different Positions of the Ecliptick in respect of the Horizon, which are always changing, will produce great Inequalities in the *Moon*'s Motion, and will make her constantly alter her Place, both as to Longitude and Latitude: So likewise as the *Moon* ascends or descends, the *Parallaxes* will always be changeable; and except we often compute them, it will be hard to escape falling into an Error.

But because the Method of computing Eclipses by *Parallaxes* is that which is generally made use of by the *Astronomers*, it will be convenient likewise to explain that Method. And here I suppose the Reader to be already instructed in the Doctrine of the *Parallaxes*, either by other Astronomical Books, where it is explained at Length, or by what we shall after this say upon that Subject: And that being understood, the Principles on which the Calculation is founded are easily apprehended, though the Practice of the Rules is very difficult and tedious.

Lecture XIV.

The common Method of computing Eclipses of the Sun.

FIRST of all, the visible *Conjunction*, and the Way the *Moon* is then to take in the Heavens, are to be determined. For in this Case, the true and visible *Conjunction* are very different both as to their Times and Places. The true Place of the *Moon* is that which is seen from the Center of the *Earth*, the visible Place is that which is seen from our Habitation on its Surface. Let the Semicircle A B C represent an Hemisphere of the Globe of the *Earth*, whose Center is T; from whence draw the Right Line T L S through the *Moon* at L, and the *Sun* at S, at a much greater Distance; and therefore since the Centers of the *Sun* and *Moon* are seen in the same Right Line from the Center of the *Earth*, they will appear in the same Point of the Heavens, and they will be in true *Conjunction*. But a *Spectator*, on the Surface of the *Earth* at A, will observe the Centers of the *Sun* and *Moon* in distinct Points of the Heavens, their Distance being the Arch S E. The Point where the Right Line T L, drawn through the Centers of the *Earth* and *Moon*, meets with the Heavens, is called the true Place of the *Moon*. But where a Right Line passing through the Eye of the *Spectator* on the Surface, and the Center of the *Moon*, meets with the Heavens, that Point is called the visible or apparent Place of the *Moon*. Let these Points be S and E; the Arch S E, which is the Distance between the true and apparent Place, is called the *Moon*'s *Parrallax*. Now because the Points T and L, in respect of the immense Distance of the *fixed Stars*, coincide, the Arch S E will be the same, whether its Center be conceived to be in L, or in T; and therefore the Arch S E is the Measure of the Angle S L E, or of the Angle A L T, which is equal to it. But the Angle A L T is the Angle under which the Semidiameter of the *Earth* A T, drawn through the Place of the *Spectator*, is seen from the *Moon*: And therefore the *Parallax* of the *Moon* is always equal to this Angle. And this Angle is biggest when the Semidiameter is directly seen from the *Moon*; for then the Angle L A T is

Plate XII. Fig. 1.

The Moon's *true Place.*

The Moon's *visible Place.*

The Moon's Parallax.

is a Right Angle, and the *Moon* is seen in the Horizon, and therefore the *Horizontal Parallax* is greatest of all. But if the *Moon* should be in the *Vertex* F, the Angle ALT would there vanish; and the *Moon*'s true Place would coincide with its apparent Place, whence there would be no *Parallax*. Lecture XIV.

SINCE the *Parallax* of any celestial Body is always equal to the Angle under which the Semidiameter of the *Earth*, passing through the Place of the *Spectator*, is seen from that Body, there will be no sensible *Parallax* of the *Sun*. For, as I have frequently said, the *Earth* seen from the *Sun* appears no bigger than a Point, and under no sensible Angle: But the *Moon*, when it is in the Horizon, has a sensible *Parallax*, and sometimes it is greater than a Degree.

The Sun *has no* Parallax.

HENCE it follows, that the *Parallax* always shews the *Moon* more depressed, or at a greater Distance from our *Vertex*, than it really is: This Depression will change its Place according to the Ecliptick, and make the *Moon* appear to have a Longitude and Latitude different from what it has, when seen from the Center of the *Earth*. For in the Figure, let the Circle HCZ be the Meridian, or the Circle passing through the *Vertex* of the *Spectator* and the *Pole*; let Z be the *Vertex*, HED the *Horizon* of the Place, CE the Ecliptick; in which let the *Moon* according to its true Place be in L, without any Latitude; let ZT be Vertical Circle passing through the *Moon*; and because the *Parallax* always depresses the *Moon* in the Vertical, the apparent Place of the *Moon* will be further distant from the *Vertex* Z, than the true Place is. Let the apparent Place be O, and since the true Place is L, the *Parallax* will be LO; which being in the Vertical, or Circle of Altitude, is called the *Parallax* of Altitude. Through O let there a Circle pass, which is perpendicular to the Ecliptick, meeting with the Ecliptick in *m*: That Point will be the apparent Place reduced to the Ecliptick, and the Arch L*m* is called the *Parallax of Longitude*; and O*m*, the visible Distance

Plate XII. Fig. 2.

Lecture XIV.

of the *Moon* from the Ecliptick, is called the *Moon's Parallax of Latitude*. For determining the *Phases* of Eclipses, as they are to be seen from a given Place, it is necessary to know at that Time the true Places of the *Moon* and *Sun*, which may be computed by Astronomical Tables from any given Moment of Time. Moreover, we must know the apparent Place of the *Moon*, which is to be determined from a true Place, by a Computation of the *Moon's Parallax*; which being premised, the Times and *Phases* are thus found out.

Plate XII. Fig. 3.

A Calculation of the visible Eclipse.

LET $p\,k$ be a Portion of the Ecliptick; S the Place of the *Sun* therein, at the Time of the true *Conjunction*, and l the apparent Place of the *Moon* reduced to the Ecliptick; $l\,o$ the visible Latitude of the *Moon*, and lS will be its visible Longitude from the *Sun*. At a small Portion of Time before the true *Conjunction*, find out again the visible Place of the *Moon* in the Ecliptick; which let it be at p, and let $p\,q$ be the *Moon's* visible Latitude: Draw $q\,o$, which produce, and let it meet with the Ecliptick in k; and $q\,k$ will be the visible Way of the *Moon* from the *Sun* at the Time of the *Conjunction*. In the Triangle $q\,o\,n$, right-angled at n, we have $o\,n$ the Difference of Longitude of the *Moon* from the *Sun* in p and l, and $q\,n$ the Difference of Latitude; whereby we can find the Angle $q\,o\,n$, or $q\,k\,p$, which is equal to it, and which is the Inclination of the visible Way of the *Moon* to the Ecliptick. From thence also we can find the Side $q\,o$, and by them we can find the Lines $o\,t$, $t\,k$, Sk and St. For $p\,l$ is to $q\,o$, as $l\,s$ is to $o\,t$: And in the Triangle $o\,l\,k$, by having $o\,l$ and the Angle k, we can find $o\,k$ and $l\,k$, and thereby Sk and St. Now, when the Center of the *Moon* is seen at t, then is the visible *Conjunction* of the *Sun* and *Moon*. And therefore we say, As $q\,o$ is to $o\,t$, or as $p\,l$ is to lS, so is the Time that the *Moon* moves through the Space $q\,o$, to the Time it moves through $o\,t$; and then we shall have the Time between the true and visible *Conjunction*. From S upon

upon the Way of the *Moon* let fall a Perpendicular S *m*, and in the right-angled Triangle S *k m*, we have S *k* and the Angle *k*; therefore we can find out S *m*, which is the least visible Distance, or the nearest Approach of both *Sun* and *Moon*. If this Distance be greater than the Sum of the Semidiameters of the *Sun* and *Moon*, there will be no Eclipse visible in that Place; but if it be less, the Difference reduced into Digits will shew the Quantity of the Eclipse. Having the Side S *m*, and the Angle *t* S *m*, which is equal to the Angle *k*, we can find out the Line *t m*, and thence the Time the *Moon*, in her visible Way, takes to describe the Line *t m*, which is the Time between the visible *Conjunction*, and the Moment of greatest Obscuration. Lecture XIV.

THE Beginning of the Eclipse is thus determined: Let *p k* be a Portion of the Ecliptick, as before; S the Center of the *Sun*; let *q k* be the visible Way of the *Moon* S *m* its shortest Distance from the *Sun*. Draw from the *Sun* to the Way of the *Moon*, the Right Dine *s q*, equal to the Semidiameters of both *Sun* and *Moon*: And then, when the Center of the *Moon* comes to *q*, the Eclipse will begin to be visible, and the two Margins of the *Sun* and *Moon* will seem to touch one another. In the rectangled Triangle *q* S *m*, we have the Side S *q*, equal to the Sum of the Semidiameters of the *Sun* and *Moon*, and S *m* the nearest Approach of their Centers; from whence we can find out the Angle *q* S *m*, the Angle of Incidence, and also the Side *q m*; and thereby we have the Time wherein the *Moon* describes the Line *q m*, and from thence the Time between the Beginning of the Eclipse and the Time of the greatest Obscuration.

Plate XII. Fig. 4. *A Calculation of the Beginning and End of the Eclipse.*

AFTER the same Method we may find the Time of the visible End of the Eclipse. But here we must begin again, and compute anew, by the *Parallaxes*, the visible Way of the *Moon* from the *Sun*, after the *Conjunction*, which will not be the same it was before. For the Inclination of the visible Way

is

Lecture XIV. is constantly changing, because the Quantity of the *Parallax* is in perpetual Flux, as the *Moon* rises and falls in Altitude. Seek therefore, about an Hour after the *Conjunction*, the visible Longitude of the *Moon* from the *Sun*, and its visible Latitude, and from them compute the Inclination of the *Moon*'s Way from the *Sun*; which being found, by the same Method we found the Time of the Beginning of the Eclipse, we may likewise find the Time of its End.

A Determination of the Phasis *for any Moment of Time.*

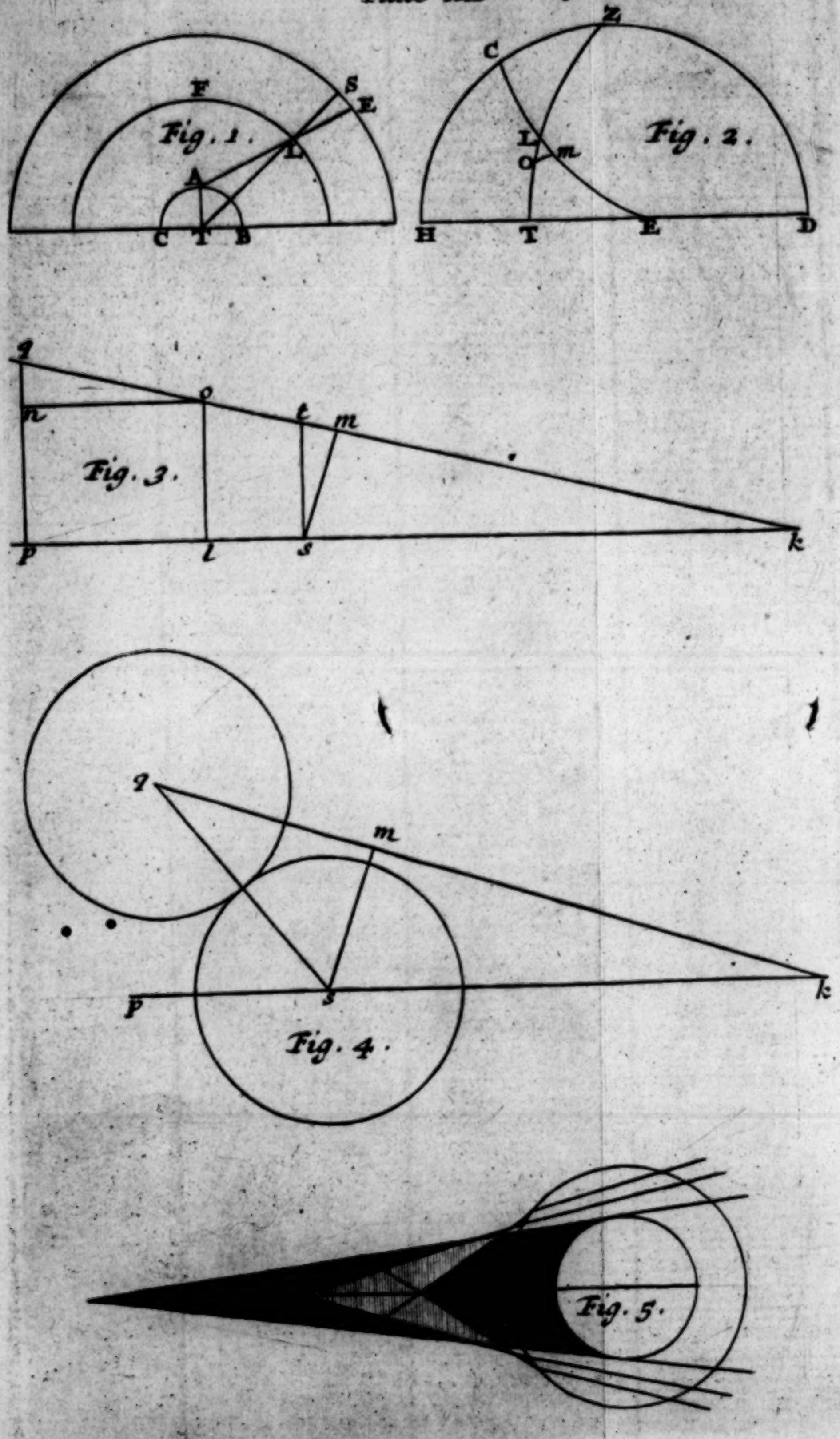
If the *Phasis* of an Eclipse for any Moment of Time be required, find for that Time the Place of the *Moon* in her visible Way; and at that Center, and with a Distance equal to the Semidiameter of the *Moon*, describe a Circle; likewise at the Center S, with a Distance equal to the Semidiamater of the *Sun*, describe another Circle; the Intersections of these two Circles will shew the *Phasis* of the Eclipse, and the Quantity of Obscuration, as also the Position of the Cusps or Horns.

Before we make an End of this Doctrine of Eclipses, it will be requisite to explain one notable Appearance, and to shew the Reason of it.

In total Eclipses of the *Moon*, even when she is near the Center of the Shadow, her Body is frequently to be seen of a pale and languid Colour, which could not be without her being illuminated with some Light; and some will wonder from whence arises this Light. Some suspected that it was the native and proper Light of the *Moon* herself. Others derived it from the *Planets* and *Stars*; for the Interposition of the *Earth* intercepts all the Light of the *Sun*, and seems to bring a thick Darkness upon the whole Space taken up by the conical Shadow. But we must consider, that the *Earth* is surrounded with a Sphere of Air of a considerable Depth and Density, which has a refractive Power, whereby it turns the Rays of the *Sun* out of their Way when they fall upon it, and makes them enter the conical Shadow; which therefore will be illuminated by that small Quantity of

14

Plate XII fronting . p . 155



of Light which falls obliquely on our Atmoſphere, and imparts to all the Bodies within it a faint Light, the which will illuminate the *Moon*, even when it is in the Midſt of the Shadow, and make it viſible to our Eyes, as the Figure ſhews. Lecture XV. Plate XII. Fig. 5.

LECTURE XV.

Of the Phænomena *or Appearances ariſing from the Motions of the* Earth, *and the two inferior Planets* Venus *and* Mercury.

HITHERTO we have contemplated the Motions of the *Earth* and *Moon*, and have given an Account of many Appearances that ariſe from them. The *Moon* indeed is no primary Planet, but a ſecondary, which does no other Ways go round the *Sun*, the true Center of our Syſtem, than by accompanying our *Earth*, to whom ſhe properly belongs, in her annual Courſe round the *Sun*.

The ſix primay Planets. BUT the chief and primary *Planets* of our Syſtem, which perform their Circulations round the *Sun*, without regarding any other Body, are in Number ſix, *viz. Mercury* ☿, *Venus*, ☿ the *Earth* ⊕, *Mars* ♂, *Jupiter* ♃, and *Saturn* ♄, whoſe Motions and Appearances are now to be explained. And, *firſt*, we have already demonſtrated, that the Orbits of *Venus* and *Mercury* include the *Sun*, and that they are included within the Orbit of the *Earth*; and ſince

Lecture XV.

since they finish their Calculations in less Time than the *Earth* does, it is manifest that these *Planets*, seen from the *Sun*, will appear in the Heavens sometimes nearer, and sometimes further from the *Earth*; and that sometimes they may from thence appear in the same Point, and sometimes in opposite Points of the Heavens, with the *Earth*. And because *Venus* and *Mercury* are carried faster about than the *Earth*, a *Spectator* in the *Sun*, after seeing either of them in *Conjunction* with the *Earth*, will see it recede from the *Earth*, which follows with a slower Motion, and get by Degrees a good Way to the *East* of the *Sun*.

As these *Planets*, seen from the *Sun*, change their Positions in respect to the *Earth*, so likewise we, seeing them from the *Earth*, observe that they change their Positions in respect to the *Sun*, and are sometimes nearer, sometimes further removed from him; and sometimes they appear in *Conjunction* with the *Sun*. But the *Conjunctions* of these *Planets* seen from the *Earth*, do not only happen when the *Earth* and they are seen together in *Conjunction* from the *Sun*, but also when a *Spectator* in the *Sun* sees the *Earth* and them in Opposition: Even then the *Sun* and they, seen from the *Earth*, appear to be in *Conjunction*. For let S be the *Sun*, A B C the Orbit of the *Earth*, FHB the Orbit of *Venus*; and let the *Earth* be in T, and *Venus* in V, in the Line which joins the Centers of the *Earth* and *Sun*: In this Position *Venus*, seen out of the *Sun*, is in *Conjunction* with the *Earth*, as the *Sun* from the *Earth* is seen conjoined with *Venus*.

Two Cases of Conjunctions.

Plate XIII. Fig. 1.

But if the *Earth* were in T, and *Venus* in F, a *Spectator* in the *Sun* will see *Venus* and the *Earth* in Opposition, or in opposite Points of the Heavens. But a *Spectator* in the *Earth* will see *Venus* not in Opposition to the *Sun*, but in *Conjunction* with him. In the first Case of these *Conjunctions*, *Venus* is between the *Sun* and the *Earth*; in the other the *Sun* is situated between the *Earth* and *Venus*; and *Venus*

goes

goes above the *Sun*: The first is called the *inferior Conjunction*, the second the *superior*.

After either of these two Sorts of *Conjunctions*, *Venus* will seem daily to remove further from the Neighbourhood of the *Sun*, and will daily seem to go further off from him; but still she keeps within certain Bounds, for she never comes to be opposite to the *Sun*; nor does she even arrive at a *Quadrantile Aspect*, which is 90 Degrees; or a *Sextile Aspect*, which is 60 Degrees distant from the *Sun*. And *Venus* is seen at her greatest Distance from the *Sun*, when the Line which joins her Center with the *Earth*, touches the Orb of *Venus*, as about D. For when this *Planet* is farther advanced to H, its Place in the Heavens is seen to be nearer to the *Sun*, than it was before at D. Now before she came to D, she always receded more and more from the *Sun*: And after she has left D, she every Day comes nearer to the *Sun*: It is necessary, that between the Times of her Recess and Approach, she become stationary in respect to the *Sun*, and for some Time appear to keep the same Distance from him; at which Time the visible Motion of *Venus* will be equal to that of the *Sun*. The Arch of a great Circle, intercepted between *Venus* and the *Sun*, is called the *Elongation* of that *Planet* from the *Sun*,

The Elongation of a Planet *from the* Sun.

But here it is to be observed, that only in a Circle, which has the *Sun* for its Center, the greatest Elongation happens, when the Right Line which joins the *Earth* and *Planet* touches. For in an Elliptick Orbit it may be, that the Elongation from the *Sun* may grow still greater, even after it has left the Place where the Line joining the *Earth* and *Planet* touches its Orbit: For after that, the true Distance of the *Planet* from the *Sun* may increase, whilst the Distance of the *Sun* and *Planet* from the *Earth* does not increase, but they may rather decrease. And therefore in Two Triangles, the greater Base will subtend the greater Angle. But because the Orbits of the *Planets* are nearly circular, such small Differences may be here neglected.

The Elongation is not always greatest, when it is in a Line touching the Planet*'s Orbit.*

The

Lecture XV.

The greatest Elongation of *Venus* is found by Observation to be about 48 Degrees, by which in a circular Orbit, we may know the Distance of *Venus* from the *Sun*, in respect of the *Earth*'s Distance from the same: For S T is to S D, as the *Radius* is to the *Sine* of the Angle S T D, which is the greatest Elongation.

Hence also it is manifest, that *Venus*, from the Time of her superior *Conjunction*, where she is furthest from the *Earth*, to the Time of her inferior *Conjunction* with the *Sun*, where she approaches nearest it, is always seen more *Easterly* than the *Sun*; and all that Time *Venus* sets later than the *Sun*, and is seen after Sun-setting; and then she is called the *Evening-Star* or *Vesperus*, being a Forerunner of Night and Darkness. But from the inferior *Conjunction*, till she comes again to the superior, she is always observed to be to the *Westward* of the *Sun*, and consequently must set before him in the Evening, and rise before him in the Morning, and then she is only to be seen before Sun-rising; when she is called the *Morning-Star*, or *Phosphorus*, her Appearance foretelling that Light and Day are at Hand.

Let us now suppose *Venus* and the *Earth* to be seen out of the *Sun* in *Conjunction*, and the one to be at V, the other in T, in the same Point of the Ecliptick: In which Position *Venus* and the *Sun* are seen from the *Earth* likewise in *Conjunction*. After this, *Venus* circulating faster than the *Earth*, being come again to V, and having finished her Course, and by an angular Motion round the *Sun*, describing four Right Angles, will not have overtaken the *Earth*; who in the mean Time has proceeded farther in her proper Orbit. And therefore *Venus* must still move farther on to come in a Right Line between the *Sun* and the *Earth*. Let S L M be the next Right Line in which *Venus* is seen from the *Sun* together with the *Earth*, so that *Venus* may be in L, when the *Earth* is in M: Now before *Venus* can overtake the *Earth*, she must not only finish

Lecture XV.

The Time between two Conjunctions *of the same Kind.*

finish her own Calculation, or four Right Angles, but also so much angular Motion more, as the *Earth* has made in the mean Time round the *Sun*. Now the angular Motion of *Venus* and the *Earth*, performed in the same Time, are reciprocally as the periodical Times of *Venus* and the *Earth*, And therefore, as the periodical Time of the *Earth* is to the periodicall Time of *Venus*, so is the angular Motion of *Venus* (which is equal to four Right Angles, and moreover to the angular Motion the *Earth* makes from the Time of one *Conjunction* to the next) to the angular Motion of the *Earth*. And therefore by Division of Proportion, as the Difference between the periodical Times of the *Earth* and *Venus* is to the periodical Time of *Venus*, so are four Right Angles to a fourth Quantity; which shews the angular Motion of the *Earth*, from the Time of her *Conjunction* with *Venus*, to the Time of the next *Conjunction* of the same Kind. Now the periodical Time of the *Earth* is 365 Days and 6 Hours, or 8766 Hours. And the Period of *Venus* consists of 224 Days 16 Hours, or 5392 Hours, whose Difference is 3374 Hours. Say then, As 3374 is to 5392, so are four Right Angles, or 360 Degrees, to a fourth Number of Degrees, which is 575; which Motion is equal to a Circulation and a half, and besides 35 Degrees; which angular Motion the *Earth* makes in the Space of one Year and 218 Days. And therefore, if *Venus* should be this Day in *Conjunction* withthe *Sun*, in the inferior Part of her Orbit, she will not come to the same *Conjunction* again till after a Year and seven Months and twelve Days. And if one *Conjunction* be in the Beginning of *Aries*, the next will fall out when the *Sun* is in *Scorpio*. There is the same Distance of Time between any two other similar Positions of *Venus* and the *Sun*. For Example, between two superior *Conjunctions*, or between two such Situations of *Venus*, where she has a given Elongation from the *Sun* the same Way.

THIS

Lecture XV.

Another Way of doing the same.

THIS *Problem*, and another of the same Nature, about the *Conjunctions* of the *Sun* and *Moon*, are otherwise solved by the *Astronomers*; for they find out the diurnal Motion of *Venus* seen from the *Sun*, and likewise the Diurnal Angular Motion of the *Earth*; and the Difference of these Motions is the relative Diurnal Motion of *Venus* from the *Earth*, or the Quantity by which *Venus* is seen to recede from the *Earth* every Day, by a *Spectator* in the *Sun*. Thus the middle Motion of the *Earth* is every Day about 59 Minutes and 8 Seconds: *Venus*'s middle Motion in a Day is 1 Degree 36 Minutes and 8 Seconds, whose Difference is 37 Minutes. Say therefore, As 37 Minutes is to 360 Degrees, or to 21600 Minutes, so is one Day to that Space of Time, wherein *Venus*, having left the *Earth*, has receded from her 360 Degrees; that is, to the Time in which she returns to the *Earth* again, which is the Time between two *Conjunctions* of the same Kind, which will be found to consist of 583 Days.

Plate XIII. Fig. 2.

BUT these *Conjunctions* are here computed according to the Middle Motions of the *Planets*, supposing them to move always equably, or with the same Angular Velocity; and they are therefore called *mean Conjunctions*. But because *Venus* and the *Earth* are really carried in Elliptick Orbits, in which their Motions are constantly variable, sometimes going faster, and sometimes slower; it may be, that the true *Conjunctions* shall happen some few Days sooner or later than the Computation we have given; yet having the Time of the *mean Conjunction*, the true *Conjunction* is from thence to be computed after this Manner: Let A B C be the Ecliptick, in which A is the Point where the *Planets* are to be in *Conjunction* according to the mean Motion. For the Time of this *Conjunction*, compute by Astronomical Tables the true Places of the *Earth* and *Venus* in the Ecliptick; and suppose *Venus*'s true Place in the Ecliptick to be D, and the *Earth* in T, by which we shall find the Distance of the *Earth* and *Venus* seen

ſeen from the *Sun:* But we have, for that Time, the angular Motions of theſe two *Planets* for any given Space of Time; for Example, for ſix Hours; and the Difference of theſe two Motions will give the Acceſs of *Venus* to the *Earth*, or her Receſs from it in ſix Hours. Say then, As this Difference of Motions is to D T, ſo is ſix Hours to the Time between the mean *Conjunction* and the true; which Time, added to, or ſubtracted from, the Time of mean *Conjunction*, as *Venus* is to the *Eaſt* or *Weſt* of the *Earth*; ſhews the Time of their true *Conjunction*.

Lecture XV.

Venus *conſtantly changes her Diſtance from the* Earth.

It is plain from the Inſpection of the Figure, that though *Venus* does nearly always keep the ſame Diſtance from the *Sun*, yet ſhe is continually changing her Diſtance from the *Earth*; and her Diſtance is greateſt when ſhe is ſeen in her ſuperior *Conjunction* with the *Sun*; and it is the leaſt when ſhe is in her inferior *Conjunction:* And the Difference is ſo great, that it equals the whole Diameter of *Venus*'s Orbit; ſo that the Diſtance of *Venus* from the *Earth*, when ſhe is in her ſuperior *Conjunction*, is to her Diſtance from the *Earth* in the inferior, as 1 to 6: And therefore *Venus* approaches the *Earth* ſix times nearer in the one Poſition, than in the other; and juſt ſo much are the apparent Diameters of *Venus* changed, as we obſerve them to be. But theſe greateſt and leaſt Diſtances are ſomewhat changeable, upon the Account of the Elliptical or Excentrick Orbits: For *Venus* is then moſt remote from the *Earth*, where the ſuperior *Conjunction* happens when *Venus* and the *Earth* are both in their *Aphelions*. And the Diſtance of *Venus* and the *Earth* is the leaſt of all, when the inferior *Conjunction* falls out when *Venus* is in her *Aphelion* and the *Earth* in her *Perihelion*.

The Phaſes *of* Venus *like thoſe of the* Moon.

Because *Venus* is an opake Globe without any Light of her own, and only ſhines with the borrowed Light of the *Sun*, that Face of *Venus* will only appear bright, which is turned towards the *Sun*, while

Lecture XV. while the Opposite remains in Darkneſs; and for want of Light is altogether inviſible. Wherefore, if the Situation of the *Earth* be ſuch, that this dark Side of *Venus* be turned towards the *Earth*, *Venus* will become inviſible, except by chance ſhe appear like a black Spot in the Diſk of the *Sun:* But if the whole illuminated Face of *Venus* be turned towards the *Earth*, as it is when ſhe is near her ſuperior *Conjunction*, then ſhe appears like a full ſhining Orb; and according to the different Poſitions of the *Earth*, *Venus*, and the *Sun*, *Venus* will have different Forms, and appear with different Faces and Figures, and will undergo the ſame Change and Viciſſitudes in her Appearances that the *Moon* does.

Plate XIII. Fig. 3. Let ABCDEFGH be the Orbit of *Venus*, T L a Portion of the Orbit of the *Earth*, in which the *Earth* is at T, and let *Venus* be in A in her ſuperior *Conjunction* with the *Sun*; it is manifeſt in this Situation of theſe two *Planets*, that the Face of *Venus*, which is illuminated by the *Sun*, is likewiſe turned towards the *Earth*; and then *Venus* will appear to us like a full, lucid Circle, as the *Moon* does at Full: But when ſhe has gone from thence to the Poſition B, ſome Part of her obſcure Hemiſphere will be turned towards the *Earth*, and will loſe ſomething of her Fulneſs, and ſeem to us to be gibboſe. When *Venus* comes to the Poſition C, but half her illuminated Side is turned towards the *Earth*, and then ſhe is ſeen like a half Circle, as the *Moon* is when ſhe enters in her firſt or laſt Quarter. But *Venus*, when ſhe arrives at the Poſition D, has but a ſmall Part of her illuminated Side turned towards the *Earth:* And becauſe ſhe is of a ſpherical Figure, which to us, becauſe of its great Diſtance, appears like a Plane; the illuminated Part which we ſee, will appear to end in Points or Horns, whoſe Direction is always oppoſite to the *Sun*. But *Venus*, when ſhe is in the Poſition E, that is, in her inferior *Conjunction* with the *Sun*, has her dark Side totally turned towards the *Earth*; and then

then ſhe quite diſappears, unleſs ſhe happen to be in her *Node*, or near it; then ſhe will appear like a black Spot to paſs over the Body of the *Sun*, which delightful Spectacle was never ſeen by mortal Eyes but once; and it was our Countryman Mr. *Horrox*, who alone enjoyed that Pleaſure. *Venus* will undergo the ſame *Phaſes* while ſhe paſſes through F, G to H; *viz.* about F ſhe is horned, in G a half Circle, in H gibboſe, and in A again full.

These Appearances of *Venus*, though they are not to be diſcerned by the naked Eye, yet they are diſtinctly and plainly to be perceived with a Teleſcope. Before the Invention of this noble Inſtrument, when *Copernicus* firſt revived the antient *Pythagorean Syſtem*, and propoſed it to the Learned in *Aſtronomy*, to whom he maintained that the *Planets*, among which he reckoned the *Earth*, did move round the *Sun*, which was immoveable in the Center; it was objected to him, that if the Motions of the *Planets* were ſuch as he ſuppoſed them to be, then *Venus* ought to undergo the ſame Changes and *Phaſes* as the *Moon* does. *Copernicus* anſwered, that perhaps the *Aſtronomers* in After-ages would find, that *Venus* does really undergo all theſe Changes. This Prophecy of *Copernicus* was firſt fulfilled by that great *Italian* Philoſopher *Galileus*, who directing his Teleſcope to *Venus*, obſerved her Appearances to emulate the *Moon*, as *Copernicus* had foretold: And theſe Obſervations did ſurprizingly confirm the old Syſtem revived by *Copernicus*.

The Prophecy of Copernicus.

Phaſes *firſt obſerved by* Galileus.

If the Centers of the *Sun* at S, *Earth* at T, and one of the inferior *Planets* at O, be joined with Lines, they will from the Triangle T S O: And if through the Center of the *Planet* there paſs two Planes one perpendicular to the Line T O, and the other to the Line S O; the one will cut off the Hemiſphere, which is turned towards the *Earth*; the other, that which is turned towards the *Sun*, and by him illuminated. And the exterior Angle of the Triangle T S O, which is at the *Planet*, that is, the Angle

Plate XIII. Fig. 4.

Lecture XV.

POS, will be equal to the Angle *m* O *q*, which measures *m q* the Portion of the illuminated Semicircle that is turned towards the *Earth*. For the Angle S O *r* is a right Angle, and so is the Angle *m* O T, which are therefore equal; but the Angles *r* OP and *p* O *q* are likewise equal, being vertical to each other; and therefore, taking away Equals from Equals, there will remain the Angle S O P, equal to the Angle *m* O *q*; which Angle is measured by the Arch *m q*. And therefore the Part of the illustrated Semicircle which is towards the *Earth*, and is to be seen from thence, does always measure the exterior Angle S O P of the Triangle S O T. Now this Arch, as seen from the *Earth*, is projected into its own versed Sine upon the Disk, as we shewed before in the *Moon*. And hence the Illumination of *Venus* seen from the *Earth*, is to her full and total Illumination, all other Things remaing the same, as the versed Sine of the exterior Angle at *Venus* is to the Diameter of the Circle.

The Quantity of Illustration.

Venus *appears not with her greatest Lustre when she shines with a full Face.*

ALTHOUGH *Venus* in A shines upon the *Earth* at T with a full Face or Orb, yet she does not appear there with her greatest Brightness and Lustre; for her Splendor is diminished on the Account of her greater Distance from the *Earth*; and it is lessened in a greater Proportion than the conspicuous Part of the illuminated Disk is increased. For the Lusture of *Venus* decreases in the duplicate Proportion of the Distance increased. But the visible illuminated Part of her Face increases only according to the versed Sine of the Angle S O P; and therefore the greatest Brightness of *Venus* is not when she is in A, but rather when she is about O. For suppose *Venus* at O four Times nearer the *Earth*, than when she is in A; in that Case every determined Part of the illuminated Disk will give sixteen times more Light, than the same Part does at A: But in O it may happen, that only a fourth Part of the illuminated Disk can be seen from the *Earth*; and therefore the Brightness of *Venus* is more increased by her Distance being diminished,

minished, than the same Brightness is lessened on Account of a smaller Portion of her illuminated Disk being visible from the Earth.

Lecture XV.

Where the Illustration is greatest.

If you desire to know in what Position *Venus* appears with the greatest Lustre, the great *Geometer* and *Astronomer*, Dr. *Edmund Halley*, my Colleague, has given us an elegant Solution of this *Problem* in the *Philosophical Transactions*, Numb. 349; wherein he has shewn, that *Venus* is brightest when she is about 40 Degrees removed from the *Sun*; and that then but only a fourth Part of her lucid Disk is to be seen from the *Earth*. And in this Situation *Venus* has been many Times seen in the Day-time, even in full Sunshine. This Beauty and Brightness of *Venus* is very admirable; who having no native Light of her own, and only enjoying the borrowed Light of the *Sun*, should yet break out into so great a Lustre, that the like is not to be observed in *Jupiter*, nor even in our *Moon*, when she is in the same Elongation from the *Sun*. 'Tis true, the *Moon*'s Light is much greater, upon the Account of her apparent Magnitude, than that of *Venus*; yet it is but a dull, and, as it were, dead Light, which has nothing in it of that Vigour and Briskness that does always accompany the Beams of *Venus*.

The Plane of Venus's *Orbit does not lie in the Ecliptick.*

If the Plane of the Orbit of *Venus* coincided perfectly with the Plane of the Ecliptick, *Venus* would always seem to move in the *Ecliptick*, and no where recede from it. But *Venus*'s Orbit does not lie in the Plane of the Ecliptick, but is in a Plane which is inclined to it, in an Angle of 3 Degrees and 24 Minutes, and cuts the Plane of the Ecliptick, in a Line which passes through the *Sun*'s Center, that is called the *Line of the Nodes*. And the two Points, where the Orbit of the *Planet* produced cuts the Ecliptick, are named the *Nodes*. And therefore *Venus* is never seen, either from the *Sun* or the *Earth*, in the Ecliptick, but when she is in the *Nodes*; in all the other Points of her Orbit, she is sometimes nearer to the Ecliptick, sometimes further from it; and seen from the *Sun*, she makes her greatest Excursion, when she is 90 Degrees distant from both the *Nodes*.

The Line of the Nodes.

Lecture XV.

PlateXIII. Fig. 5.

LET TAB be a Circle in the Plane of the Ecliptick, L *n* V N the Orbit of *Venus*, cutting the Plane of the Ecliptick in the Line *n* N; we must conceive the one half of this Orbit of *Venus* *n* LN to be raised, or to stand above the Plane of the Ecliptick, and the other half *n* V N to fall below that Plane; and when *Venus* is in N or *n*, she is then in the Ecliptick: But when she arrives at P, she is seen to deviate from it: but in L, the Arch N L being a Quadrant seen from the *Sun*, she appears to recede the furthest from the Ecliptick; And this Point L is called the *Limit*, determining her greatest Excursion; for from thence departing, she again approaches the Ecliptick. If from the Place of *Venus*, as in P, we let fall on the Plane of the Ecliptick, a Perpendicular P E, and draw S E, the Angle P S E will measure the Distance of *Venus* from the Ecliptick, which is called *Venus*'s *Heliocentrick Latitude*, or such as it is seen from the *Sun*. Now this Latitude, having the Place of the *Planet* in its Orbit, is thus investigated: Let the Arch N E be a Portion of the Ecliptick, NP a Portion of the *Planet*'s Orbit produced to the Heavens: Let P be the Place of *Venus*, N the *Node*; and let a Circle pass through the Place of the *Planet* perpendicular to the Ecliptick; the Arch P E of this Circle, intercepted between the *Planet* and the Ecliptick, is the Distance of the *Planet* from the Ecliptick, or the Measure of the Angle P S E. Now in the spherical Rectangular Triangle P N E, besides the Right Angle at E, we have the Side N P, the Distance of the *Planet* from the *Node*, also the Angle N the Inclination of the Plane of the Orbit to the Ecliptick; wherefore by *Trigonometry* we can find out P E, which is the *Heliocentrick Latitude* of the *Planet*. This *Heliocentrick Latitude*, when the *Planet* comes to the same Point of its Orbit, is always the same and unchangeable: But the *Geocentrick Latitude*, or the Distance of the *Planet* from the Ecliptick, as it is seen from the *Earth*, even though the *Planet* be in the same Point of her Orbit, is not constantly the same, but alters

The Heliocentrick Latitude.

The Geocentrick Latitude.

alters according to the Position of the *Earth*, in respect to the *Planet*. For let B T A *t* be the Orbit of the *Earth*, *n* P N, as before, the Orbit of the *Planet*, which suppose to be at P; from which let fall on the Plane of the Ecliptick the Perpendicular P E: In whatever Part of her Orbit the *Earth* is, this Line P E will always subtend the Angle which measures the *Geocentrick Latitude* of the *Planet*. Suppose therefore the *Earth* at T, and *Venus* in P, where she comes nearest to the *Earth*; in which Position *Venus* is seen in her inferior *Conjunction* with the *Sun*, and her *Geocentrick Latitude* is measured by the Angle P T E. But if *Venus* should be in the same Situation P, and the *Earth* were at *t*, and from thence *Venus* were observed in her superior *Conjunction* with the *Sun*, where she is at her greatest Distance from us, her *Geocentrical Latitude* would be answerable to the Angle P *t* E, which is much less than the Angle P T E; because the Distance P *t* is greater than P T. What we have here said of the Latitude of *Venus*, is likewise true of that of *Mercury*, and upon the same Account. Hence it is plain, that the inferior *Planets*, all other Things remaining the same, have a greater Latitude when they are near the *Earth*, than when they are further off. And it may happen, that the *Geocentrical Latitude* of *Venus* may be greater than her *Heliocentrick*, which will be when she is between the *Sun* and the *Earth*, and she is nearer to us than to the *Sun*. But *Mercury* keeping always at a greater Distance from the *Earth*, than he has from the *Sun*, his *Geocentrick Latitude* will constantly be less than his *Heliocentrick*, which, when at the biggest, is about 7 Degrees; for so much is the Inclination of his Orbit to the Plane of the Ecliptick.

Lecture XV.

Plate XIII. Fig. 6.

Since none of the Orbits of the *Planets* lie in the Plane of the Ecliptick, but all of them cut it in a Line passing through the *Sun*, no *Planet* can be above twice in the Time of its Period in the Ecliptick, which is when they are in their *Nodes*;

 at

Lecture XV. at all other times every one of them will deviate, some more, some less, from the said Plane of the Ecliptick: But yet there are certain determinate Boundaries which they never transgress. And therefore, if we imagine in the Heavens a Zone, or broad Circle of 20 Degrees Breadth, that is, 10 Degrees on each Side of the Ecliptick, which lies exactly in the Middle of this Space or Zone; such a Space will constantly contain all the *Planets* in its Compass, and is called the *Zodiack*, from the Images of living Creatures, or the Constellations which fill that Part of the Heavens: The *Earth* keeping always, as it were, in the King's Highway, never turns out from its Course in the Ecliptick; but the *Moon*, and the other five Wanderers, will make Excursions from it, for several Degrees, sometimes to the *North-side*, and sometimes to the *South-side* of the Ecliptick; and yet they always keep within the Bounds of the *Zodiack*.

The Zodiack.

The Motion of Venus *in the* Zodiack.

HITHERTO we have considered the Motion and *Phases* of *Venus*, as they have a Relation to the *Sun* and *Earth*: Let us next consider the Motions of *Venus* in the Heavens, as they are observed from the *Earth*, and the Way she takes in the *Zodiack*. For which Purpose let A B C be the Orbit of *Venus*, T G F the Orbit of the *Earth*, the Circle L M O the *Zodiack* among the *fixed Stars*: And, first, suppose the *Earth* in T, and *Venus* in A, near her superior *Conjunction* with the *Sun*; it is evident, that a *Spectator* on the *Earth* will see *Venus* at A, as if she were in the Point of the *Zodiack* L: If the *Earth* had no Motion while *Venus* moves from A to B in its Orbit, it would seem to describe the Portion of the *Zodiack* L M: But in the mean time the *Earth* also moves; and when *Venus* is in B, the *Earth* is come to the Point of its Orbit H; from whence the *Spectator* looking upon *Venus* at B, observes her in the *Zodiack* at N; so that she will seem to have run through the Space L M N in the *Zodiack*: And *Venus* will appear to have gone more *Eastward* than

Plate XIII. Fig. 7.

than ſhe would have done, had the *Earth* ſtood ſtill without any Motion in its Orbit. But when *Venus* comes to C, the *Earth* has moved on to G; ſo that *Venus* is ſeen in the Line G O drawn from the *Earth*, which touches her Orbit; in which Poſition her apparent Motion in the *Zodiack* will be very nearly equal to the apparent Motion of the *Sun*. From thence let *Venus* move on from C to A, and in that Time the *Earth* will have come from G to K, and then *Venus* will be ſeen near her inferior *Conjunction* with the *Sun*; in which Poſition ſhe will be obſerved in the *Zodiack*, as if ſhe were at P: But before ſhe was ſeen at O; and therefore ſhe will here appear to have gone backwards in the *Zodiack* through the Arch O P, or to have moved from the *Eaſt* to the *Weſt*, contrary to the Order of the Signs. And becauſe in C ſhe was obſerved to go *Eaſtwards* as faſt as the *Sun* does; but in A ſhe is ſeen to have a quick Motion backward: There muſt be ſome Place of her Orbit between C and A, where ſhe appears to us neither to go forward nor backward, but to ſtand ſtill, and continue in the ſame Place of the Heavens: In which Caſe ſhe is ſaid to be *Stationary*, or to ſtand ſtill.

Lecture XV. Venus *Direct.* Venus *Retrograde.* Venus *Stationary.*

Let *Venus* now arrive at E, and the *Earth* at the Point of its Orbit F: *Venus* will then be ſeen in the Point of the Ecliptick Q, and will appear to have moved further backward in the Ecliptick, or towards the *Weſt*. But when *Venus* is ſeen from the *Earth* in a Line which touches her Orbit, ſhe will then ſeem to have a progreſſive Motion, equal to the apparent Motion of the *Sun* from *Weſt* to *Eaſt*: And becauſe before, her apparent Motion was backward, or from *Eaſt* to *Weſt*, and now forward the contrary Way, from *Weſt* to *Eaſt*, there muſt be ſome Place between the two contrary Motions, where ſhe will neither appear to go backward nor forward; but for ſome Time to ſtand ſtill, and keep the ſame Poſition in the Heavens. While the *Earth* comes to D, and *Venus* arrives at C, ſhe will appear in that

Time

Lecture XV.

Time to have moved through the Arch QR of the *Zodiack*, and to have a quicker Motion towards the *East*. Hence *Venus*, when ſhe is in her ſuperior *Conjunction* with the *Sun*, is always ſeen to move directly according to the Order of the Signs; but when ſhe is in her inferior *Conjunction*, and between the *Earth* and the *Sun*, then ſhe is ſeen to have a backward Motion, and to be carried againſt the Order of the *Signs* from *Eaſt* to *Weſt*.

The Appearances of Mercury *like thoſe of* Venus.

WHATEVER we have demonſtrated concerning the Motions of *Venus* is likewiſe true, and to be underſtood of the Motions of *Mercury*; but the *Conjunction* of *Mercury* with the *Sun*, his Directions, Stations, and Retrogradations, are more frequent than in *Venus*; for *Mercury* circulating faſter, and in a leſſer Orbit than *Venus*, does oftener overtake the *Earth* than ſhe. Hence it is plain, that the Motions of theſe two *Planets*, ſeen from the *Earth*, are very irregular and unequal, ſince they are ſometimes ſeen to have a Motion forward; ſometimes they appear immoveable or ſtationary; after this they change their Courſe, and move backward; and after ſuch a Regreſſion they again take up their Stations, and keep for ſome Time the ſame Place in the *Zodiack*. Whereas a *Spectator* in the *Sun* will always obſerve theſe *Planets* to go forward with a Motion regulated after a certain Rate: For the apparent Inequality of theſe Motions, ſeen from the *Earth*, is ſuch as exactly anſwers to a regular Motion round the *Sun*. And therefore it is manifeſt, that the *Sun*, and not the *Earth*, is the Center of theſe *Planets* Motions.

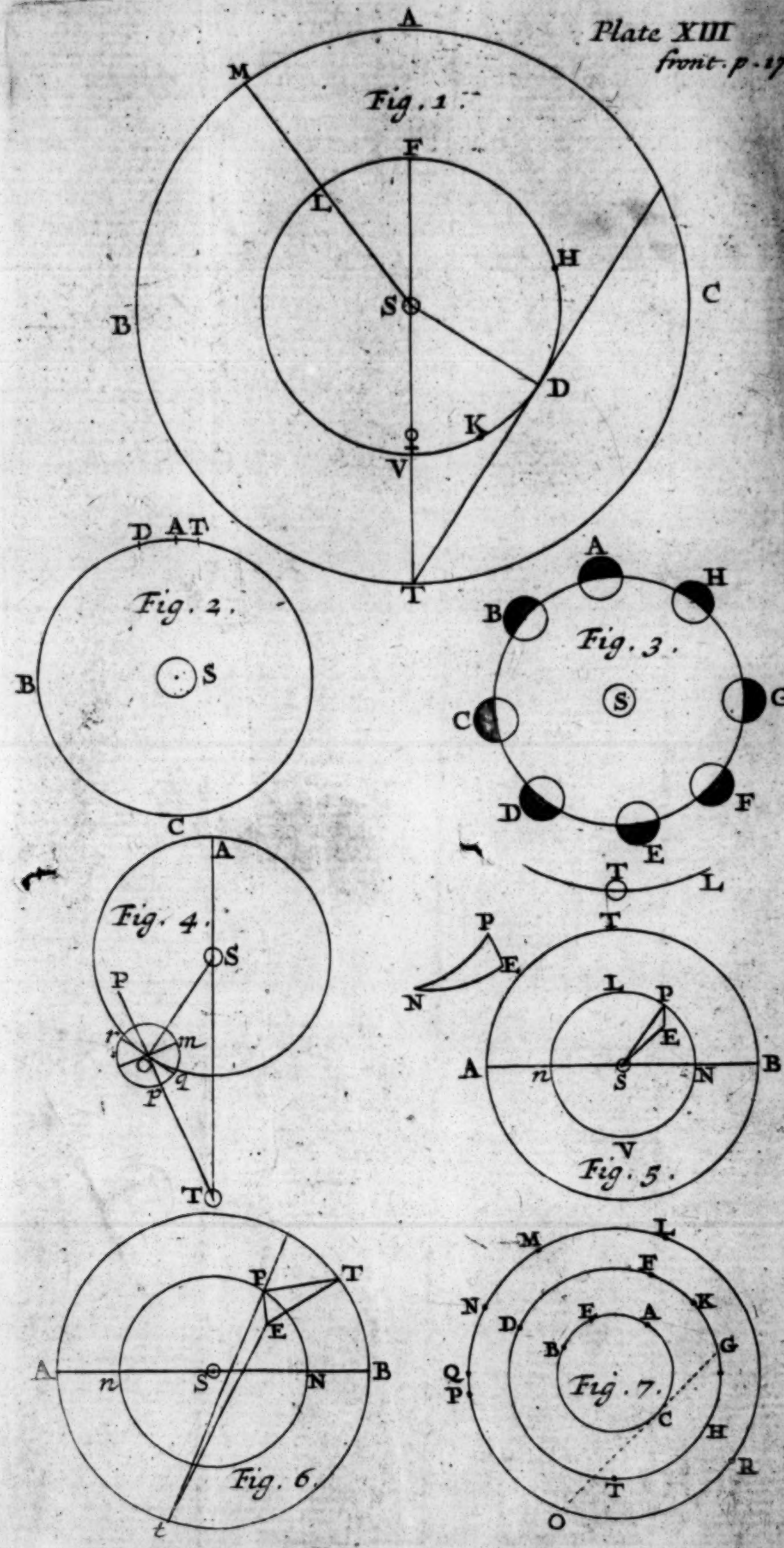
The Orbits of Mercury *and* Venus *Elliptical.*

WE ſhewed before, that the Orbit of the *Earth* was not a Circle, but an Ellipſe; the ſame Thing is true of the Orbits of *Venus* and *Mercury*, and of all the other *Planets*, which are really Ellipſes, and not Circles, that have one common *Focus* in which the *Sun* reſides, about whom the *Planets* perform their Circulations with Motions, which though not perfectly equable, yet they are all regulated by a certain, unchangeable and conſtant Law, which none

of

15

Plate XIII
front. p. 170



of them transgress; for every *Planet* moves in the *Perimeter* of his own Ellipse, so that the Line or Ray passing from its Center to the Center of the *Sun*, does always describe or sweep an Elliptick Space or *Area* proportional to the Time; or, which is the same Thing, in equal Time it sweeps an equal *Area*. Hence the *Planets* must move more slowly in their *Aphelia*, and quicker in their *Perihelia*: And these *Aphelia* are not like the *Apogeon* of the *Moon*; but they are either at Rest without Motion, or, if they have any, it is so slow, that it is not easily perceived in the Time of a Man's Age. And here it is to be observed, that of all the *Planets*, *Mercury* has the most Excentrick Orbit; for therein the Excentricity is to the mean Distance as 2051 to 10000.

Lecture XVI.

LECTURE XVI.

Of the Motions of the three superior Planets, Mars, Jupiter *and* Saturn, *and the Appearances arising from them.*

These superior Planets *may have any Position or Aspect in respect of the* Sun.

WE have now dwelt long enough on the Explications of the Motions of the two inferior *Planets*; let us next contemplate the Superiors. For which Purpose let A B C T be the Orbit of the *Earth*, and let *Saturn*, *Jupiter* and *Mars*, turn round the *Sun* in different Orbits at their proper Distances, and perform their Circulations, each in its proper Period; and let P Q V be a Portion of the *Zodiack* in which these *Planets* are observed to perform their Motions: *First*, It is plain that all these *Planets*, seen from the

Plate XIV. Fig. 1.

Sun,

Lecture XVI.

Sun, may be observed either in *Conjunction* with the *Earth*, or in *Opposition* to it. Thus *Saturn* may be in ♄, when the *Earth* is in M, in the Line which joins the Centers of the *Sun* and *Saturn*; in which Case the *Earth* and *Saturn* from the *Sun* are seen in *Conjunction*: But the *Earth* may likewise be in the same Right Line produced the contrary Way, as in B, where from the *Sun* these two *Planets* will be seen in *Opposition* to each other. But in this Situation, the *Sun* seen from the *Earth*, will appear to be in *Conjunction* with *Saturn*. *Secondly*, It is evident, that these *Planets*, seen from the *Earth*, may have any Aspect, or obtain any Position in respect to the *Sun*, and may have any desired Elongation from him; which cannot be in the inferior *Planets*, who are always confined to the Neighbourhood of the *Sun*. For from the *Earth* T, there may be drawn a Line T P, which will cut all the Orbits of the superior *Planets*, and may make, with T S the Line which joins the *Sun* and *Earth*, any Angle required, as S T P. And therefore, when the *Earth* is in T, *Saturn* may be in F, whose Elongation from the *Sun* will then be the Angle S T F. Moreover, when the *Earth* and any superior Planet are seen from the *Sun* in *Conjunction* together, that *Planet*, observed from the *Earth*, will appear in *Opposition* to the *Sun*; and an Inhabitant of our terraqueous Globe will see the *Sun* and it, in opposite Parts of the Heavens.

The Times between two Conjunctions, *or two* Oppositions, *in the superior* Planets.

Let now any superior *Planet*; as for Example, *Saturn*, be seen from the *Sun* in *Conjunction* with the *Earth*: After *Conjunction* the *Earth* having a quicker angular Motion than *Saturn*, an Inhabitant or *Spectator* in the *Sun* will see the *Earth* daily to recede more and more from *Saturn*. And because the *Earth*, according to its mean Motion, does every Day describe an Arch of the Ecliptick of 59 Minutes 8 Seconds, and *Saturn* moves only 2 Minutes in a Day, the *Earth* will appear from the *Sun*, to recede every Day from *Saturn* the Space of an Arch of 57 Minutes 8 Seconds. If we say then, as 57 Minutes 8 Seconds

Seconds is to 360 Degrees, or to 21600 Minutes, so is one Day to a fourth Quantity; we shall have the Number of Days in which the *Earth* will be again observed from the *Sun*, to be in *Conjunction* with *Saturn*, which is 378 Days. But when the *Earth* and *Saturn* are seen from the *Sun* in *Conjunction*, the *Sun* and *Saturn* from the *Earth* appear in *Opposition*. And therefore the Time between two *Oppositions* of the *Sun* and *Saturn*, immediately following one another, computed according to their middle Motions, is 378 Days, or 1 Year and 13 Days. And there is the same Time between two *Conjunctions* of *Saturn* and the *Sun* seen from the *Earth*, or between any two similar Aspects or Elongations from the *Sun*. And the Time between the *Opposition* and *Conjunction* of *Saturn* with the *Sun*, is the half of this Time, or 189 Days.

By the same Method we shall find, that the Time between two *Conjunctions* or *Oppositions* of *Jupiter* and the *Sun* consists of 398 Days, or a Year, and 33 Days. But *Mars*, after an *Opposition*, does not again come into the same Situation, till after two Years and 50 Days.

When the *Planets* are in *Opposition* to the *Sun*, they rise when the *Sun* sets, and set when he rises; and then, after their Departure from the *Opposition* to the *Sun*, they remain to the *Eastward* of the *Sun*; and after Sun-set they are to be seen in the Evening, till they come in *Conjunction* with him, when they set and rise together. Afterwards, as they recede from the *Sun*, they become more *Westerly* than he, and are then only to be seen in the Morning before the *Sun* is up; for in the Evening they set before the *Sun*, till they at last come to be opposite to the *Sun*, when again they rise at Sun-set.

The Planes of their Orbits are inclined to the Ecliptick.

As in the inferior *Planets*, so the superior have not their Orbits in the Plane of the Ecliptick; for the Planes of all their Orbits cut the Plane of the Ecliptick in Lines which pass through the *Sun*, which are called, the *Lines of the Planets Nodes*: And the

Points

Lecture XVI. Points where these Lines meet with the Ecliptick, are called the *Nodes*. And therefore the superior *Planets* are never precisely in the Ecliptick, but when they are in the *Nodes*: In all the other Points of their Orbits they are further or nearer to the Ecliptick, according to their Distance from the *Nodes*; and their Distances are greatest, when they are at equal Distances from both *Nodes*; which Points are called the *Limits*, where the greatest *Heliocentrick* Latitudes, which measure the Inclinations of the Orbits to the Ecliptick, are as follow: *Saturn*'s greatest *Heliocentrick* Latitude is 2 Degrees 30 Minutes *Jupiter*'s is 1 Degree 20 Minutes; and that of *Mars* is 1 Degree 52 Minutes.

The Heliocentrick *and* Geocentrick *Latitudes.*

HAVING the Place of a *Planet* in its Orbit, or, which is the same Thing, its Distance from the *Node*, by the same Method we find out its *Heliocentrick* Latitude, as we did in the inferior Planets *Mercury* and *Venus*. But the *Geocentrick* Latitudes, or the Distances of the *Planets* from the Ecliptick, as they are seen from the *Earth*, depend much upon the Position and Distance of the *Earth*. For where the *Heliocentrick* Latitude continues the same, yet according to the various Positions the *Earth* may have, the visible Latitude of a *Planet* seen from thence will be various. For let T ♂ *t* be the Orbit of the *Earth*; and the Orbit of any superior *Planet*; as for Example, that of *Mars*, suppose to be ♂ M, whose Plane is inclined to the Ecliptick, and cuts it in the Line of *Nodes* *n* N. Let *Mars* be in ♂ and the *Earth* in T, so as *Mars* may be observed in *Opposition* to the *Sun*; and from ♂ let fall on the Plane of the Ecliptick the Perpendicular ♂ E; this Line will subtend the Angle which measures the *Geocentrick* Latitude. And therefore, when the *Earth* is in T, the visible Latitude is measured by the Angle ♂ T E, But if the *Earth* was in *t*, so that *Mars* was seen in *Conjunction* with the *Sun*, its visible Latitude will be the Arch which measures the Angle ♂ *t* *E*, which is much less than the

Plate XIV. Fig. 2.

Angle

Angle ♂ T E, and is nearly less in the same Proportion as the Distance T ♂ is less than the Distance *t* ♂. When the *Earth* is in T, the *Geocentrick* Latitude of *Mars* is greater than its *Heliocentrick*; but when it is in *t*, the *Heliocentrick* is greater than the *Geocentrick*; and according to the various Positions of *Mars* and the *Earth*, his visible Latitude will be changeable; so that all other Things being alike, the Latitude is greater, the nearer he comes to the *Opposition* of the *Sun*; and the less, as he approaches to a *Conjunction* with the same.

Lecture XVI.

It is also evident, that none of the superior *Planets* can be seen from the *Earth* in the *Sun*'s Disk, as the inferior *Mercury* and *Venus* are; but yet they may be all of them covered by the *Sun*, and lie behind him, when they come in *Conjunction* with him, and are near their *Nodes*.

Jupiter *and* Saturn *have always a round full Face.*

Since the Faces of all the *Planets* which are turned towards the *Sun*, shine only with a reflected and borrowed Light; and because the *Earth*, seen from *Jupiter* or *Saturn*, is always to be observed near the *Sun*'s Body, the Faces of these *Planets* which are turned towards the *Sun*, will also be towards the *Earth*; whence the Inhabitants of our Globe do always behold these *Planets* shining in full Orbs or Circles. But *Mars* having an Orbit which lies very near the *Earth*, its Face, which is towards the *Sun*, will not always be totally turned towards the *Earth*: but when in his Quadrature, or when there is about a fourth Part of the Ecliptick between the *Sun* and him, as suppose the *Earth* in M or B, and *Mars* in N or R, then some Part of the illuminated Face will be turned from the *Earth*; and therefore *Mars* will not appear in a complete Circle, but will be seen as deficient or gibbose; but when he comes to be in *Conjunction* or *Opposition*, he then re-assumes his round Figure, his illuminated Face being totally turned towards the *Earth*; and particularly, when, in *Opposition* to the *Sun*, he looks brightest and biggest.

Mars *in his Quadrature, gibbose.* Plate XIV. Fig. 1.

Lecture XVI.

In Opposition *the superior* Planets *are biggest.*

For all the superior *Planets* appear much bigger when they are in *Opposition* to the *Sun*, than when they are in *Conjunction*, being much nearer to the *Earth* in the one Position than in the other; insomuch that the Difference of their Distances in these two Positions, is as great as the Diameter of that Orb in which the *Earth* goes round the *Sun*; which Difference bears a considerable Proportion to the Distance of *Mars* from the *Sun*, and greater than it does to the Distances of the other *Planets*; and therefore will produce a great Difference in his apparent Magnitude: For *Mars* is five times nearer to us when he is in *Opposition*, than when he is in *Conjunction* with the *Sun*. And therefore, since the visible Disk and Lustre of a *Planet* increases in a duplicate Proportion of that wherein the Distance is diminished, *Mars* will appear 25 times bigger and brighter when he is in *Opposition*, than when he is in *Conjunction* with the *Sun*.

The apparent Diameter of the Sun *seen from* Jupiter *and* Saturn.

Because *Jupiter* is five times further off the *Sun* than the *Earth* is, the apparent Diameter of the *Sun* seen from *Jupiter* will be five times less than it is seen from the *Earth*, and will be no bigger than 6 Minutes, which to us is 30 Minutes. And the Disk of the *Sun* will appear 25 times less to the Inhabitants of *Jupiter*, than it does to us, who will likewise receive but the 25th Part of the Light and Heat from him that we enjoy. But *Saturn* being ten times further from the *Sun* than we, the apparent Diameter of the *Sun* seen from him, will be no bigger than 3 Minutes, and will be but little more than twice the Diameter of *Venus*, when she approaches nearest to the *Earth*: And therefore the Disk of the *Sun*, as it would appear to a *Saturnian Astronomer*, will be a hundred times less than we see it; and both its Light and Heat are there diminished in the same Proportion; and therefore the warmest Regions in *Saturn*, even under his Æquator, are much colder than our Frigid Zones.

Their Degrees of Heat compared with our Heat, which we receive from the Sun.

All

16

Plate XIV
front. p. 176

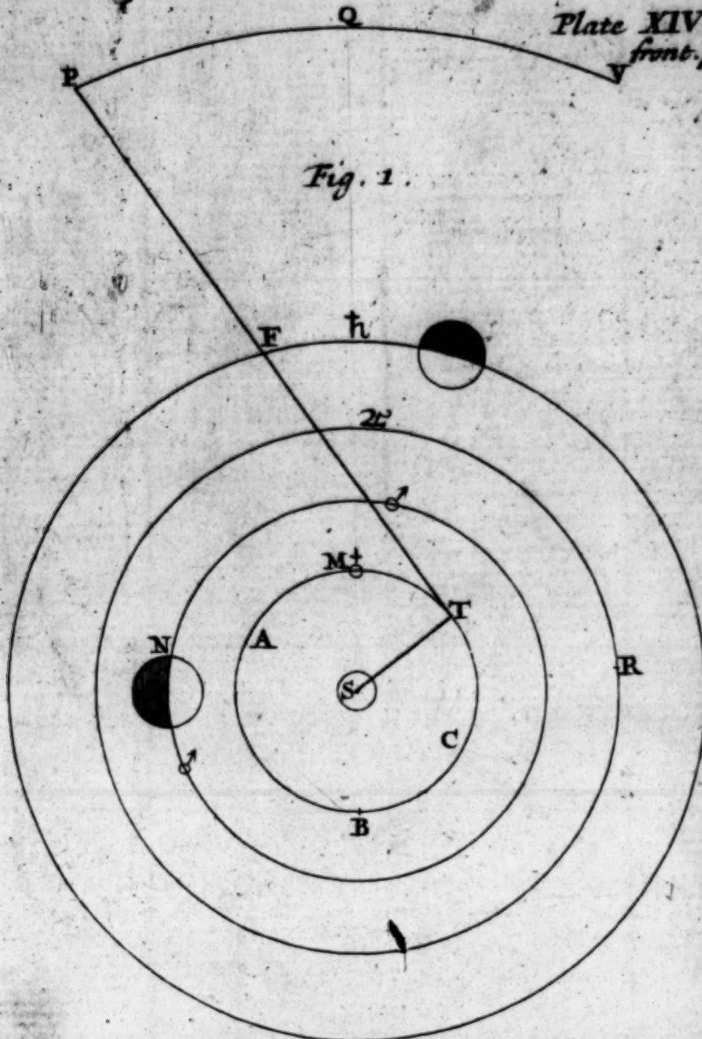


Fig. 1.

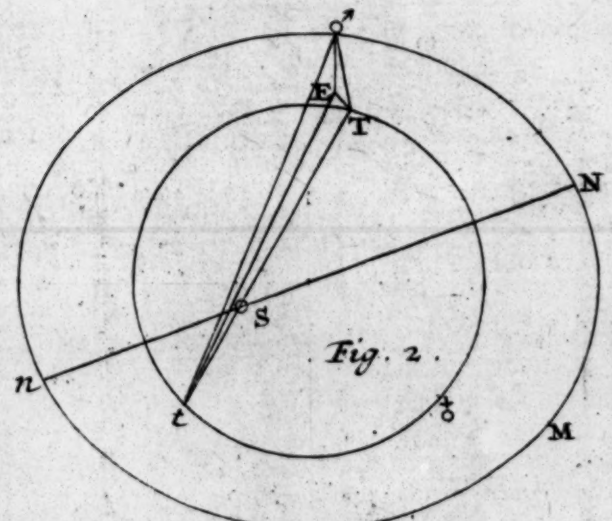


Fig. 2.

ALL the superior *Planets* observed from the *Sun*, will appear to move regularly the same Way, and to proceed in their Orbits, according to the same Law; which is the equable Description or sweeping of Elliptick *Area's* round the *Sun*; by which Means their angular Motions round the *Sun* will appear somewhat unequal: For in their *Aphelia*, they proceed more slowly; in coming to their *Perihelia*, they accelerate their Motions. But these *Planets*, observed from the *Earth*, have very different Appearances, and irregular Motions in the *Zodiack*. Sometimes they seem to move forward from *West* to *East*, according to their real Motions; then they by Degrees slacken their Pace, till at last they lose all their Motion, and seem to stand still. After some small Time they are again set a moving, but seem to take a contrary Course to what they had before; and go backward directly in Opposition to their real and true Motions. And thus having for some Way gone backward, or from *East* to *West*, they come again to be immoveable and stationary. These great Changes of their Courses and Motions are not real in the *Planets*, but are occasioned by the Motion and Position of the *Earth*, from whence the *Astronomer* observes them.

Lecture XVI.

The Motions of the Planets seen from the Sun, are very regular.

But from the Earth they are observed to be very irregular.

LET PQO be a Portion of the *Zodiack*, ABCD the Orbit of the *Earth*, EMGHZ the Orbit of a superior *Planet*: For Example, of *Saturn*; and suppose the *Earth* in A, and *Saturn* in E; in which Position he will appear in the *Zodiack* at the Point O. If *Saturn* remained there without any Motion of his own, when the *Earth* comes to B, he would be seen in the Point of the *Zodiack* L, and would appear to have described the Arch of the *Zodiack* OL, and to have moved according to the Order of the Signs, from *West* to the *East*. But because in the mean Time, while the *Earth* is passing from A to B, *Saturn* does likewise move in his own Orbit from E to M, where he is seen in *Conjunction* with the *Sun*, he will appear to have described the Arch of the *Zodiack* OQ,

Plate XV. Fig. 1.

Lecture XVI.

which is greater than the Arch O L; whence the superior *Planets*, when they are in *Conjunction* with the *Sun*, appear to have a Motion forward much quicker that the other Times, and that for a two-fold Cause; which is because they really have a Motion forward from *West* to *East*; and likewise because the *Earth*, in the opposite Part of the Heavens, is carried the same Way round the same Center. And therefore these *Planets*, when they are at their greatest Distance from us, and in *Conjunction* with the *Sun*, appear to have a quicker Motion than usual to the *East*, according to the Order of the Signs: In which Position a *Planet* is said to be direct, or to have a direct Motion. When the *Earth* comes to C, while *Saturn* describes the Arch M G, he will then be observed in the *Zodiack* at R. But the *Earth* being advanced to K, and *Saturn* to H, so as the Line K H joining the *Earth* and *Saturn*, continue for some Time parallel to itself, or very nearly so; then our *Astronomers* will observe *Saturn* all that while in the same Point of the *Zodiack* at P, and with the same *fixed Stars*, he then appearing Stationary. But the *Earth* being come to D, and *Saturn* coming into *Opposition* to the *Sun* in X, he will appear in the *Zodiack* at V, and will seem to have gone backwards through the Arch P V. And therefore the superior *Planets*, when they are in *Opposition* to the *Sun*, are always retrograde, or appear to have a backward Motion from *East* to *West*, which is contrary to the Order of the Signs. But when the *Earth* comes again to A, and *Saturn* remaining near to Z, again that *Planet* will seem there to ocupy his Station, and to remain without Motion. at last, after the *Earth* has left that Situation, *Saturn* will appear to begin again to move forward.

When the superior Planets *are direct and swift.*

When Stationary.

WHAT we have here shewed concerning *Saturn*, is likewise to be understood of *Jupiter* and *Mars*, who are likewise observed to have all these Variations and Changes in their Motions, as sometimes to move quickly forwards, then to stand still,

and

and after that to fall backward; then again they become Stationary; and in a short Time after they go forward with direct Motion. But the Regressions or backward Motions of *Saturn*, are more frequent than those of *Jupiter*; because the *Earth* more frequently overtakes *Saturn* whose Motion is slower than *Jupiter's*, who is not a little quicker in his Motion. And for the same Reason *Jupiter's* Regressions do oftner happen than those of *Mars*; because *Mars* moving faster, describes a greater Space in the *Zodiack*; so that there is more Time necessary for him to come in *Opposition* to the *Sun*, than what *Jupiter* needs for that Purpose.

Lecture XVI.

When Retrograde.

Let A C be a Portion of the *Earth's* Orbit, which is touched by the right Line A N, in which we will suppose the superior *Planets* to be seen from the *Earth*, viz. *Mars* in ♂, *Jupiter* ♃, and *Saturn* in ♄; and let K L M N be a Portion of the *Zodiack*. Then the Place of *Mars* seen from the *Sun* is K, which is called his true or heliocentrick Place. But an *Astronomer* on the *Earth* will observe him at the Point N, which is called his *apparent* or *geocentrick Place*; so likewise *Jupiter*, seen from the *Sun*, appears in L, which is his true Place; but from the *Earth* his apparent Place is N. After the same Manner the true Place of *Saturn* seen from the *Sun*, the Center of his Motion, is M; but his Place in the *Zodiack*, that is visible from the *Earth*, is N. The Arches K N, L N, M N, the Differences between the true and apparent Places of the superior *Planets*, are called the *Parallaxes* of the annual Orb in these *Planets*. Through the *Sun* S draw S O parallel to A N, and by the 29th of the 1*st* of *Euclid*, the Angles A ♂ S, A ♃ S, A ♄ S, will be respectively equal to the Angles K S O, L S O and M S O. But the Angle A N S is equal to the Angle N S O, whose Measure is the Arch N O, which will therefore be the Measure of the Angle A N S, which is the Angle under which the

The Parallax *of the annual Orb.*

Plate XV. Fig. 2.

 Semidi-

Lecture XVI.

Semidiameter A S of the *Earth*'s Orbit is ſeen from the ſtarry Heavens. But this Semidiameter is nothing in reſpect of the great Diſtance of the Heavens or Stars; for from thence it would appear under no ſenſible Angle, and look like a Point. And therefore in the Heavens the Angle N S O, or the Arch N O vaniſheth, and the Points N and O coincide; and the Arches K O, L O, M O, are of the ſame Bigneſs with the Arches K N, L N, and M N, which are therefore the Meaſures of the Angles A ♂ S, A ♃ S, A ♄ S. But theſe Angles are as the apparent Semidiameters of the Orbit of the *Earth* ſeen from the reſpective *Planet*: And therefore in each of the ſuperior *Planets* the *Parallax* of the annual Orbit is equal to the Angle under which the Semidiameter of the *Earth*'s Orbit is ſeen from that *Planet*; and the nearer any of them is to the *Earth* or *Sun*, ſo much the bigger is that Angle. And therefore this *Parallax* in *Mars* is greater than in *Jupiter*, and again in *Jupiter* greater than it is in *Saturn*. But in the *fixed Stars* there can be no *Parallax* of the annual Orb obſerved, it being ſo very ſmall.

The Retrogreſſions of Mars *greater than thoſe of* Jupiter; *and* Jupiter's *greater than* Saturn's.

IT is alſo evident from hence, that the Retrogreſſions of *Mars* are greater than thoſe of *Jupiter*, though they do not happen ſo often; ſo likewiſe *Jupiter* has his Retrogreſſions greater than thoſe of *Saturn*; and that upon a double Account: Firſt, becauſe *Mars* is nearer to the *Earth* than *Jupiter*, and *Jupiter* nearer than *Saturn*; and likewiſe becauſe they move faſter.

The Diſtances of the Planets *from the* Sun, *found by the Parallax of the annual Orb.*

HAVING the *Parallax* of the annual Orb in any *Planet*, we can from thence eaſily find his Diſtance from the *Sun*, in reſpect of the *Earth*'s Diſtance from him. For in *Mars*, becauſe the Angle A ♂ S is given, being meaſured by the *Parallax* of the annual Orb, and the Angle ♂ A S is found by Obſervation, being the viſible Elongation of *Mars* from the *Sun*: If we make the Proportion, as the *Sine* of the annual *Parallax* is to the *Sine* of the Elongation, ſo let S A the Diſtance of the *Earth* from the *Sun* be

be to a Fourth, which will be ♂ S, the Distance of *Mars* from the *Sun*. This annual *Parallax*, by which the *Planets* seem sometimes to move faster, sometimes slower, in the Heavens, sometimes to go *Eastward*, and sometimes *Westward*, produces in their Motions an Inequality; which, by the *Astronomers*, is called their *second* or *optical* Inequality, to distinguish it from their first Inequality, which the *Planets* really have, by which they move in their Orbits with Motions that are not always the same. In the *Oppositions* or *Conjunctions* of these *Planets* with the *Sun*, this second Inequality or *Parallax* vanishes; and their *Geocentrick* Places and the *Heliocentrick* coincide; or a *Spectator* in the *Sun*, and another in the *Earth*, would observe the *Planet* in the same Point of the Heavens.

Lecture XVI.

Their second or optical Inequality.

THE Angles A ♂ S, A ♃ S, A ♄ S, are nearly the greatest Elongations of the *Earth* from the *Sun*, if she were observed from the respective *Planets*, when the Line N ♂ A touches the *Earth*'s Orb in A. In *Mars* the Angle A ♂ S is about 42 Degrees; and therefore the *Earth*, seen from *Mars*, never goes so far from the *Sun* as we see *Venus* does. In *Jupiter* the greatest Elongation of the *Earth* from the *Sun* will be observed to be but 11 Degrees, and therefore is not so much as half the Distance we observe *Mercury* to depart from the *Sun*. In *Saturn*, the Angle A ♄ S, or the greatest Elongation of the *Earth* from the *Sun* that can be seen from that *Planet*, is but 6 Degrees, and not much above a fourth Part of the greatest Elongation we observe in *Mercury*. And since *Mercury* is but seldom seen by us, a Sight of the *Earth* from *Saturn* may be a rare and unusual Spectacle: Perhaps the *Saturnian Astronomers* have not yet discovered, that there is such a Body as our *Earth* in the Universe.

EACH of the two outmost of the *Planets* have a good Company of Attendants; for *Jupiter* keeps no fewer than four constantly by him, and *Saturn* five in his Retinue, which is a Sight no less wonderful than delightful. These *Satellits*, like our *Moon*, do always

The Attendants or Satellits *of* Jupiter *and* Saturn.

Lecture XVI. always accompany their primary *Planets* in their Circuits round the *Sun*; and in the mean Time they perform their proper Circulations about their Primaries; and therefore they will have the ſame *Phaſes* and Figures that our *Moon* ſhews us: When they are in *Oppoſition* to the *Sun*, they appear to *Saturn* and *Jupiter* bright and full; from thence receding, they aſſume a gibbous Shape. When they come to a Quadrantile Aſpect, they look like Half-Moons; before the *Conjunction* they ſhew themſelves in horned Figures; and when they come to be joined in the ſame Line with the *Sun*, they totally diſappear.

THESE *Satellits*, ſeen from the *Earth*, though they go, at the furtheſt, but a little Way from their Primaries, yet ſometimes they approach them nearer, and ſometimes remove a little further from them. Plate XV. Fig. 3. Let ABT be the Orbit of the *Earth*, in the Middle of which the *Sun* reſides. Let E F be a Portion of the Orb of *Jupiter*; in which let *Jupiter* be in a ♃, who keeps in the Middle of the Orbits of his four Attendants. Theſe *Satellits* or *Moons*, when they deſcribe the inferior Parts of their Orbits L M N, ſeen from the *Earth* or *Sun*, will appear to have a Motion *Weſtward*; but while they are moving through the ſuperior Portions G H K, we obſerve them to move *Eaſtward*, according to their true Motions. Now when their viſible Motions is *Eaſtward*, they are twice hid from us; once in O behind the Body of *Jupiter*, that is, in the right Line which joins the Centers of the *Earth* and *Jupiter*; and again they vaniſh and become inviſible, when they fall into the Shadow of *Jupiter*, or are in the right Line which joins the Centers of the *Sun* and *Jupiter*; and then they ſuffer Eclipſes, which is always when they are at their Full, as ſeen from *Jupiter*: Theſe Eclipſes happening in the ſame Manner as they do to our *Moon*, by the Interpoſition of the *Earth* between the *Sun* and it.

WHEN *Jupiter* is to the *Eaſt* of the *Sun*, and is ſeen in the Evening after Sun-ſetting; that is, when the *Earth* is in A, they are firſt hid behind *Jupiter*,

Lecture XVI.

Jupiter, because of their visible *Conjunction* with *Jupiter*, before they fall into his Shadow; and their second disappearing is in the Eclipse, upon their entering the Shadow. But when *Jupiter* is more *Westerly* than the *Sun*, as he appears after *Conjunction*, when he is only seen in the Morning; that is, when the *Earth* is about B, then they fall into *Jupiter's* Shadow at V, and are eclipsed, before they are hid behind his Body in P. But when these *Moons* have a retrograde Motion, that is, when they are seen to go *Westward*, and describe the inferior Parts of their Orbits, then they only once disappear in Q, when they cannot be distinguished from the Body of *Jupiter*: But when the *Satellits*, seen from the *Sun*, are in their inferior *Conjunction* with *Jupiter*, or as seen from *Jupiter*, they are in *Conjunction* with the *Sun*, their Shadows will fall upon *Jupiter*, and some Part of the Disk of *Jupiter* will be in an Eclipse, and a *Spectator* within the Shadow would observe a total Eclipse of the *Sun*. We have already given the Distances and Periods of all the *Jovial* and *Saturnian Moons*, at the End of our third Lecture.

The Parallax *of the annual Orb, and the Distance of* Jupiter *from the* Sun, *determined by the Eclipses.*

By the Motions and Eclipses of these *Moons*, the *Parallax* of the annual Orb in *Jupiter*, and his Distance from the *Sun*, may be easily known. For, let POR be the Orbit of any *Satillit*; for Example the outermost; and suppose the *Earth* in the Point of its Orbit A, the Time must be observed when the *Satellit* lies hid behind *Jupiter's* Body in O: For which Purpose the Moment of Time must be carefully marked when he first disappears, and then also the Moment he becomes again visible; the middle Moment between these two, is the Time when the *Satellit* is in O, or in the Line which passes through the Centers of the *Earth* and *Jupiter*. After the same Manner observe when the *Satellit* is in the middle of an Eclipse, or in the middle of *Jupiter's* Shadow, that is, when it is in V; by this Means we shall have the Time it takes to describe the Arch OV. And because his Motion about *Jupiter* is

equable,

Lecture XVI. equable, and his periodical Time known, we can from thence find out the Arch O V; for this *Planet* revolves about *Jupiter* in 402 Hours. Let us suppose the Time he takes to move from O to V be 12 Hours; say, As 402 Hours are to 12 Hours, so are 360 Degrees to a fourth Quantity, which will be found to be 10 Degrees 44 Minutes. And therefore the Arch O V is 10 Degrees 44 Minutes. But this Arch OV is the Measure of the Angle O ♃ V, or of the Angle which is equal to it A ♃ S; and the Arch which measures this Angle is the *Parallax* of the annual Orb, which therefore is known. In the Triangle therefore A ♃ S we have the Angle at ♃, and also the Angle at A the Elongation of *Jupiter* from the *Sun*, which may be had either by a Calculation from Astronomical Tables, or by Observation. Besides, we have the Side A S, the Distance of the *Earth* from the *Sun*, which we assume to consist of 100000 Parts. Since therefore in this Triangle we have all the Angles and one Side, by *Trigonometry* we shall find the other Sides, and particularly S ♃ the Distance of *Jupiter* from the *Sun*; so likewise we may find A ♃ the Distance of *Jupiter* from the *Earth*, which is always variable. But for the nice Determinations of these Distances, it may be needful to have several, and those very accurate Observations, made by the Skilful, and taken by the Help of the best Telescopes.

Whether Light be propagated in an Instant, or in Time?

BY the Eclipses of *Jupiter*'s *Moons* we are able to give a Solution of a Problem, which is the most noble and curious in natural Philosophy, which cannot but raise our Wonder and Amazement; that is, Whether Light be propagated to us in an Instant; or if its Motion be successive, and if it takes some Time to arrive from the *Sun*, or any distant Object, to us! Now these Eclipses do shew us, that there is no instantaneous Motion in Light, though it comes from the Heavens to us with a prodigious quick Motion, and incredible Celerity.

FOR

For if the Motion of Light were in an Instant, when the *Earth* is at T, at his greatest Distance from *Jupiter*, an *Astronomer* here would observe an Eclipse of a *Satellit* at the same Moment of Time he would do, were the *Earth* at X at her nearest Distance to *Jupiter*: For, according to this *Hypothesis*, Light is propagated in the same Distance through all Spaces indefinitely, whether near, or never so much remote. But if Light takes up any Time for its Propagation through Space, it will sooner pass through a shorter Space than a greater. And therefore an Observator at X, being nearer to *Jupiter* than one at T, by the Distance X T, which is almost equal to the Diameter of the *Earth*'s Orbit, will sooner observe the Eclipse of a *Satellit*, than a Spectator can do at T. And therefore from the Difference of those Times, which is proportional to X T the Difference of Distances, we can collect the Velocity of Light; and so this Matter is in Reality. For whenever the *Earth* is at its nearest Distance from *Jupiter*, the Eclipses are found to happen sooner than they do when they are observed from T at a greater Distance, where they fall out sensibly later than they ought to be, according to our Astronomical Computations. These quicker and slower Returns of Eclipses having been observed for many Years by Mr. *Romer* with much Care and Diligence, upon them he founded this Argument for demonstrating the successive Propagation of Light; and by them he proved, That Light, like all other Bodies in Motion, had a determined Degree of Velocity, and took a determined Time to move through a given Space. To which Opinion the most Part of the *Astronomers* and *Philosophers* do now give their Assent.

This Question determined by the Observation of the Eclipses of Jupiter*'s* Moons.

The Particles therefore of Light, though their Minuteness be indefinite, and not easily to be imagined, yet they have a progressive Rectilinear Motion, and are not diffused as by the Waves of any *Medium* or Fluid. *Romer* determines the Velocity of Light to be such, that it reaches us here from the

Sun

Lecture XVI.

Sun in the Space of a 11 Minutes: But that Distance does not seem to be less than 50000000 Miles; which Space Light passes through in so small a Time, that so prodigious a Velocity cannot easily be conceived by us, which so much exceeds the Velocity of the swiftest Bodies we know. For though the *Earth* has a very quick Motion round the *Sun*, yet its Velocity, compared with the Velocity of Light, is no more than that of a Snail, in Comparison of the Swiftness of the *Earth*.

The Longitude of Places determined by the Observations of those Eclipses.

FROM the Eclipses of *Jupiter*'s *Moons* we have likewise this Advantage, that when they are observed in different Places of the *Earth*, the Longitude of Places are by such Observations determined. But that this Method of finding the Longitude may be the more easily understood, we must first lay down some few Principles.

IF through the Poles of the *Earth* and any Place, there be drawn a great Circle upon its Surface, this Circle, by the Rotation of the *Earth*, will be turned round the *Earth*'s *Axis*: And when the Plane of this Circle produced, passes through the Body of the *Sun*, all the Inhabitants which live under this Circle will then observe the *Sun* to come into their Meridian, and they will have Mid-day; from whence this Circle has the Name of a Meridian, from the *Latin* Word *Meridies*, which signifies Mid-day. Now if we imagine another Meridian placed more *Westwardly*, which, with the former, makes an Angle of 15 Degrees, the Plane of this Meridian will pass through the *Sun* one Hour later than the former did; and therefore, when the Inhabitants under this Meridian reckon Mid-day, the Inhabitants under the first will reckon one Hour after Mid-day. If there be a Meridian which makes an Angle of 30 Degrees with the first we mentioned; then, when they that live under this Meridian have Mid-day, those that live under the first, will reckon two of the Clock after Mid-day; and so for every 15 Degrees of the Æquator which lies between the two Meridians, so many Hours more do they reckon, who live under

the

the more *Eastern* Meridian, than they who live under the *Western*. And after the same Manner for every Degree of the Æquator between Meridians, the *Eastern* People are four Minutes sooner in their Reckoning than the *Western*; and for every 15 Minutes of a Degree, they reckon one Minute in Time. As for Example, If the Arch of the Æquator between the two Meridians consist of 85 Degrees, dividing 85 by 15, the Quotient $5\frac{2}{3}$ shews, that under the more *Easterly* Meridian they reckon the fifth Hour and 40 Minutes, when they under the *Western* Meridian have Mid-day; and when the *Eastern* People have Mid-day, those to the *West* will reckon their Time to be the sixth Hour and 20 Minutes in the Morning; and the Difference between the Hours which are reckoned under these two Meridians, will always be $5\frac{2}{3}$, if the Arch of the Equator intercepted between them be 85 Degrees. Lecture XVI. 4

ON the contrary, having the Difference of the Hours which are reckoned under two different Meridians for the same Moment of Time, we shall by this Difference find the Arch of the Æquator intercepted between them; which Arch is called the *Difference of Longitude* of the Places under those Meridians, when the Longitudes are computed from one fixed and settled Meridian, which is called the *first Meridian*: And this Arch is found by multiplying the Difference of the Hours by 15, and the Product shews the Degrees. So likewise, if the Minutes of Time be multiplied by 15, and the Product, if it exceed 60, be divided by 60, the Quotient and Residue will give the Degrees and Minutes that are further to be added to the former, and which make up the Difference of Longitude of Places. For Example, Suppose the Difference of the Hours to be 7 and 22 Minutes; 7 multiplied by 15 is 105, and 22 by 15 is 330 Minutes; which divided by 60, gives 5 Degrees 30 Minutes: And therefore the whole Difference of Longitude is 110 Degrees 30 Minutes. These Things being noted,

Lecture XVI.

If in two different Places the Beginning of an Eclipſe of any of *Jupiter*'s *Moons* be obſerved, and the Times marked when this Beginning happened; according to the Times of the reſpective Places, the Difference of Hours, converted into Degrees and Minutes of the Æquator, will ſhew the Difference of Longitude of thoſe Places.

If we had *Ephemerides* of the Motions and Eclipſes of *Jupiter*'s *Moon*, accurately computed for any Meridian; inſtead of an Obſervation in another Place, we might conſult the *Ephemerides*, which tell when the Eclipſe is to be obſerved in that Place; and we might take from them the Hours and Minutes when the Eclipſe happens in that Place; and this Time, compared with the Time the Eclipſe is obſerved in any other Place, will give the Difference of Times in thoſe two Places: And from thence we can find out the Difference of their Longitudes as before. The Longitude of Places may likewiſe be found by Obſervations of Eclipſes of *Moon*, or the *Appulſes* of the *Moon* to the *fixed Stars*, obſerved from ſeveral Places: But theſe are Appearances that are more ſeldom to be obſerved, than are the Eclipſes of the *Satellits* of *Jupiter*.

Upon Land the Eclipſes are eaſily obſerved, and the Longitude found, but not at Sea.

Upon Land and firm Ground the Eclipſes are eaſily obſerved; and if they could be as eaſily obſerved at Sea, the Art of Navigation would be brought almoſt to Perfection, and liable to no Errors in Computation; but at Sea the Motion and Toſſings of the Ship render all Obſervations of ſuch Eclipſes impracticable. And therefore, if any could find a Method for determining the Longitude of a Ship at Sea at any Time, he would then oblige the Seamen with a Diſcovery, by them more deſired than any thing elſe in Navigation; and which would be ſo uſeful to the Publick, that the Parliament hath thought fit to allow a large Reward of 20000 Pounds to the Diſcoverer. Upon which many, tempted by ſo great a Reward, have ſpent much Labour and Thought, to make the Diſcovery, but to no Purpoſe: For no Man has hitherto been able to

lay

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fronting p. 188

Plate XV

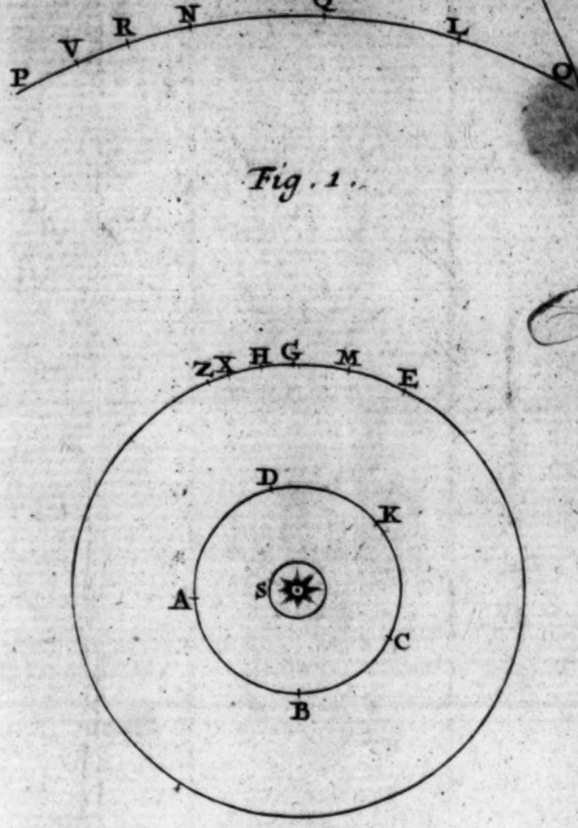

Fig. 1.

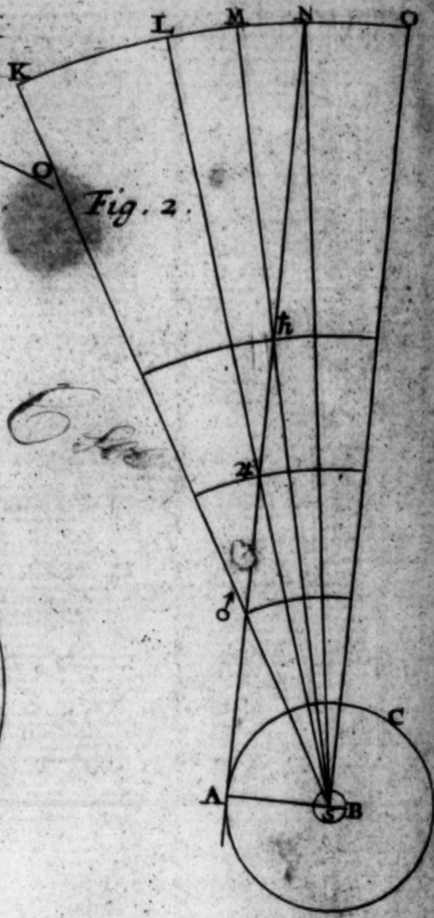

Fig. 2.

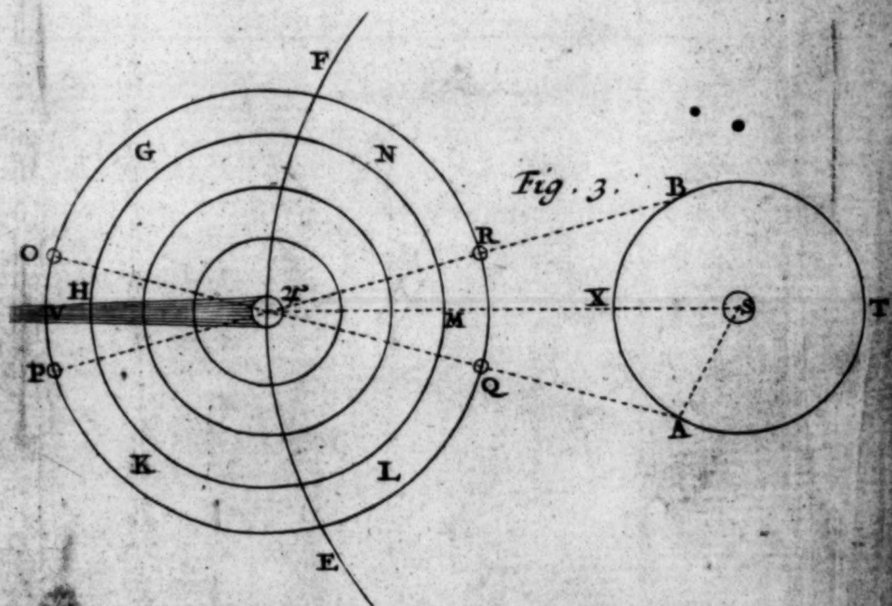

Fig. 3.

lay hold on the Reward, though they have proposed many different Methods and Ways of attaining it. Many being much in love with their own Inventions, imagining that they had certainly found it, have demanded the Reward promised to the Discoverer; but yet most of these Men have been so ignorant, that they have scarce known what it is to find the Longitude.

Lecture XVII.

LECTURE XVII.

Of COMETS.

BESIDES the ordinary *Planets*, which are always in our Neighbourhood, and within our View, there are another Sort of *Planets*, which may be called *Temporary*; which are conspicuous only for a Season, after which they again withdraw, and are no longer visible. The ancient Philosophers allowed them a Place in the heavenly Regions, and ranked them in Stations far above the *Moon*. For *Aristotle*, *Seneca*, *Plutarch* and others testify, That the *Pythagoreans*, and the whole *Italian* Sect, maintained that a Comet was a Kind of *Planet* or *wandering Star*, which appeared again after a long Interval of Time. *Hippocrates Chius* was of the same Opinion, as *Aristotle* informs us: The same was the Opinion of *Democritus*, as we are told by *Seneca* in his *Natural Questions*, *Book* VII. *Chap.* 3. For, says he, " *Democritus*, the most curious and " subtle of all the Ancients, suspected, that there " were many more *Stars* which moved, understand- " ing by them the Comets; but he neither esta- " blished their Number or their Names, the Courses

Comets are a Sort of Planets.

The Philosophers Opinion of Comets.

" of

Lecture XVII.

" of the five *Planets* not having as yet been disco-" vered." Again, *Seneca* assures us, That *Apollonius Myndius*, one of the most skilful Philosophers in the Search of Natural Causes, did assert, That the *Chaldeans* reckoned Comets among the other *wandering Stars*, and that they knew their Courses. *Appolonius* himself maintained, That a Comet was a *Star* of its own Kind, as the *Sun* and *Moon* are, but that its Course was not yet known: That by its Motions it mounts very high in the Heavens, and only appears when it descends into the lower Part of its Orb. And *Seneca* himself embraces this Opinion: " I cannot believe, says he, that a Comet is a Fire " suddenly kindled, but that it ought to be ranked " among the eternal Works of Nature. A Comet " has its proper Place, and is not easily moved from " thence; it goes its Course, and is not extinguished " but runs off from us. But you will say, if it were " a *wandering Star*, it would keep in the Zodiack: " But who can set one Boundary to all the *Stars*? " Who can restrain the Works of the Divinity to a " narrow Compass? For each of those Bodies which " you imagine to be the only that have Motion, " have very different Circles; why therefore may " there not be some that have peculiar Ways of " their own, wherein they recede far from the rest? " But that their Courses may be known, it is ne-" cessary to have a Collection of all the ancient Ob-" servations about Comets; for their Appearances " are so rare, that their Orbits are not yet deter-" mined; nor can we, as yet, find if they have " their Periods, and if they return again in a cer-" tain Order." At last he thus prophesies: " The " Time will come wherein these Things, which are " now hid from us, will be discovered; which Ob-" servation, and the Diligence of After-Ages will " find out; for it is not one Age that is sufficient " for so great Matters. The Time will be when " Posterity will wonder that we were ignorant of " Things so plain: One will arise who will demon-" strate in what Regions of Space the Comets wan-

" der,

"der, why they recede so far from the other *Planets*, how great, and what Sort of Bodies they "are."

Lecture XVII.

BUT for all this, the whole Sect of *Peripateticks*, fearing that Generations and Corruptions should be introduced into the Heavens by placing the Comets in them, thrust all the Comets down into the sublunary Regions, and would maintain, that they were nothing but a Kind of Meteors. But the *Phænomena*, or the Manner these Comets appear in, will not suffer them to have a Place so low, and so near to us. For it is clear that they are not generated in our Atmosphere, because they are certainly far higher than it reaches: For Comets are to be seen at the same Time from different Parts of the *Earth*, which are at great Distances from one another; which cannot happen to any Body that resides within our Atmosphere, which is not extended upwards above fifty Miles.

The Peripateticks supposed Comets to be Meteors generated in the Air.

BUT that Comets are not only above the Air, but also beyond the *Moon*, is plain; because Comets seen from different Places, are observed to be at the same Distance from a *fixed Star* which is near them. As for Example, The Comet which *Tycho Brahe* observed at *Uraniburg*, was likewise seen by *Hagecius*, at *Prague* in *Bohemia*, at the same Time, which two Places differ 6 Degrees in Latitudes, and are nearly under the same Meridian; and both measured the Distance of this Comet from the *Star* we call the *Vultur*; that is, how much it was below it towards the Horizon; for both the *Vultur* and it were in the same Vertical Circle, and both Observators found their Distance the same, and consequently they both viewed the Comet in the same Point of the Heavens; which could not be, unless it had been higher than the *Moon*.

Comets are higher than the Moon.

Plate XVI. Fig. 1.

LET the Circle A B G represent the *Earth*, in which let *Uraniburg* be in A, and *Prague* at B: Let D be the Place of the Comet: Let F C E be the Firmament of the *fixed Stars*, in which let the *Star* F be the *Vultur*; the Place of the Comet seen from *Uraniburg*

A Demonstration that Comets are higher than the Moon.

Lecture XVII. *Uraniburg* among the *Stars* is E, and its Diſtance from the *Vultur* is the Arch F E; but the Comet ſeen from *Prague*, appears in C; and its Diſtance from the *Vultur* is the Arch F C, which is leſs than the Arch F E. But by Obſervation is has been found, that this Comet, ſeen from both theſe Places, ſeemed to be at the ſame Diſtance from the *Vultur*; and therefore the Arches F E and F C are equal, or rather the ſame. So great therefore is the Diſtance of the Comet from the *Earth*, that the Arch C E vaniſheth, and is altogether imperceptible: But the *Moon*, ſeen from theſe two Places, would appear to have different Diſtances from the *Vultur*; and ſo therefore would a Comet were it as near as ſhe is: This Comet therefore was further diſtant off than the *Moon*.

The true and the apparent Place of a Comet. A Comet at D ſeen from the Center of the *Earth* would appear in G; but from the Surface of the *Earth* at A it is obſerved in E: The firſt is called the *Comet's true Place*, and the ſecond its *apparent Place*; and the Diſtance G E between the true and apparent Place, is called the *Parallax* of the Comet: By it a Comet is always depreſſed more towards the Horizon, than it is in its true Place.

Its Parallax. Now the *Parallax* of any *Star* is always equal to the Arch that meaſures the Angle, that the *Earth's* Semidiameter, paſſing through the Place of the Obſervator, is ſeen under from the Comet; as we ſhewed before, when we treated of the *Parallax* of the *Moon*.

Now if there be no ſenſible *Parallax*, the Angle, under which the Semidiameter of the *Earth* is ſeen from the Comet, will not be ſenſible; and therefore the Comet muſt needs be at a vaſt Diſtance from us, ſince the *Earth* ſeen from thence appears no bigger than a Point.

A Way to find if a Comet has a ſenſible Parallax. By the Help only of a Thread, in a Matter of ſo great Nicety, we may find out if a Comet have any ſenſible *Parallax*: For a Comet, juſt before it diſappears, goes ſo ſlowly, that it ſcarce ſeems to move; and it may be twice obſerved in this Manner;

ner: First, when it is very high above the *Horizon*, take any two *Stars* between which the Comet lies in a right Line parallel to the *Horizon*, which, by extending the Thread directly before the *Stars* may be easily tried; afterwards, when the Comet approaches near to the *Horizon*, by extending the Thread we must again try, if it still keeps in a right Line between the same two *fixed Stars*. Now if there be any sensible *Parallax* which depresses the Comet, it cannot be seen in the same Right Line as before; and therefore if it keeps the same Position as to those *Stars*, it is a convincing Argument, that the Comet has no sensible *Parallax*, and must therefore be at a prodigious Distance from us. We need not here fear any Error rising from Refraction, which always raises the *Stars*, and makes them appear more elevated above the *Horizon* than they are; for this Refraction equally affects both Comet and *Stars*; and therefore it will not change their Positions in respect of one another.

Lecture XVII.

Another Method for the same.

A Comet may likewise be observed when it is near the *Eastern* Part of the *Horizon*, and in a right Line with two *Stars* that are both in the same Circle; which is perpendicular to the *Horizon*: And afterwards, when the *Stars* rise higher, and are not in the same vertical Circle as before, if it appear still to be in the same Line with them, it can have no sensible *Parallax*; and therefore its Course must be very high in the Heavens. But if it should be found more depressed than to appear in the right Line that joins the *Stars*, then the Comet must needs have a *Parallax*. If, while these Observations are making, the Comet should have a proper Motion of its own, there must be made an Allowance for that Motion, according to the Time between the two Observations.

Comets are affected with the Parallax *of the annual Orb.*

As the want of a diurnal *Parallax* was an Argument for placing Comets above the *Moon*, so their being subject to the *Parallax* of the annual Orb is a convincing Proof of their descending into the planetary Regions; for Comets which have a Mo-

Lecture XVII.

tion forward, according to the Order of the Signs, near the Time of their disappearing, are all of them either slower than usual; or even Retrograde, if the *Earth* be between the *Sun* and them; or they are quicker than ordinary in their Motions, when the *Sun* is between the *Earth* and them: And they appear in *Conjunction* with the *Sun*, as the *Planets* are observed to do. On the contrary, those Comets which have their proper Motions Retrograde, or contrary to the Order of the Signs, are quicker than usual when they begin to withdraw themselves, and disappear when the *Earth* is between the *Sun* and them; or else they slacken their Pace, and seem to move more slowly, when the *Earth* is in the opposite Position. These Changes in their Motions arise from the Motion of the *Earth*, and its various Position, as in the *Planets*, who, according as the Motion of the *Earth* agrees with theirs, or is contrary to it, sometimes appear to go with a retrograde Motion; sometimes they go slower, and sometimes with a quicker Motion.

When a Comet is seen Retrograde.

If the *Earth* move the same Way as the Comet does, and hath an angular Motion round the *Sun* quicker than it, so that the right Lines which constantly join the *Earth* and Comet, all converge to Points beyond the Comet: This Comet seen from the *Earth*, upon the Account of his slower Motion, will appear Retrograde; but if the Motion of the *Earth* be less than that of the Comet, the Motion of the *Earth* takes off from the visible Motion of the Comet, and then the visible Motion of the Comet seems to be slower. But when the *Earth* and Comet have contrary Motions, the Comet's apparent Motion is thereby accelerated.

When the Motion of a Comet seems slower, when quicker.

We infer the same Thing from the Curvature of a Comet's Way; for they generally seem to move in great Circles almost as long as their Motion is swift. But at last, when that Part of their apparent Motion which arises from the *Parallax* of the annual Orb, bears a greater Proportion to their whole

whole apparent Motion, then they use to deviate from moving in a great Circle; and when the *Earth* moves one Way, they go the contrary: This Deflection or Deviation arises chiefly from the *Parallax* of the annual Orb, and exactly answers to the Quantity of the *Earth*'s Motion. And by Observation, it has been found in some Comets so great, as sufficiently to prove that they have descended far below *Jupiter:* And in their *Perigeons* and *Perihelions*, when they are nearest to us, they often come within the Orbit of *Mars*, and even the Orbits of the inferior *Planets*.

Lecture XVII.

WHEN the Comets recede from the *Earth*, and approach the *Sun*, their Lustre and Light is increased although their apparent Diameters be diminished upon Account of their further Distance from us.

The Figures of Comets.

THE Figures of Comets are observed to be very different; for some of them throw forth Beams like Hair every way round them, and these are called *Hairy Comets*. Others again have a long Beard, or rather a fiery Tail, opposite to the Region in which the *Sun* is seen; and they are called *Bearded*, or *Comets with Tails*. Their Magnitude has also been observed to be very different; many of them, without their Hair, appear no bigger than *Stars* of the first Magnitude. But some Authors have given us an Account of others, which were much greater; such was that which appeared in the Time of the Emperor *Nero*, which, as *Seneca* relates, was not inferior in Magnitude to the *Sun* itself. So the Comet, which in the Year 1652 *Hevelius* observed, did not seem to be less than the *Moon*, though it had not so bright a Splendor; for it had a pale and dim Light, and appeared with a dismal Aspect. Most Comets have a dense and dark Atmosphere surrounding their Bodies, which weakens and blunts the *Sun*'s Rays that fall upon it; but within it appears the Kernel or solid Body of the Comet, which, when the Clouds are dispersed, gives a splendid and brisk Light.

Their Magnitude.

Lecture XVII.

Comets have their apparent diurnal Motion from East *to* West.

COMETS, since they are at such a Distance from the *Earth*, like all other *Stars*, must have the apparent Motion round the *Earth* from *East* to *West*, which arises only from the Rotation of the *Earth* round its *Axis*. But besides this they have a real aud proper Motion of their own, by which they are continually shifting their Place in the Heavens, and have their proper Courses in the Celestal Regions. The Antients were not ignorant of such a Motion; for they never had reckoned them among the wandring *Stars*, unless they had known that, like the *Planets*, they had their peculiar Courses: *Seneca* acknowledged and observed that they had such a Motion, and said, that their Way was in a Right Line; or, as the *Astronomers* use to say, in a great Circle. For in the seventh Book of his natural Questions, Chap. 8. he says, "That the Course "of a Comet is easy and quiet, that it takes a determined Way: That Comets do not proceed in "a confused and tumultuous Manner, as some be- "lieve, nor are they driven by turbulent and un- "certain Causes." In his 29th Chap. he mentions two Comets, one of which, in the Space of six Months, passed thro' one half of the Heavens. Another, in the Time of the Emperor *Claudius*, was first observed towards she *North*, which by Degrees arose directly higher and higher, till it quite disappeared.

They have likewise a proper Motion of their own.

The Method of finding the Course of a Comet.

BY the Means of a Celestial Globe, in whose Sursace the *Stars* are rightly placed and painted, by a Mechanical Method, the Way of a Comet may be easily traced in the Heavens. Let there be every Day observed four *Stars* which are round the Comet, and let them be such as the Comet may be in the right Lines which join the two opposite *Stars*; which may easily be found out by the Means of a Thread placed before the Eye, and extended over-against the *Stars* and Comet: For Example, let the Comet's Place be A, between the four *Stars* B, C, D, E, so that the Line joining the *Stars* B and D, may pass thro' the Body of the Comet; and so likewise the Line

Plate XVI. Fig. 2.

Line passing through the *Stars* C and E. And therefore upon a Globe, in which are marked these four *Stars* in their proper Places, extend one Thread thro' the *Stars* B and D, and another thro' the *Stars* C, E; and the Intersection of the Threads will give you the Place of the Comet. If this be daily done, and the Place of the Comet be every Day taken, by this Means we shall manifestly find out the Course a Comet takes in the Heavens, which will be found to be a great Circle; for all the Points thus marked will be found to fall on the Periphery of a great Circle: And having any two Points of this Circle, we shall find its Inclination to the Eclriptick, and the Places of the *Nodes:* For it is only observing where a Thread stretched through the two Points cuts the Ecliptick.

Another Way of doing the same.

THERE is another Way of finding out the proper Course of a Comet, by observing every Day its Distance from two *fixed Stars*, whose Longitudes and Latitudes are known; from which Distances we can compute the Places of the Comet; and these Places being marked on the Surface of a Celestial Globe, will manifestly shew that the Course of a Comet is in a Portion of a great Circle, excepting that the Motion of the *Earth* will make it appear to deviate a little from it.

The Course of a Comet appears nearly to be in a great Circle.

The Motion of Comets is in Planes which pass thro' the Eye or the Sun.

HENCE it is manifest, that the Motion of a Comet is in a Plane, which passes through the Eye of the Spectator, or, more exactly, which passes through the *Sun*; for all visible Motion that is made in such a Plane, however it be inclined to the Ecliptick, will always appear to be in the Periphery of a great Circle. Moreover, the Motion of a Comet is regular and orderly; and tho' it is unequal, yet there is a certain exact Order observed in the very Inequality of Motion. The proper Motion of Comets is not the same in all; but each has its peculiar Course. Some go from the *West* to the *East*, others from *East* to *West*, contrary to the Order of the Signs, and their Direction is contrary to the Way the Planets take,

Their Courses various.

Lecture XVII.

who all move from *West* to *East:* All of them that are exactly observed, turn *Southwards* or *Northwards*, with different Inclinations to the Ecliptick; and they are not like the *Planets* to be comprehended within the *Zodiack*, but they quickly depart out of it, and with various Motions pass through all the Regions of the Heavens, some with a quicker, some a slower Motion. The greatest Velocity that any we have yet seen has had, was that which was observed by *Regiomontanus*; which Comet moved in one Day full forty Degrees. Some are swiftest in the Beginning of their Appearance, and slacken their Pace as they begin to vanish. Others again in the Beginning and End of their Appearance have a slow Motion; but in the middle Time they are carried with a greater Velocity.

Comets deviate from a Course in a great Circle.

It has been observed, that some Comets, for a few Days before they disappeared, did not keep their Course exactly in a great Circle, but did somewhat deviate from it; so that the Angle of the Comet's Orbit and the Ecliptick, was found to be different at last from what it was at first: But this Deflection was only apparent, and did not arise from the real Motion of the Comet, but from that of the *Earth*, as we shewed in the inferior and superior *Planets*; whose Distance and Inclination to the Ecliptick is various, according to the different Position of the *Earth*, whereas, if they were observed from the *Sun*, any one of them would always appear to move in the same great Circle.

The true Line that a Comet describes.

Although the Motion of a Comet appears to be in a great Circle, yet its true Way may be quite different from a Circle, and may be in very various and different Lines, as either a right Line, an Elliptick, Parabolick, or Hyperbolick Curve; or it may be any other Curve described in the same Plane: For all Motions in whatever Line the moving Body takes, when it lies in a Plane passing through the Eye, will always be observed to be performed in a great Circle. Many *Philosophers*, and not a few *Astronomers*, have maintained that the Comets

Motions

Motions are rectilinear; but that which answers best to their Appearance, is a Motion in a parabolick or elliptical Orbit. And if their Orbits be elliptical, they are extremely excentrick; so that their greater *Axis* bears a very considerable Proportion to their lesser; upon which Account they differ very much from the *Planets*, which, though they move in elliptical Orbits; yet they are so little excentrick, that they differ but a small Matter from Circles. Now the *Sun* resides in the common *Focus* of the Orbits of both Comets and *Planets*. And the Comets observe the same Law in their Circulations round the *Sun* as the *Planets* do; that is, they move at such a Rate in their Orbits, that the Line which joins the *Sun* and them, does always describe Areas or Spaces proportional to the Times; and therefore upon the same Account as the *Planets*, they likewise must have a Gravity or Propension towards the *Sun*.

Lecture XVII.

The Comets when they become visible. When invisible.

WHEN the Comets come to the inferior Parts of their Orbits, and descend towards the *Sun*, or are just ascending from him, then only they become visible; afterwards departing from the *Sun*, and ascending higher in their Orbits, they run out into far distant Regions, and withdraw themselves from our Sight; for upon the Account of their going further off the *Sun*, the Light they receive from him is thereby much weakened; and because likewise of their greater Distance from us, their apparent Diameters become constantly less, till at last they vanish into a Point, and become invisible. In their *Aphelions*, whither they run out into far distant Regions, because of the great Excentricity of their Orbits, they have a very slow Motion; but in their *Perihelions*, where they come near the *Sun*, they move with a quick Pace.

LET S be the *Sun*, A P D G the elliptick Orbit of a Comet; T C E the Orbit of the *Earth*. If we should suppose the *Semi-Axis* of the Comet's Orbit to be 100 Times greater than the *Semi-Axis* of the *Earth*'s Orbit, or which is the same, than its

Lecture XVII. Plate XVI. Fig. 3.

mean Diſtance from the *Sun*, that Comet would not complete its Revolution in leſs than 1000 Years; for the Squares of the periodical Times of the *Earth* and Comet, muſt be as the Cubes of their mean Diſtances from the *Sun*; and the Comet becomes viſible only for that Part of its Period, wherein it deſcends towards the *Sun*, and approaches near the *Earth*, as in F; and then, after it hath paſſed its *Perihelion* conſtantly riſing higher from the *Sun* about G, it will begin to vaniſh, and will not be viſible without a Teleſcope. If the *Aphelion* Diſtance be to the *Perihelion*, as 1000 is to 1 the Velocity of a Comet in the *Perihelion* will bear the ſame Proportion to the Velocity at the *Aphelion*. For the Area A S B muſt be but equal to the Area P S D, if the Arches A B and P D be deſcribed by the Comet in equal Times; and then the Arch P D muſt be greater than A B, in the ſame Proportion as A S is greater than P S. This is the Proportion of their abſolute Velocities. But their angular Velocities about the *Sun* are in a duplicate Proportion of theſe Diſtances, or as 1000000 to 1; ſo that while the Comet in its *Perihelion* deſcribes one Degree with its angular Motion, when it aſcends to its *Aphelion*, it will deſcribe in an equal Time but the $\frac{1}{1000000}$ of a Degree; ſo that there it may have ſo ſlow a Motion, that it will require ſeveral Years before it can compleat a Degree of angular Motion.

The ſmall Portion of the Ellipſe which a Comet deſcribes while it is ſeen by us, may be eſteemed as a Parabola.

SINCE the elliptick Orbits of Comets are all of them very Excentrick, thoſe Portions of them wherein they become viſible to us, may paſs for Parabolas. For if one of the *Focus's* of an Ellipſe recede infinitely from the other, this Ellipſe will thereby be changed into a Parabola; as when the two *Foci* come together and coincide, the Ellipſis is changed into a Circle. Now by conſidering that Portion of a Comet's Orbit which is near the *Perihelion*, as a Piece of a Parabola near its *Vertex*, the Calculation of their Motions becomes much eaſier; and upon that *Hypotheſis* our moſt ſkilful *Aſtronomer* and *Geometer*,

Dr.

Dr. *Halley*, has constructed and calculated a Table, by which the Motions of all Comets are easily computed, and the Calculations founded upon this *Hypothesis* do exactly agree with the Observations made on them. Dr. *Halley* himself, having computed the Motions of several Comets, and compared them with Observations made by others, has found there was so nice a Correspondence between them, that the Calculation scarce ever differed from the Observation above three Minutes. By which Examples it is abundantly manifest, that this Theory satisfies all the Appearances and Motions of Comets, with no less Exactness than the Motions of the *Planets* are accounted for and foretold from the Theories we have of them, whose computed Places do sometimes differ from Observations as much as in Comets. And altho' the Motions of Comets are much more unequal than those of the *Planets*, yet this Theory does wonderfully answer all their Appearances: And therefore since it is built upon the same Laws as the Theory of the *Planets*, and the Motions of one governed by the same Physical Causes as they of the other are; and since it accurately answers all Observations of *Astronomers*, it cannot but be the true Theory.

Many Comets move from East *to* West.

ALTHO' all the *Planets* have their proper Motions from *West* to *East*, yet many Comets have been observed to hold on in a contrary Course, and have been seen to go from *East* to *West*, with a very great Degree of Velocity. Such was the Course of the Comet which *Regiomontanus* observed in the Year 1472, that described 40 Degrees of a great Circle in one Day. Hence we can positively conclude, that there are no *Vortices*, or Whirlpools of fluid Matter in the Heavens, which, according to the Opinion of some *Philosophers*, carry the *Planets* round the *Sun*: For if there were any such Whirl-pools, when the Comets come down and enter within the Region of the *Planets*, they must be necessarily driven out of their Course by the rapid Motion

Therefore there can be no Vortices.

Lecture XVII. Motion of the Solar Vortex, as by a mighty Torrent, which near the *Earth* is of ſuch Force, that it carries it above 20000 Miles in an Hour: And who can think, that ſo rapid a Stream would not affect the Comets, and when they have a Motion contrary to its Motion, ſoon deſtroy it? For what can reſiſt ſo violent a Torrent of fluid Matter? Now many Comets have been obſerved to take a Courſe directly contrary to this Stream, and which perform their Motions with the greateſt Freedom and without the leaſt Reſiſtance, juſt after the ſame Manner as they would do in a void Space, where there is nothing to withſtand them. But this is plainly repugnant to the Nature of a *Vortex*; for that *Medium* which can put the *Planets* in Motion, would without all Queſtion, ſet all other Bodies which ſwim in it a going the ſame Way. But ſince there is nothing like this obſerved in Comets, we muſt acknowledge that in the Heavens there is no Reſiſtance, and therefore no *Medium* or Fluid, which, compared with our Air, hath any ſenſible Denſity: For our Air gives a very conſiderable Reſiſtance to all Bodies that move in it.

There is no Fluid in the Heavens which has a ſenſible Denſity.

LET not therefore the *Carteſians* and *Leibnitians* talk to us any more about their *Vortices*; for the Appearances of the Celeſtial Bodies are ſuch as that we can by no means admit of them; ſo that they who labour to explain the Motions of the Heavens by them, do only amuſe us with Trifles and Impoſſibilities; and it is to no purpoſe to trouble ourſelves any longer with their Fancies, ſince there is Demonſtration againſt them.

SINCE the Reſiſtance of a *fluid Medium* ariſes chiefly from its Denſity, it from thence neceſſarily follows, that where there is no ſenſible Reſiſtance of the *Medium*, there the *Medium* muſt have no ſenſible Denſity; and therefore ſince, in the Heavens the Comets ſuffer no ſenſible Reſiſtance, but exert their Motions with the greateſt Freedom, as if they were in a perfect Void or *Vacuum*, there likewiſe the Denſity of the *Medium* muſt be the leaſt that can be, or next

next to Nothing. And who knows but the *Medium* in the Heavens may be so rare and fine, that if you except the *Planets*, and their *Atmospheres*, the Matter which is diffused thro' all the rest of the Planetary Region, or our solar System, may not be so much as that which is contained in an Inch of our common Air. For this we have demonstrated to be possible in our Physical Lectures.

Lecture XVII.

THE *Philosophers* after this need trouble us no longer with their Metaphysical Quirks against a *Vacuum*; for they seem to be very like the Quibbles of the antient *Sophists* against the Possibility of Motion: And as *Diogenes* confuted the *Sophists* by rising and walking, so we may answer the *Cartesians* by bidding them look up into the Heavens; and there, notwithstanding their nice and subtle Arguments, they will find, from the Appearances and Motions there observed, a manifest Demonstration for the Necessity of a *Vacuum*.

A Vacuum or Void is proved.

FEW Comets have been observed before their Descent to the *Sun*, and their Return from their *Perihelion*. For before they have been considerably heated in the Neighbourhood of the *Sun*, they scarcely project a Tail to make them remarkable: But after they have been well heated in their *Perihelion*, then they generally send forth a large shining and fiery tail, which seems to consist of a very fine, rare and luminous Matter, which is attenuated by the great Heat of the *Sun*, and projected with an immense Force from the Body of the Comet. The Cause of this Projection perhaps may be very like that whereby a great Quantity of fine lucid Vapour was lately thrown out from the *Earth*, to an immense Height above the Air, so that it was visible through the greatest Part of *Europe*; and in Figure and Lustre looked very like the Tails of Comets, but the Matter being spent, it soon vanished.

The Tails of Comets.

IT is very remarkable, that all Comets have their Tails in Opposition to the *Sun*; that is, if the *Sun* be in the *West*, the Tail is projected *Eastward*, or, if

The Tails of Comets always opposite to the Sun.

Lecture XVII.

if the *Sun* be in the *East*, the Tail looks *Westward*; at Midnight the Direction of the Tail is to the *North*. These Tails grow bigger as they descend to the *Sun*, and at the *Perihelions* they are biggest; and as they go further off from the *Sun*, and cool by Degrees, the Tail lessens, till at last it is contracted within the Comet's Atmosphere.

COMETS which have short Tails, do not throw forth the Matter of them with a very quick Motion, in a continual Stream from their Bodies; for then they would soon be dissipated and vanish: But these Tails seem to be rather permanent and fixed Columns of Vapour and Exhalations, which being propagated from the Body with a slow Motion upwards and retaining still the Motion which they had impressed in them to go along with the Comet, they still continue to move on with the Body through the Celestial Regions. From hence we may likewise conclude, that in the Heavens there is no Resistance; for in them not only the solid Bodies of *Planets* move, but also the thin and fine Vapours which arise from Comets feel no Resistance; but move with the greatest Freedom, and for a long Time preserve their Motions.

THE great Comet which appeared in the Year 1680, after its Departure from the *Perihelion*, projected such a Tail as extended itself more than forty Degrees in the Heavens; nor can this be a Wonder, for it was so near the *Sun*, that its Distance from his Surface at the *Perihelion* was but a sixth Part of the Diameter of the *Sun*'s Body; and therefore the *Sun* seen from the Body of the Comet, would appear to fill the greatest Part of the Heaven, and its apparent Diameter would not be less than 120 Degrees; and therefore the Heat it received from thence must be prodigiously intense beyond Imagination; for it exceeded above 3000 Times the Heat of red hot Iron. And therefore we must allow, that the Bodies of Comets, which can bear so great a Heat, must be very dense, hard and durable Bodies: For if they were nothing

nothing but Vapours and Exhalations raised from the *Earth* and *Planets*, as some have dreamt, this Comet, at so near an Approach to the *Sun*, must have been quite destroyed and dissipated.

Lecture XVIII.

LECTURE XVIII.

The Spherical Doctrine, or, Of the Circles of the Sphere.

The Eye of a Spectator is always in the Center of his own View.

SINCE every Spectator, in whatever Place of the vast Expansion of the Universe he resides, is always in the Center of his own View, when he looks up at the Heavens, he will see it as a Concave Spherical Surface, whose Center is the Eye, which Surface is every way bespangled with an innumerable Multitude of shining *Stars:* And the Spectator will likewise observe, that all the Heavenly Bodies perform their Motions, whether real or apparent, in this Surface. Now since the Distance of the *Earth* from the *Sun* is but a Point, as it were, in Comparison of the immense Distance, of the *Starry* Firmament, in whatever Point of it's Orbit the *Earth* is placed, there will be the same Prospect of the Heavens, the same Position and Magnitude of the *Stars* and Figures of the Constellations, as a Spectator would observe, did he reside in the *Sun*; and therefore it is the same Thing as to these Appearances, whether the Center of the Universe or Heavens, be placed in the *Sun* or *Earth*: And if we imagine several Circles to pass through the *Earth*, and to have its Center for theirs, and others parallel to them to pass through the *Sun* these Circles in the Heavens will seem to coincide, because their Distance will vanish in respect of the immense

It is no Matter whether the Center of the Heavens be considered to be in the Sun *or* Earth.

Distance

Lecture XVIII. Distance of the *fixed Stars*; and those Circles which are drawn through the *Sun* and *Earth* on parallel Planes, will appear to pass thro' the same *Stars* in the Heavens.

A great Circle. FOR the better determining the Places of all sorts of *Stars*, and observing of their Motions, it is requisite to imagine several Circles described in the Heavens, some of which are great Circles, others of a less Size. A great Circle is the greatest that can be described on the Surface of the Sphere, and divides it into two equal Portions, and has likewise the same Center that the Sphere has; and therefore all great Circles, having the same Center, must cut each other into equal Portions or Semicircles.

The lesser Circles. THE lesser Circles divide the Spheres into unequal Portions, and have not the same Center that the Sphere has; and they take their Denomination from some great Circle to which they are parallel, as the Æquator, Horizon, or Ecliptick.

The Poles *of Circles.* EVERY Circle of the Sphere hath two *Poles*, which are Points on the Surface of the Sphere, which are at equal Distances from all the Points of the Circle; and they are placed in the Surface where a Line from the Center, perpendicular to the Plane of the Circle, meets with the Surface of the Sphere, when the Line is produced both ways.

Circles moveable and immoveable. SOME Circles of the Sphere depend only upon the Place of the Spectator, and have a Regard to his Position; others again are produced by Motion: The first are called moveable Circles, because, as the Place of the Spectator is changed, so are they, and move along with him. The second are called immoveable, and are supposed to be fixed to the same Points of the Heavens.

THE Circles which owe their Origin to Motion, are chiefly the Ecliptick and Æquinoctial, and their Parallels. For because the *Earth* is carried round the *Sun* in a Year, the Spectator in the *Sun* will see the *Earth* describe a great Circle in the Heavens or *Starry* Firmament, which we call the Ecliptick; and it is the very same Circle which we in the *Earth* observe

observe the *Sun* to move in by an apparent Motion, likewise in the Space of a Year, as we shewed before. The Ecliptick is divided into twelve equal Parts, which are called the twelve Houses or Signs, and they have their Names from the neighbouring Constellations: They begin at the Vernal Intersection of the Æquator and Ecliptic, and are reckoned from the *West Eastwards*, as the *Sun* seems to move. The first three Signs are ♈ ♉ ♊, which arise from the Æquinoctial, and ascend *Northwards* to the Point of the Summer Solstice. The next three are ♋ ♌ ♍, which begin from *Cancer*, aud descend again towards the Æquinoctial, till they come to the Autumnal Intersection. The third Ternary of Signs consists of ♎ ♏ ♐, which begin at *Libra*, and departing from the Æquinoctial *Southward*, reach the Winter Solstice. ♑ ♒ ♓ make the fourth, which begin at *Capricorn*, and end in the Vernal Æquinox, Each Sign is divided into 30 Degrees, and consequently the whole Ecliptick into 360. The *Sun* is always observed in this Circle, and never deviates in the least from it, as the *Planets* do, which go sometimes on one Side of it, sometimes on the other, through a Space of about eight Degrees; and therefore, if we imagine a broad Circle or Zone of about sixteen Degrees in Breadth, which the Ecliptick cuts in the Middle, this will be the Space wherein the *Planets* perform all their Motions, and by the *Greeks* it is called the *Zodiack*, by the *Latins Signifer*, or the Sign-bearer, because of the Signs placed within it.

Lecture XVIII.

The Ecliptick and its Parts.

The Zodiack.

Secondaries of the Ecliptick.

If we imagine an indefinite Number of great Circles to be drawn thro' the Poles of the Ecliptick, and intersecting of it, these Circles are called Secondaries of the Ecliptick, for by them every *Star* and Point of the Heavens are reduced to the Ecliptick, and have their Places in regard to it determined in the Heavens. For the Place of any *Star* reduced to the Ecliptick, is that Point where the Secondary passing thro' the *Star* intersects the Ecliptick. The Arch between this Point and the Beginning of ♈, or the Vernal Intersection, and counted *Eastward*,

Lecture XVIII.

The Longitude and Latitude of a Star.

is called the Longitude of that *Star*, as the Arch of the Secondary between the *Star* and the Ecliptick is called the Latitude of that *Star*, and is either *North* or *South*; for the Ecliptick divides the *Starry* Firmament into *Hemispheres*, *North* and *South*.

SINCE the *Earth* turns round its own *Axis*, from thence it comes that the Inhabitants thereof see the Heavens and all the *Stars* revolve round the *Earth*, in the Space of twenty-four Hours, from *East* to *West*; which apparent Motion is called the diurnal or daily Revolution of the Heavens, and was conceived to be by the Force of a moving Sphere called the *Primum Mobile*, or First Mover, which carried the whole Heavens round with it about an *Axis*, which coincides with the *Axis* of the *Earth* produced, as if the *Earth* itself had no Motion, but the Heavens were volvible. The great Circle which is exactly between the two *Poles* of the *Earth*, or at equal Distance from both, is called the *Earth*'s Æquator; and if we imagine the Plane of this Circle extended or produced to the Heavens, it will there make the Celestial Æquinoctial Circle, or the Æquator in the Heavens; and all the *Stars*, and every Point of the Heavens except the two *Poles*, will seem to describe by their apparent Revolution either this Circle, or a lesser parallel to it; which Circles are either bigger or lesser, according as the *Stars* which seem to describe them, are more removed or nearer to the *Poles*.

The Primum Mobile.

The Æquinoctial.

THE Æquinoctial and Ecliptick being both great Circles, will cut each other into Semi-circles, and their common Intersection will always keep parallel to itself, and will constantly be directed to the same Point of the Heavens; for we here abstract and consider as nothing that very small Motion, whereby the *Axis* of the *Earth* falls backward, and this Intersection equally with it. And therefore, whenever the *Sun* is observed in the Point of the Ecliptick, where this Intersection is, that is, when the *Earth* is really in the opposite, the *Sun* then by this apparent diurnal Motion will de-

scribe

ſcribe the Equinoctial Circle in the Heavens; and there being two Points of Interſection, the *Sun* will be obſerved to revolve in the Equinoctial, twice every Year; that is, when he is in the two Interſections, Vernal and Autumnal, at which Times all the Inhabitants of the *Earth* will have their Days and Nights equal; upon which Account this Circle has got the Name of the Æquinoctial. The Angle which the Ecliptick and the Æquator make at the Points of Interſection, is about 23½ Degrees. The *Sun* leaving theſe Interſections by an apparent Motion, declines every Day from the Equinoctial Circle more and more towards the *North* or *South*, till he comes to the ninetieth Degree from the Interſections, where he appears to be 23½ Degrees diſtant from the Equinoctial, which is his greateſt Declination: For from thence he begins to return again towards the Equinoctial; and therefore the two leſſer Circles, which the *Sun*, at his greateſt Declination, ſeems by his diurnal Motion to deſcribe, are called the *Tropicks*, from a *Greek* Word which ſignifies to return: This upon the *North* Side of the Equinoctial, is called the *Tropick* of *Cancer*; the other, on the *South* Side, the *Tropick* of *Capricorn*. How this apparent Motion of the *Sun*, and conſtant Change of his Declination, ariſe from the real Motion of the *Earth*, the *Sun* himſelf being all the while at Reſt, we have already explained in our ſeventh *Lecture*.

Lecture XVIII.

The two Tropicks.

The two polar Circles.

There are two remarkable leſſer Circles of the Sphere, which are parallel to the Equinoctial; and theſe are deſcribed by the apparent diurnal Motions of the two *Poles* of the Ecliptick round the *Poles* of the Equinoctial, from which they are diſtant 23½ Degrees. They are called the two polar Circles; this in the *Northern* Sphere is named the *Arctick* Circle, from the two *Bears* that lie near it; the other Circle on the *South* is called the *Antarctick*, or Circle oppoſite to the *Arctick*.

Secondaries of the Equinoctial.

If thro' the *Poles* of the World, or of the Equinoctial, there be conceived innumerable great Circles

P to

Lecture XVIII.

to be drawn, they are called Secondaries of the Equinoctial, by the Help of which the Position of every Point of the Heavens, in regard to the Equinoctial, is determined, as before they were determined by the Secondaries of the Ecliptick, in regard to the Ecliptick: And the right Ascension of a *Star*, or Point in the Heavens, is an Arch of the Equinoctial, between the Beginning of *Aries* and the Point where the Secondary, passing thro' the *Star*, cuts the Equinoctial. The Declination of a *Star* or Point is the Arch of the Secondary intercepted between the *Star* and the Equinoctial, which is likewise, as the Latitude, either *North* or *South*, as the *Star* declines towards the *North* or *South Pole:* From hence these Secondaries are called Circles of Declination. And the two chief of them are the two Colures, one of which, passing thro' the two Equinoctial Intersections, is called the Equinoctial Colure; and the other, which cuts the former at right Angles, and passes thro' the *Poles* of the Ecliptick, is called the Solstitial Colure, because it intersects the Ecliptick in the Points which are at the greatest Distance from the Æquator; to which when the *Sun* comes, he does not sensibly, for some Days, change his Declination, but seems to stand without approaching to, or receding from the Equinoctial; and therefore these Points are called Solstices.

The right Ascension and Declination of a Star.

The two Colures.

THAT Circle which is on the Surface of the *Earth*, exactly in the Middle between the two *Poles*, is the *Earth*'s Æquator; and by the Production of it we shewed, that the Celestial Equinoctial was formed. And as the Places of all the *Stars* in the Heavens are determined by their Longitude and Latitude, in regard to the Ecliptick and its Secondaries; so by the terrestrial Æquator and its Secondaries, drawn thro' the *Poles* of the *Earth*, the Positions of Cities and Places upon the Surface of the *Earth* are determined according to Longitude and Latitude. A Secondary of the Æquator, passing thro' any Place on the *Earth*'s Surface, is called the Meridian of that Place; because that when, by the Rotation of the *Earth* round

The Meridian of a Place.

its *Axis*, the Plane of that Meridian comes to paſs thro' the *Sun*, then the Inhabitants under this Meridian have Mid-day. The Longitude of any City or Place is an Arch of the Æquator intercepted between a certain Point where ſome fix'd Meridian paſſes, which is called the firſt Meridian, and the Meridian of the Place. The ancient *Geographers* made the firſt Meridian paſs thro' ſome known Place, which was as far *Weſtward* as they knew, and from thence they reckoned the Longitude of all Places conſtantly *Eaſtward:* But ſince by Navigation it has been diſcovered that there is no Place in the *Earth* that can be eſteemed the moſt *Weſterly*, ſo that there is not another more *Weſtward* beyond it, this way of computing the Longitude from a firſt Meridian hath been generally laid aſide; and now each *Geographer* determines the Longitude of Places in regard to the Longitude of the chief City of the Country where he dwells. The Latitude of a Place is an Arch of the Meridian of that Place intercepted between the Place and the Æquator, and is either *North* or *South*, as the Place lies on the *North* or *South* Side of the Æquator.

Lecture XVIII.

The Longitudes and Latitudes of Places.

Of the Inhabitants of the *Earth* compared with one another, in regard to their Meridians and Parallels, ſome are called *Periœci*, that live under the ſame Parallel, but in the oppoſite Semicircles of the ſame Meridian; both of them have the Seaſons of the Year the ſame, the *Sun* by its annual apparent Motion coming to or receding from the Vertex of both Places at the ſame Time of the Year; but they change their Turns of Night and Day, ſo that when it is Mid-day to the one, it is Mid-night to the other. Others again are called *Antœci*, whoſe Habitations lie in the ſame Semicircle of the Meridian, but in oppoſite Parallels, and both of them have Mid-day and Mid-night at the ſame Inſtant of Time; but the Seaſons of the Year are different, it being Summer to one, when it is Winter to the other. Laſtly, there are the *Antipodes*, whoſe Habitations being ſituated in both oppoſite Parallels and oppoſite Meridians,

The Periœci.

The Antœci.

The Antipodes.

Lecture XVIII. have their Feet directly opposite to one another in a Line passing thro' the Center of the *Earth*; and they have not only their Days and Nights directly contrary but also the Seasons of the Year: When it is Summer in the one, it is Winter in the other Place; and when Mid-day in the first, the second reckons Midnight.

The five Zones.

THE four lesser Circles in the Surface of the *Earth*'s Globe, which lie directly under the Circles of the same Name in the Heavens, *viz.* the two *Tropicks* and *Polars*, divide the *Earth* into five Portions, which are called *Zones*; of which the Torrid is that which is bounded on each Side by a *Tropick*, and was believed by the Ancients to be not habitable, by reason of the violent Heats: But our modern Travellers and Voyagers have discovered this Tract of the *Earth*'s Surface to be the most fruitful and delightful of all, abounding not only with the Things that are necessary, but such likewise as conduce to the Satisfaction and Pleasure of Life; upon which Account it is very well stored with Inhabitants. There are two cold *Zones* which the polar Circles comprehend; in the Middle of the one lies the *Arctick Pole*, of the other the *Antarctick*, and in them the Cold is so excessive, that they are scarcely habitable. Besides these, between the Frigid and the *Torrid* Zone, on each Hand lie the two temperate ones; that on the *North* is possessed by us, the other our *Antipodes* keep. *Virgil* elegantly describes these five *Zones*.

The Amphiscii.

Quinque tenent Cœlum Zonæ, *quarum una corusco*
Semper Sole rubens, & torrida semper ab igne;
Quam circum extremæ dextra lævaq; trahuntur,
Cœrulea glacie concretæ atq; imbribus atris.
Has inter mediamq; duæ mortalibus ægris
Munere concessæ Divum.

The Amphiscii.

THE Inhabitants of the torrid *Zone* are called *Amphiscii*, having their Meridian Shadows at different Times

Times of the Year projected towards both *Poles*; but when the *Sun* comes to be Vertical to them, then they have no Shadow, and are *Ascii* or Shadowless, nothing that stands perpendicularly upwards having a Shadow at Noon. We who live in the temperate *Zones* may be named *Heteroscii*, having our Meridian Shadow projected only towards one *Pole* throughout the whole Year. But the miserable People of the two Frigid *Zones*, are called *Periscii*, because the *Sun* not setting upon them, their Shadow turns quite round in the Space of twenty-four Hours.

Lecture XVIII.

The Ascii.

The Heteroscii.

The Periscii.

The two Circles which we conceive to be immoveable, and determined by their respect to the Spectator, are the *Horizon* and Meridian. The *Horizon* is that great Circle which any one, when he is placed in a large extended Plane, or in the open Sea, observes, terminating or bounding his Sight every where round him, by which the visible Heaven is distinguished and separated from the invisible. This *Horizon*, being discovered by our Senses, is called the sensible *Horizon*, from which the rational *Horizon*, parallel to it, is distant by the Semidiameter of the *Earth*, thro' whose Center it passes: For the *Astronomers* reduce the Appearances of the Heavens to a spherical Surface, which is not concentrical to the Eye, but to the *Earth*.

The Horizon *sensible and rational.*

'Tis true, these two *Horizons*, produced to the *fixed Stars*, will appear to coincide into one, since the *Earth*, compared to the Sphere in which the *fixed Stars* appear, is but a Point; and therefore the two Circles which are but a Point distant from each other, may be well considered as coinciding into one. There are two *Poles* of the *Horizon*; the one is the Point of the Heavens which is directly over the Head of the Spectator, and is called the *Zenith*; the other directly opposite, under his Feet, is named the *Nadir*; and innumerable Circles, drawn thro' these *Poles* to the *Horizon*, are styled *Vertical Circles* or *Azimuths*. Among them there are two particularly remarkable, one of which is the Meridian, and the other is the

Zenith *and* Nadir, *the* Poles *of the* Horizon.

Lecture XVIII. *Prime Vertical*; the first passes thro' the *Poles* of the Equator and the *Zenith*, and cuts the *Horizon* in the Points of *North* and *South*; the other passing thro' the *Zenith* cuts the former at right Angles, and marks upon the *Horizon* the Points of *East* and *West*. These Circles divide the *Horizon*, at their Intersections with it, into four Quarters, each of which is again subdivided into eight Parts; and consequently the whole *Horizon* is divided into thirty-two Parts, which are called the *Rhumbs* or Points of the Compass.

The Vertical *or* Azimuth *Circles.*

The Altitude or Depression of a Star.

THE Altitude or Depression of any *Star* is an Arch of the Vertical intercepted between the *Horizon* and the *Star*.

The Azimuth of a Star.

AGAIN the *Azimuth* of a *Star* is an Arch of the *Horizon*, intercepted between the Points of *North* or *South*, and the Point where the Vertical passing thro' the *Star* cuts the *Horizon*, and is either *Easterly* or *Westerly*. The rising or setting Amplitude of a *Star* is an Arch of the *Horizon* intercepted between the Points where the *Star* riseth or setteth, and the Points of *East* or *West*; and this Amplitude is either *North* or *South*, according as the *Star*, at rising or setting, is to the *North* or *South* of those Points.

The Amplitude.

AS in the *Horizon* all the *Stars* first appear and disappear, so in the Meridian Circle they all arise to their greatest Height or Altitude, where they are said to *Culminate*; so likewise they are at their greatest Depression below the *Horizon*, when they arrive at the same Meridian. Now since the Meridian makes Right Angles both with the Æquator, and *Horizon*, it will divide the Segments of the Æquator and all Parallels, as well those that lie above the *Horizon* as those which are below it, into equal Portions; and therefore the Time between the rising of a *Star*, and its Culmination or Arrival at the Meridian, will be equal to the Time between this Culmination and its Setting; and because the *Sun* every Day describes some Parallel by its apparent diurnal Motion, when the *Sun* comes to the Meridian at any Time, it will be then Mid-day; and Midnight, when he arrives

The Culmination.

at

at the ſame Meridian below the *Horizon*; and from thence this Circle has its Name. The *Ninetieth* or *Nonageſimal Degree* is that Point of the Ecliptick which is 90 Degrees diſtant from the Interſections of the *Horizon* and Ecliptick, and the Altitude of this Point is the Meaſure of the Angle of the *Horizon* and Ecliptick. The *Medium Cœli* or Mid-Heaven, is that Point of the Ecliptick which culminates, or is in the Meridian. In the aſcending Signs from ♑ to ♋, the *Nonageſimal* Degree is to the *Eaſt* of the Meridian, but in the deſcending Signs, from ♋ to ♑, the *Nonageſimal* lies *Weſtward* of the Meridian. Lecture XVIII.

ALTHO' we have here conſidered the *Horizon* and Meridian as immoveable Circles taking the apparent Motion of the Heavens as real; yet if we ſpeak according to Truth, and the Nature of Things, the *Horizon* and Meridian are the only moveable Circles, and the *Sun* or a *Star* riſes not when it aſcends, but when the Plane of the *Horizon* deſcends below it, ſo that the *Star* or *Sun* becomes viſible. So likewiſe the *Stars* ſet not by their own Motion, or going down under the *Horizon*, but by the *Horizon*'s aſcending and getting above them; which Motion of the *Horizon* ariſeth from the Rotation of the *Earth*. So alſo the *Sun* or *Stars* arrive at the Meridian, when the Plane of the Meridian of any Place, having an angular Motion round the *Axis* of the *Earth*, comes to paſs thro' the Bodies of the *Sun* or *Stars*. *The* Horizon *and Meridian truly moveable Circles.*

SINCE every Meridian finiſhes its Circulation round the *Axis*, or 360 Degrees in twenty-four Hours, it muſt each Hour have an angular Motion of 15 Degrees, which is the twenty-fourth Part of 360; and therefore, if we conceive a Circle paſſing thro' the *Poles*, which makes an Angle of 15 Degrees with the univerſal Meridian paſſing thro' the *Sun*, when by the *Earth*'s Rotation the Plane of the Meridian of any Place comes to the Plane of this Circle after it has paſſed the univerſal Meridian; then the Inhabitants in that Meridian will reckon one Hour after Mid-day, and therefore that Circle is called the *The univerſal Meridian.*

Lecture XVIII.

Circle of the first Hour. In like Manner, if another Circle be conceived cutting the Equinoctial in the thirtieth Degree from the universal Meridian, this will be the Circle of the second Hour; and when the Plane of the Meridian of the Place comes to coincide with it, it will be in that Place Two of the Clock in the Afternoon. After the same Manner, if thro' the *Poles*, and every fifteen Degrees of the Æquator, we conceive Circles to pass, they are called *Horary* Circles; and they will divide the Equinoctial into twenty-four equal Parts, and each of them in its Order will determine the Hour that is to be reckon'd in a Place, when the Plane of a Meridian of that Place comes to coincide with that Circle. So, for example, when the Meridian of the Place comes to the Circle which makes seventy-five Degrees with the universal Meridian, the Hour then counted in that Place will be the fifth from Noon. But when the Meridian of the Place makes an Angle of ninety Degrees with the Meridian passing thro' the *Sun*, then it will be the sixth hour. But, on the other Hand, if we should imagine the Meridian of the Place to be immoveable, and that a Circle passing thro' the *Poles* and the *Sun* should, together with the *Sun*, turn round the *Axis*, as it appears to do; when the Plane of this Circle, in which the *Sun* resides, coincides with a Circle that makes an Angle of fifteen Degrees with the Meridian of the Place *Westward*, it is then the first Hour after Midday; and the Circle with which the Circle passing thro' the *Sun* and *Poles* does then coincide, is called the Circle of the first Hour. The next Circle to this, which makes an Angle of thirty Degrees with the Meridian of the Place, is the *Horary* Circle for Two of the Clock; and that which makes an Angle of forty-five Degrees with the Meridian of the Place, is the *Horary* Circle for Three of the Clock, &c.

The Horary Circles.

Plate XVI. Fig. 4.

In any Place of the terraqueous Globe, the Height of the *Pole* above the *Horizon* is equal to the Latitude

Latitude of the Place. Let the Circle HZQ be the Meridian, HO the *Horizon*, ÆCQ the Æquator, Z the *Zenith*, and P the *Pole*: The Elevation of the *Pole*, or its Distance from the *Horizon*, is the Arch PO, the Latitude of the Place or its Distance from the Æquator is ZÆ. And because the Arch PÆ between the Pole and the Æquator is a Quadrant, or fourth Part of a Circle, and the Arch ZO, from the *Zenith* to the *Horizon* is likewise a Quadrant, these two Arches ZÆ and PO must be equal. Take away the Arch ZP, which is common to both, and there will remain the Arch ZÆ equal to the Arch PO; that is, the Latitude of the Place, is equal to the Altitude or Height of the *Pole* above the *Horizon*.

Lecture XVIII.

The Height of the Pole *above the* Horizon *is always equal to the Latitude of the Place.*

HENCE we have a Method of measuring the Circumference of the whole *Earth*, and of knowing how many Miles it is round the *Earth*: For if we go directly *Northward* till the *Pole* be elevated one Degree higher, and then if we measure the Length of the Way we have gone *Northward*, and have the Number of Miles it contains, we shall have the Number of Miles in a Degree of a great Circle of the *Earth*'s Globe; and this Number multiplied by 360, the Degrees in the whole Periphery, it will give the Length of the Circumference of the *Earth* in Miles. By the most accurate Observations, the Length of a Degree is found to be 69 *English* Miles, which was commonly reputed to be only 60 Miles.

LECTURE

LECTURE XIX.

Of the Doctrine of the Sphere.

The Right Position *of the* Sphere.

THE Angle contained between the Æquator and the Horizon, is measured by the Arch of the Meridian Æ H, which is the Complement of the Latitude to a Quadrant. And therefore if that Angle be right, the Latitude of the Place will be nothing, or the Place will be in the Æquator; or the Equinoctial Circle will pass thro' the Vertex of the Place, and all the Parallels of the Æquator will be perpendicular to the Horizon; and therefore this Position of the *Sphere* is called a *Right Position*, in which all these Parallels are cut by the *Horizon* into equal Portions; whence the *Stars* are as long above the *Horizon*, as they lie hid under it: Here likewise the Poles lie in the *Horizon* without any Elevation, as is manifest by the Figure, where the Point of the Æquinoctial Æ is in the Vertex, and the Poles P *p*, in the *Horizon*. If we go from the Æquator towards either of the Poles, the Æquator will then appear to depart from the Vertex or *Zenith*, and to come nearer to the *Horizon*, making with it an oblique Angle; whence such a Situation is called an *oblique Position of the Sphere*, and the Pole towards which we move, doth rise more and more above the *Horizon*, the nearer we approach it; its Elevation being alway equal to the Latitude of the Place, while the other continues as much depressed below it. The Figure does clearly shew this sort of Position, which we and all that live in the temperate Zones obtain, where the Æquator Æ Q is bisected by the *Horizon*, as it is in a right *Sphere*. Where-

Plate XVI Fig. 5.

An oblique Sphere; *or an* oblique Position of the Sphere.

Fig. 6.

Wherefore when the *Sun*, by his apparent diurnal Motion, describes this Circle, it makes the Day equal to the Night; but the Parallels of the Æquator, in this *Oblique Position*, are not cut into two equal Parts by the *Horizon*; but those which are towards the elevated Pole, each of them have a greater Portion above than under the *Horizon:* And as each Parallel is nearer the Pole, so much the larger Portion of it stands above the *Horizon*. But when the Distance of the Parallel from the Pole becomes less than the Elevation of the Pole, or the Latitude of the Place, then that Parallel, and all those included within it, are wholly above the *Horizon*, no Part of them ever setting under it. The contrary happens in the Parallels which lie towards the depressed Pole, a smaller Portion of them being above the *Horizon*, and the greater Part lying under it. And those Parallels, which are nearer to the depressed Pole than the Latitude of the Place, remain perpetually, together with the *Stars* included within them, under the *Horizon*, and are never visible to us. Hence it is necessary, since the *Sun* each Day describes, by his apparent diurnal Motion, some Parallels, that from the Vernal Æquinox to the Summer Solstice, the Days growing longer and longer, will be continually longer than the Night; after the Solstice, tho' the Days continue till the Autumnal Æquinox to be longer than the Nights, yet they become shorter and shorter, and at the Æquinox, they but just equal the Nights: From thence to the Winter Solstice, the Days continually become shorter than the Nights, and are the shortest when the *Sun* is in that Solstice; but as the *Sun* leaves it, they increase again, and in the Vernal Æquinox the Day is as long as the Night.

Lecture XIX.

In an *Oblique Sphere* the *Stars* all obliquely rise and set. And as the *Right Ascension* of a *Star* is the Arch of the Æquator contained between the first of *Aries*, and that Point which comes to the *Meridian* with the *Star*, or that Point which in a *Right Sphere* rises with the *Star:* So the Oblique Ascension is the Arch

Oblique Ascension.

Lecture XIX. Arch of the Æquator between the first of *Aries*, and that Point of the Æquator which rises together with the *Star* in an *Oblique Sphere*, and numbered from *West* to *East*, which, according to the Obliquity of the *Sphere*, is various. The Difference between the Right and the Oblique Ascension is called the *Ascensional Difference*.

The Ascensional Difference.

In an *Oblique Sphere* there is one Parallel as much distant from the elevated Pole, as the Place is from the Æquator, which is called the Circle of *Perpetual Apparition*, or the biggest of all those which constantly appear; which is such, that all *Stars* inclosed within it never either rise or set; though they sometimes rise higher, sometimes descend lower towards the *Horizon*. Towards the other Pole, there is another Circle opposite to this, which is the Circle of *Perpetual Occultation*, within which all the *Stars* that are contained never rise, but constantly lie hid under the *Horizon*, and so are not to be seen.

The Circle of Perpetual Apparition.

If the Æquator makes no Angle with the *Horizon*, but those two Circles coincide; in such a Situation the Pole and Vertex coincide, and all the Parallels of the Æquator become Parallels to the *Horizon*; and such a Situation is called a *Parallel Sphere*, in which no *fixed Stars* do ever rise or set, but turn round in Circles parallel to the *Horizon*. And when the *Sun* enters the Æquinoctial, it then glides the whole Day along the *Horizon*. When he rises towards the elevated Pole, he never sets, but makes a very long Day of six Months: But when he goes from the Æquinoctial towards the depressed Pole, he never rises, and then there is a constant Night of six Months Length. This Position of the *Sphere* belongs only to them who live at the Pole, if any are so miserable as to have such a Place for their Habitation.

A Parallel Sphere.

Plate XVI Fig. 7.

The ancient Geographers divided the *Earth* by Climates and Parallels; for they who live under the Æquinoctial, being in a Right *Sphere*, have their Days and Nights equal: If we remove from thence towards

Climates and Parallels.

towards either Pole, the Days in Summer become longer than the Nights, and the nearer we approach the Pole, the greater is the Difference between Day and Night, when the Day is at the longest; till we come under the Polar Circles, where there is no Night at all. Hence the Geographers did so divide the *Earth* by such Parallels, as made the longest Day increase by Quarters of an Hour; that is, each Parallel was so far distant from the next, that the longest Day in the more remote from the Æquator, was a Quarter of an Hour longer than that Day at the Parallel nearer to the Æquator: And therefore reckoning the Æquator as the first Parallel, the second Parallel passed thro' those Parts of the *Earth*, where the longest Day was twelve Hours and a Quarter long. Under the third Parallel, the longest Day was twelve Hours and two Quarters. In the fourth, the longest Day was twelve Hours and three Quarters, *&c.* Now two such Parallels made up a Climate, which were therefore distinguished by the longest Day, increasing half an Hour from the one to the other. Now the Excess of the Solstitial Day above twelve Hours may grow still bigger, till we come to the Polar Circle, where the *Sun* not setting, makes the Day twenty-four Hours long, which is greater than the Æquinoctial Day of twelve Hours by twenty-four half Hours, or forty-eight Quarters of an Hour. From hence we gather, that the Number of Climates between the Æquator and Polar Circles must be twenty-four, and the Number of the Parallels forty-eight.

Lecture XIX.

The Rising and Setting of the Stars Cosmical, Achronical, and Heliacal.

BECAUSE the Civil Year of the Ancients did not keep Pace or agree with the apparent annual Motion of the *Sun*; having the Day of the Month, and the Year when any memorable Action fell out, it could not be from thence immediately known in what Season of the Year it was done. And therefore, when the Husbandmen settled the Times for the several distinct Parts of their Business, they could not point out that Time by a certain Day of their *Kalendar*;

Lecture XIX.

lendar; for the ſame Day of the Month was not always in the ſame Seaſon of the Year: But it was neceſſary to have more certain Characters and Marks to diſtinguiſh Times; and therefore the Writers of Huſbandry, Hiſtorians and Poets, had recourſe to the Riſings and Settings of the *Stars*, by them to mark out the Times. And of theſe Riſings and Settings they reckoned three Sorts, *viz*, The *Coſmical*, *Achronical* and *Heliacal*. A *Star* is ſaid to riſe or ſet *Coſmically*, which riſes or ſets when the *Sun* riſes; ſo that a *Star* which riſes or ſets in the Morning riſes or ſets *Coſmically*. A *Star* riſes *Achronically*, when it riſes while the *Sun* ſets, that is, in the Evening, when it is in Oppoſition to the *Sun*, and is viſible all Night.

A *Star* riſes *Heliacally*, when after it has been in Conjunction with the *Sun*, and on that Account inviſible, it comes to be at ſuch a Diſtance from him, as to be ſeen in the Morning before *Sun* riſing, when the *Sun*, by his apparent Motion, recedes from the *Star* towards the *Eaſt*. But the *Heliacal* ſetting is when the *Sun* approaches ſo near a *Star*, that it hides it with his Beams, which keep the fainter Light of the *Star* from being perceived. And therefore the *Heliacal* riſing and ſetting is rather an Apparition and Occultation, than a Riſing and Setting.

All the *fixed Stars* in the *Zodiack*, as likewiſe the ſuperior Planets, *Mars*, *Jupiter* and *Saturn*, riſe *Heliacally* in the Morning a little before *Sun* riſing, and a few Days after they have ſet *Coſmically*; becauſe the *Sun* in his apparent Motion gets before them, moving faſter *Eaſtward:* But they ſet *Heliacally* in the Evening a little before their *Achronical* ſetting. But the *Moon*, whoſe Motion *Eaſtward* is always quicker than the apparent Motion of the *Sun*, riſes *Heliacally* in the Evening, after the new *Moon* or the Change, when it can be firſt diſcovered at *Sun* ſetting; but the *Moon* ſets *Heliacally* in the Morning, when the *Moon* is old and approaching to a Change. The inferior *Planets*, *Venus* and *Mercury*, which ſometimes ſeem to go *Weſtward* from the *Sun*, and

ſome-

18

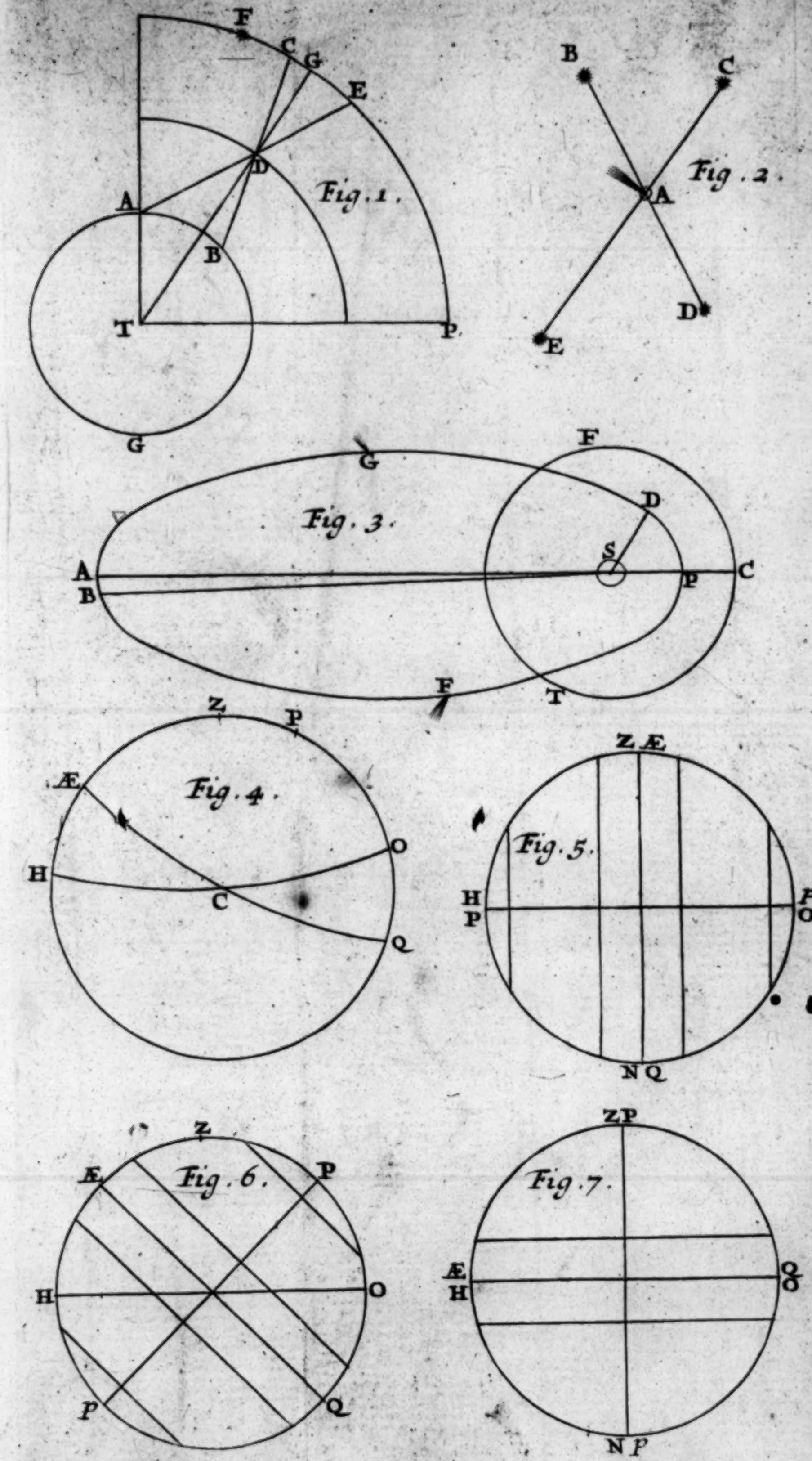
Fig. 1.
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Fig. 6.
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Fig. 7.
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ſometimes again have a quicker Motion than he *Eaſt-ward*, riſe *Heliacally* in the Morning when they are Retrograde; but when they are direct in their Motions, they riſe *Heliacally* in the Evening. Lecture XIX.

In order to obſerve the Altitude of the *Sun*, or any *Star*, we uſe a moveable Quadrant E A D, with fixed Sights A, B, or a Teleſcope placed along one of its Sides, and a plumb Line A C hanging from its Center. The Quadrant being placed in a Vertical Plane, muſt be turned upwards and downwards, till the Rays of the *Sun*, paſſing thro' the Hole of the firſt Sight next the Center, fall upon the Hole of the other; or till the *Sun*'s Image appears in the *Focus* of the Teleſcope; and then the Quadrant being kept in this Poſition, the Plumb Line or Thread, will ſhew the Arch E C, which meaſures the Altitude of the *Sun*. For produce A C to the *Zenith* Z, and let AH be an *Horizontal* Line. The Angles EAB and ZAH are equal, being both Right; but the Angles BAC and ZAS are likewiſe equal, they being vertical to each other: Wherefore taking away equal Angles, there will remain the Angle EAC equal to the Angle SAH. But the Arch EC meaſures the Angle EAC, and the Arch of the Vertical between the *Sun* and the *Horizon* meaſures the Angle SAH; and therefore this Arch or the *Sun*'s Altitude, and the Arch EC, are ſimilar or like Arches. But if the Height of a *Star* be to be obſerved, inſtead of the Irradiation of Beams as in the *Sun*, we muſt look thro' both Sights, or the Teleſcope for the *Star*; and when we can ſee it, then the Thread will ſhew the Altitude of the *Star*. The Meridian Altitude of the *Sun* or *Star* is known by obſerving when the Altitude is greateſt; for then the *Sun* or *Star* attains his greateſt Height, being then in the Meridian.

How to obſerve the Altitude of the Sun, *or of a* Star.

Plate XVII. Fig 1.

The Knowledge of the Latitude of the Place is the Foundation of all *Aſtronomical* Obſervations, without which we can know nothing; and therefore it is firſt accurately to be obtained: And becauſe we have

How to obſerve the Latitude of a Place.

Lecture XIX.

have shewed the Altitude of the *Pole* to be always equal to the Latitude, we can best find our Latitude by observing the *Pole*'s Height; but because the *Pole* is only a Mathematical Point, and no ways to be perceived by our Senses, we cannot find its Height by the same Method as we did that of the *Sun* or *Stars*. And therefore we must take another Way for finding it. And first we must find the Section of the Plane of the Meridian with the *Horizon*, which Section is called the Meridian Line. This is obtained by erecting a *Gnomon* or Perpendicular upon the *Horizon* in order to cast a Shadow; and at the Foot of the *Gnomon*, at that Point which is directly under the Top-Point, which casts the Shadow, there must be described a Circle, on whose Circumference the Shadow of the Top-Point may fall before Mid-day; and that Point of the Circumference where the Shadow comes, must be carefully marked. Again, after Mid-day observe the Point where the Shadow comes again to the same Circumference; and let the Arch between the two Points of the Shadow be bisected, or cut into equal Parts. A Line drawn from the Center to the Point of Bisection will be the Meridian Line: For the *Sun* before and after Mid-day being equally high, is equally distant from the Meridian, or is in two Vertical Circles, which make on each Side equal Angles with it. Place therefore the Quadrant on the Meridian Line, so that its Plane may be in the Plane of the Meridian; and then take some *Star* near the *Pole*, which never sets, and observe both its greatest and least Altitude. Let then the greatest be SO, and the least *s*O; the Difference of their Altitudes is the Arch *s*S, the half of which PS or P*s*, deducted from the greatest Altitude SO, or added to the least *s*O, will give PO the Altitude of the *Pole* above the *Horizon*, which is equal to the Latitude of the Place. If the Declination of the *Sun* be known, we may find out the Latitude of the Place in this Manner: Observe the Meridian Distance of the *Sun* from the Vertex or *Zenith*, which is always the Complement of

The Invention of the Meridian Line.

Plate XVII. Fig. 2.

of his Altitude; and add to this the *Sun's* Declination, when the Place and he are on the same Side of the Æquator; or subtract the Declination when they are on different Sides, the Sum or Difference is always equal to the Latitude: But when the Declination of the *Sun* is greater than the Latitude of the Place, which is known from the *Sun's* being nearer to the elevated Pole than the Vertex of the Place is, as it will often happen in the torrid Zone; then the Difference between the *Sun's* Declination and the *Sun's* Zenith Distance is the Place's Latitude. Lecture XIX.

HAVING once found out the Latitude of the Place, the Obliquity of the Ecliptick, or its Inclination to the Æquator is easily obtained, by observing about the Summer Solstice the *Sun's* least Distance from the Vertex: If this Distance be subtracted from the Latitude of the Place, when the Place is nearer to the Pole than the *Sun* is, the Remainder will shew the greatest Declination of the *Sun*, which is equal to the Obliquity of the Ecliptick. Most Astronomers make the greatest Declination of the *Sun*, or the Obliquity of the Ecliptick to be $23\frac{1}{2}$ Degrees; but the more accurate Observations of our modern Astronomers shew it to be one Minute less. *How to observe the Obliquity of the Ecliptick.*

BY the same Method the Declination of the *Sun* for any Day, or that of a *Star* may be taken; for when the *Sun* or *Star* is nearer to the Æquator than the Place, take the Difference between the Latitude of the Place and the Meridional Distance of the *Sun* or *Star* from the Vertex, and we shall have the Declination: But if the Vertex of the Place lie between the *Sun* or *Star* and the Æquator, then the Sum of these two Quantities is the Declination. *How to find the Declination of the Sun or Star.*

HAVING the Declination of the *Sun*, it is easy to find his right Ascension and Place in the Ecliptick, by the Solution of a right angled spherical Triangle. For let ÆQ be the Æquator, ÆC the Ecliptick, S the *Sun*; from which let fall on the Æquator a Circle of Declination SD, and then the Arch SD *To find the Sun's right Ascension.* Plate XVII. Fig 3.

Lecture XIX.

will be the *Sun*'s Declination. And therefore in the right-angled Triangle ÆSD we have SD and the Angle Æ: We can therefore from thence find, by spherical *Trigonometry*, the Arch ÆD, which is the right Ascension of the *Sun*, and ÆS, which gives his Place in the Ecliptick; and likewise the Angle ÆSD, the Inclination of the Circle of Declination, or of the Meridian with the Ecliptick. So likewise in the right-angled Triangle ÆSD, if we have the Side ÆD, which is the right Ascension, the Angle Æ being constant and known, we may easily find the Side SD, which is the Declination of the Point of the Ecliptick S, which comes to the Meridian together with D, and is then called the *Medium Cœli*, or Mid-heaven; as also the Angle ÆSD the Inclination of the Meridian to the Ecliptick. Or lastly, if ÆS the Longitude of the Point S be known, we can from it find its right Ascension and Declination, together with the Angle DSC, of the Ecliptick and Meridian.

The apparent Motion of the Sun *in the Ecliptick is not equable.*

If the Declination of the *Sun*, by the Method we have shewed, be daily observed, we can from thence collect the apparent Motion of the *Sun* in the Ecliptick, which is always equal to the Motion the *Earth* really has in the same Time. Now, by Observation, we find that the *Sun* does not move equably, or always with the same Velocity in the Ecliptick: And therefore the real Motion of the *Earth* must be unequal, and not constantly the same; in the Summer Solstices the *Earth* has a slower Motion, in the Winter Time it moves quicker: And this happens, because the *Earth* by its real Motion is carried in the Perimeter of an Ellipse, which has the *Sun* in one of its *Foci*; round which it revolves in such a Manner, that a Line drawn from the *Sun* to the *Earth*, and by an angular Motion carrying the *Earth* along with it, doth always sweep or describe Elliptick *Areas* or Spaces proportional to the Times in which they are described.

Having

HAVING the Place of the *Sun* in the Ecliptick, by the Help of it, and a good Pendulum Clock, we may find the right Ascensions of all the *Stars*: For which Purpose the Motion of the Clock must be so adjusted, that the Hand may run thro' the twenty-four Hours in the Time that a *Star* leaving the Meridian will arrive at it again; which Time is somewhat shorter than the natural Day, because of the Space the *Sun* moves thro' in the mean Time *Eastward*. The Clock being thus adjusted, when the *Sun* is in the Meridian, fix the Hand to the Point, from whence we are to begin to reckon our Time; and then observe when the *Star* comes to the Meridian, and mark the Hour and Minute that the Hand then shews: The Hours and Minutes describ'd by the Index, turn'd into Degrees and Minutes of the Æquator, will give the Difference between the right Ascension of the *Sun* and *Stars*; which Difference, being added to the right Ascension of the *Sun*, will give the right Ascension of the *Star*. Now if we know the right Ascension of any one *Star*, we may from it find the right Ascensions of all the others which we see, by marking the Time upon the Clock between the Arrival of the *Star*, whose right Ascension we know, to the Meridian, and another *Star*, whose Ascension is to be found. This Time converted into Hours and Minutes of the Æquator, will give the Difference of right Ascensions; from whence, by *Addition*, we collect the right Ascension of the *Star* which was to be found out.

Lecture XIX.

To find the right Ascension of the Stars.

BUT by knowing the right Ascension of one *Star*, the right Ascensions of the rest are easiest found by the following Method; where there is no need of waiting till the *Stars* come to the Meridian. We only use a Telescope, in whose *Focus* there are fixed four Threads extended, two of which, as AB, CD cut one another at right Angles, and the other two EF, GH make half-right Angles with the former two, at their common Intersection O: Then the Telescope is to be directed to a *Star*, whose right Ascension and Declination are known.

The right Ascension of one Star *being known, how from thence to find the right Ascensions of the rest.*

Lecture XIX. Plate XVII. Fig. 4.

And then the Telescope is to be constantly turned till the *Star* be seen in the Line A B, so as its apparent diurnal Motion may be along that Line; in which Position the right Line A B will represent a Portion of that parallel Circle the *Star* describes, it being in a Parallel to its Plane: And because the Line CD cuts it at right Angles, it will represent some horary Circle. Fix then the Telescope very firm in this Situation, and mark the Time according to the Clock, when the *Star*, whose Ascension is known, comes to the Line CD. Then again, observe any other *Star* with the Telescope, which will appear to move in some Line LK that is parallel to A B; and mark likewise the Time when it comes to the horary Circle CD in Q. The Difference of Times, between the Arrival of the first *Star* to the horary Circle, and the coming of this last to the same, converted into Degrees and Minutes of the Æquator, will give the Difference of their right Ascensions: And therefore if the right Ascension of one of them be found, we may from thence collect the right Ascension of the other.

Because the Angles QHO and QOH are equal being each half a Right, QH will be equal to QO. Now if we mark the Time between the coming to the Thread OH, and its touching the Thread OQ, we shall have the Time the *Star* takes to describe the Portion QH of the Parallel. This Time being turn'd into Degrees and Minutes, will give the Number of Degrees and Minutes of the Portion QH. But the Arch of the horary great Circle is equal to this Arch QH. Now in unequal Circles the Degrees and Minutes that equal Arches contain are reciprocally as the Radii or Semidiameters of the Circles, as we shall shew hereafter. As the Radius therefore of the great Circle is to the Radius of the Parallel LK (which does not sensibly differ from the Radius of the known Parallel A B) that is, as the Radius is to the Sine of the *Stars* Distance from the Pole, so are the Number of Degrees and Minutes in the Arch HQ of the Parallel, to the Number of Degrees and Minutes

Minutes in the Arch OQ of the great Circle; which therefore will be known by the Rule of Proportion. But QO is the Difference of the Declination of the *Star* which describes the Parallel AB, and of that which describes the Parallel LK; therefore from the Declination of one of these *Stars* being known, we can find the Declination of the other. And by this Method the right Ascensions and Declinations of most *Stars* may be observed. Lecture XIX.

That in unequal Circles the Numbers of similar Parts, as Degrees and Minutes, that are in Arches equal in Bigness, are reciprocal to the Radii of the Circles, may be thus demonstrated. Imagine two unequal Circles, whose Center is C, and in them equal Arches BE, AF: Draw from the Center CB, CE, cutting from the lesser Circle the Arch AD. The Arches AD and BE will contain equal Numbers of Degrees and Minutes; and because AF and BE are equal, AD will be to AF as AD is to BE. But AD is to BE as CA is to CB; therefore AD will be to AF as CA is to CB. But AD is to AF as the Number of Degrees and Minutes in AD or in BE is to the Number of Degrees and Minutes in AF. Wherefore the Number of Degrees and Minutes in BE is to the Number of Degrees and Minutes in AF, as CA is to CB; or in a reciprocal Proportion of the Radii of the Circles. Which was to be demonstrated. Plate XVII. Fig. 5.

Having the right Ascension of a Star, and its Declination, to find its Longitude and Latitude. Plate XVII. Fig. 6.

Having the right Ascension and Declination of a *Star*, its Longitude and Latitude may be found out by the Solution of a spherical Triangle. For imagine BPQ to pass through the Poles of the Æquator and Ecliptick; this Circle is the *Solstitial Colure*. Let ÆQ be the Equinoctial, EC the Ecliptick, whose Section with the Equinoctial is ♈, and let S be a *Star*, through which passes the Circle of Declination PSF, cutting the Equinoctial at F: The Arch ♈ F is the right Ascension of the *Star*, and SF its Declination. Draw through the Pole of the Ecliptick B and the *Star* the Circle of Latitude BSO,

Lecture XIX.

BSO, meeting with the Ecliptick in O: Then ♈ O will be the Longitude of the *Star*, and SO its Latitude. In the ſpherical Triangle BSP, we have the Side PS, which is the Complement of the Declination, and the Side BP, which is equal to the Arch that meaſures the Inclination of the Æquator and the Ecliptick: Beſides which, we have the Angle which is meaſured by the Arch FQ the Complement of the right Aſcenſion; and therefore we have the Angle BPS its Complement to two right Angles. And therefore in the Triangle BPS, having three of its conſtituent Parts, we may find firſt the Angle PBS, whoſe Meaſure is the Arch OC, and its Complement to a Quadrant is the Arch ♈ O, which is the Longitude of the *Star*. We can likewiſe, from the ſame Things given, find the Arch BS, whoſe Complement to a Quadrant is SO, the Latitude of the *Star*. After the ſame Method, if the Longitude and Latitude of a *Star* be known, we may from thence find its right Aſcenſion and Declination.

The Longitudes of the fixed Stars *continually increaſe, but their Latitudes are much the ſame.*

By comparing the Places of the *fixed Stars*, as they were delivered to us by the Antients, with what they now obtain, we find that their Latitudes are much the ſame they were formerly; but their Longitudes or Diſtances from the firſt of ♈ have been found to increaſe continually: Not that the *Stars* have a real progreſſive Motion; but becauſe the Equinoctial Points have a Motion backwards, and the Longitudes are computed from them. The Longitude of any *fixed Star* obſerved by the antient Aſtronomers, compared with the Longitude it has at preſent, ſhews the Quantity of this Regreſſion of the Equinoctial Points to be about one Degree in 72 Years.

By the Method we have ſhewed, the Longitudes and Latitudes of the *fixed Stars* are found, and they and their Places are ranked in a Catalogue; which being once eſtabliſhed, the Places of the *Planets* and Comets are eaſily known by Obſervation and Computation. For if the Diſtances of any *Planet* or Comet from two *fixed Stars* of known Longitude and

Latitude,

Latitude, be taken by Obſervation, we may by that Means determine the Longitude and Latitude of the Planet or Comet in the following Manner: Lecture XIX.

LET EF be a Portion of the Ecliptick, whoſe Pole is B; and let A and C be two *Stars*, whoſe Longitudes and Latitudes are known, and P a *Planet*, whoſe Diſtances from the ſame *Stars* are known by Obſervation. In the Triangle ABC, having AB and CB the Complements of the Latitudes of the two *Stars*, and the Angle ABC, whoſe Meaſure is the Arch EF, the Difference of Longitude of the two *Stars*; from thence we may find AC the Diſtance of the *Stars*, and the Angle BCA. Again, in the Triangle APC we have all the Sides from which we may find the Angle PCA; which being ſubtracted from BCA, will leave the Angle BCP. Laſtly, in the Triangle BCP, we have the Sides BC, CP, and the Angle PCB; from which we can find the Angle PBC, whoſe Meaſure is OF the Difference of Longitude of the *Star* C and the *Planet*. We can alſo find the Arch PB the Complement of the Latitude of the *Planet*. Plate XVII. Fig. 7.

AFTER the ſame Method, if we have the Diſtances of a *Planet* from two *fixed Stars*, whoſe right Aſcenſions and Declinations are known, we may find out the right Aſcenſion and Declination of that *Planet*.

LECTURE XX.

Of the Twilight, *and the Refraction of the* Stars.

The Air makes the Firmament lucid.

BESIDES other innumerable Conveniencies which we receive from the *Atmosphere*, we have this great Advantage, that while the *Sun* shines, it makes the Face of the Heavens or Firmament to appear lucid and bright; for if no *Atmosphere* surrounded and involved the *Earth*, only that Part of the Heavens would appear to shine in which the *Sun* was placed; And a Spectator, if he should turn his Back to the *Sun*, would immediately perceive it as dark as Night; and even in the Day-time, while the *Sun* shined, the least *Stars* would be seen shining, as they do now in the clearest Night; since in that Case there would be no Substance to reflect the Rays of the *Sun* to our Eyes, and all the Rays which do not fall upon the Surface of the *Earth*, passing by us, would either illuminate the *Planets* and *Stars*, or spreading themselves out into infinite Space, would never be reflected back to us.

But since there is an *Atmosphere* covering the *Earth*, which is strongly illuminated by the *Sun*, it reflects the Light back upon us, and makes the whole Heavens to shine; and that so strongly, that by reason of its Splendor, it obscures the faint Light of the *Stars*, and renders them invisible.

If

LECTURE XX.

Of the Twilight, and ... of the Stars.

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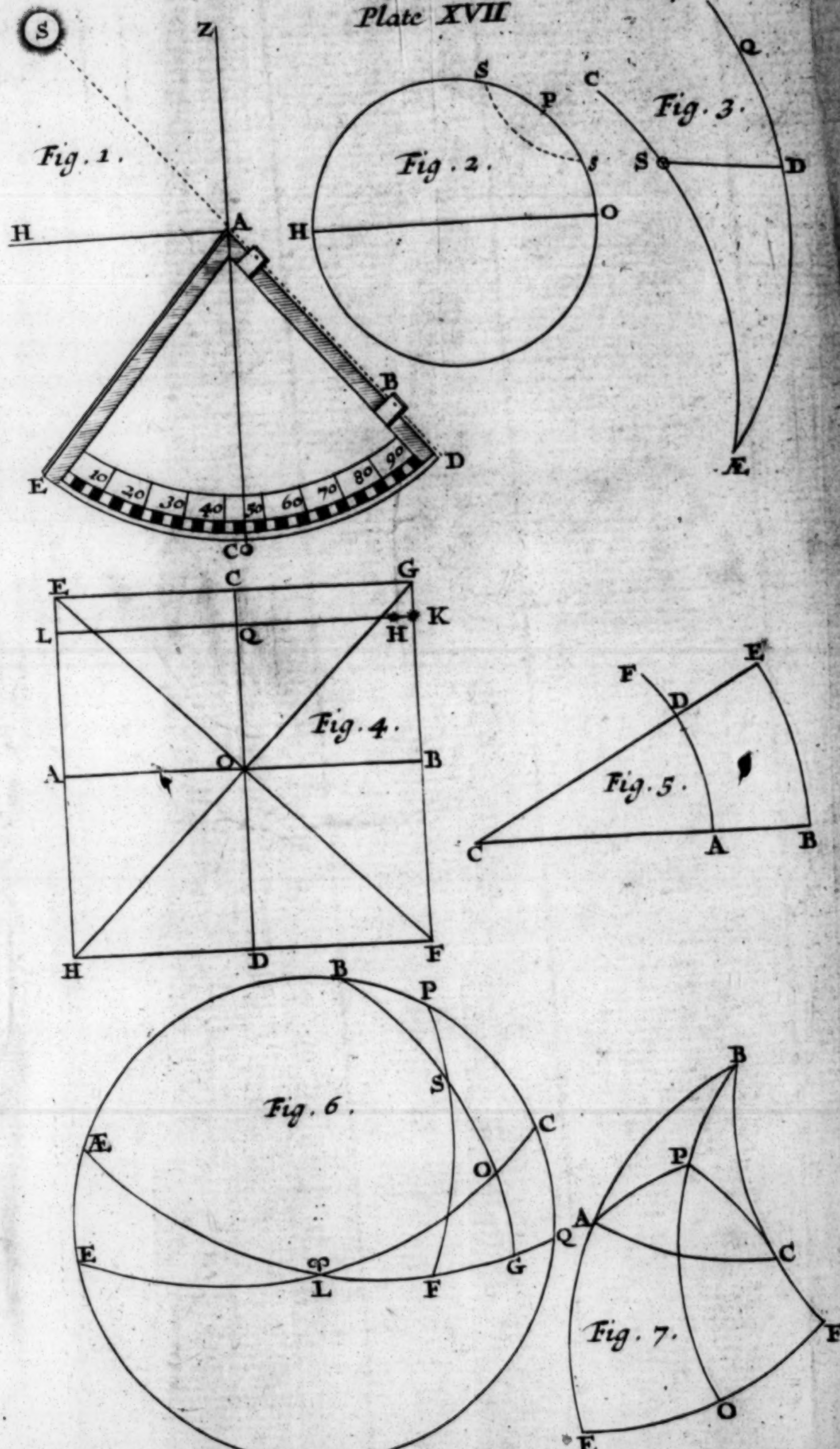
Plate XVII



If there was no *Atmosphere*, the *Sun* immediately before his setting, would shine as briskly as at Noon; but in a Moment, as soon as he is set, we should have the Face of the *Earth* in as great Darkness as it would be at Midnight: So quick a Change, and so sudden a passing from the greatest Light to the greatest Darkness, would be very inconvenient to the Inhabitants of the *Eareh*, But by means of the *Atmosphere* it happens, that tho' after *Sun*-setting we receive no direct Light from the *Sun*, yet we enjoy its reflected Light for some Time; so that the Darkness of the Night comes not suddenly, but by Degrees. For after the *Earth* by its Revolution round its *Axis* has withdrawn us from the Sight of the *Sun*; the *Atmosphere* which is higher than we are, will still be illuminated by the *Sun*; so that for a while the whole Heavens will have some of his Light imparted to it. But as the *Sun* goes still lower under the Horizon, the less is the Air illustrated by him: So that when he is got as far as 18 Degrees lower than the Horizon, he no longer enlightens our *Atmosphere*, and then all that Part thereof that is over us becomes dark.

The Cause of the Crepuscles.

So likewise in the Morning, as soon as the *Sun* comes within 18 Degrees of the Horizon, he begins again to enlighten the *Atmosphere*, and to diffuse his Light through the Heavens: So that its Brightness does still increase, till the *Sun* rises and makes full Day. This small Illumination of the *Atmosphere*, and State of the Heavens between Day and Night, is what we call the *Twilight*, which is observed in the Morning before the *Sun*'s rising, and at Night after his setting; in *Latin* it is named *Crepusculum*.

To make this plainer, imagine the Circle ADL on the Surface of the *Earth*, in the Plane of the vertical Circle in which the *Sun* is when under the Horizon. Let there likewise be another concentrical Circle CBM in the same Plane, including that Portion of the Air that reflects the *Sun*'s Beams: And suppose the Eye to be on the *Earth*'s Surface

Plate XVIII. Fig. 1.

at

Lecture XXX.

at A, whose sensible Horizon is AN. Since no Line can be drawn to A between the Tangent AN and the Periphery AD, by the 16th of the *Third Element*, it is plain, that when the *Sun* is under the Horizon, no direct Rays can come to the Eye at A: But the *Sun* being in the Line CG, a Line may be drawn from him to C, so that the Particles C may be illuminated by the direct Rays of the *Sun*; which Particles may reflect those Rays to A, where they may enter the Eye of the Spectator: And by this Means the Beams of the *Sun*'s Light illuminating an innumerable Multitude of Particles, may by them be reflected to the Spectator in A. Let the Tangent AB meet with the Surface of the Orb of Air that reflects the Light in B; and from B draw BD, touching the Circle ADL in D, and let the *Sun* be in the Line BD at S: Then the Ray SB will be reflected into BA, and will enter the Eye, because of the Angle of Incidence DBE being equal to the Angle of Reflection ABE: And that will be the first Ray that reacheth the Eye in the Morning, and then the Dawning begins; or the last which falls upon the Eye at Night, when the *Twilight* ends. For when the *Sun* goes lower down, the Particles at B can be no longer illuminated.

Another Cause of the Twilight.

THE Reflection of the *Atmosphere* does not seem to be the only Cause of the *Twilight*; but there is an Ætherial Air, or *Atmosphere*, likewise round the *Sun*, which shines after the Body of the *Sun* is set: This Orb of the *Sun*'s *Atmosphere* rising sooner, and setting later than the *Sun* itself, shines out at Mornings and Nights in a circular Figure, it being a Segment of the *Sun*'s *Atmosphere* cut by the Horizon; and its Light is quite of another Sort, than that which is made by the Reflection of our *Atmosphere*. But the Duration of the *Twilight* that arises from the *Sun*'s *Atmosphere*, is shorter much than that made by the Reflection of the *Earth*'s *Atmosphere*, which does not end till the *Sun* comes to be 18 Degrees below the Horizon,

zon, or thereabouts. But there can be no certain Bounds fixed for the Beginnings and Endings of the *Twilights*; for their Lengths depend on the Quantity of Matter in the Air which is able to reflect Light, and on the Height of the *Atmosphere*. In the *Winter* the Air, being condensed by the Cold, is low; and on that Account the *Twilights* are sooner over. In the *Summer* the Air is rarified by Heat, and therefore being higher, remains longer illuminated by the *Sun*, so that the *Twilights* last the longer: Also the Duration of the *Twilight* is shorter in the Morning than at Night. We generally reckon that the *Twilight* begins or ends, when in the Morning the *Stars* of the sixth Magnitude disappear, or in the Evening when they first come to be seen; the Light of the Air before that rendring them invisible. *Ricciolus* observed at *Bononia*, that the Morning *Twilight*, about the Time of the *Equinoxes*, lasted an Hour and 47 Minutes; but in the Evening two Hours, and did not end till the *Sun* was 20 Degrees under the Horizon: But in *Summer* the Morning *Twilight* was three Hours and 40 Minutes long; the Evening *Twilight* scarcely ending till Midnight.

The Twilights *are shorter in Winter than in Summer.*

HENCE if we have the Time of Beginning of the *Twilight* in the Morning, or the End of it at Night, we may find the Height of the Air that reflects the Light; for then the *Twilight* ends, when a Ray of Light from the *Sun* touches the Globe of the *Earth*, and is by the highest Air reflected to our Eyes: For having the Time, we can find the Depression of the *Sun* below the Horizon, and from thence the Height of the Air. For let SB be a Ray of Light touching the *Earth*, which is reflected by a Particle of Air in its highest Region, in the Horizontal Line AB; the Angle SBN is the Measure of the Depression of the *Sun* below the Horizon: And because AB is also a Tangent, the Angle AED at the Center is equal to the Angle SBN; and its Half, that is, the Angle AEB is equal to half SBN, or half the Depression of the *Sun*. Suppose the Depression

The Height of the Atmosphere *measured by an Observation of the* Twilight.

pression

Lecture XX. preſſion of the *Sun*, at the Beginning or End of *Twilight*, be 18 Degrees; then the Angle AEB will be 9 Degrees, which would be true, did the Ray SB paſs thro' the *Atmoſphere* without Refraction: But becauſe it is refracted and bent towards H, we muſt diminiſh the Angle AEB by a Quantity equal to the Horizontal Refraction, which is above half a Degree: And therefore the true Meaſure of the Angle AEB is $8\frac{1}{2}$ Degrees. Moreover, AE is to BH as the Radius is to the Exceſs of the Secant of the Angle AEB above the Radius, that is, as 100000 is to 1110. Therefore if the Semidiameter of the *Earth* be in round Numbers 4000 Miles, BH the Height of the *Atmoſphere*, which reflects the *Sun*'s Rays, will be about 44 Miles; for as 100000 is to 1110, ſo is 4000 to 44.

Under the Æquator the Time of the Twilight *is ſhort.*

IN a right Poſition of the Sphere the *Twilights* are quickly over; becauſe the *Sun* deſcends conſtantly nearer in a Perpendicular; but in an oblique Sphere they laſt longer, the *Sun* deſcending obliquely; and the more oblique the Sphere is, that is, the greater the Latitude of the Place is, ſo much the longer laſt the *Twilights*: So that all they who are in above 48 Degrees Latitude, in the *Summer*, near the Solſtices, have their *Atmoſphere* illuminated the whole Night, and the *Twilight* laſts till the Sun-riſing, without any compleat Darkneſs.

Under the Poles they laſt ſome Months.

IN a Parallel Sphere the *Twilight* laſts for ſeveral Months, ſo that the Inhabitants have either the direct or reflex Light of the *Sun* for almoſt all the Year.

The Circle terminating Twilight.

IF below the Horizon you conceive a Circle to be drawn parallel to the Horizon, and at a Diſtance from it, equal to the Depreſſion of the *Sun* at the End of the *Twilight*: This leſſer Circle is called the *Circle which terminates the* Twilights; for whenever the *Sun* by its apparent diurnal Motion reaches this Parallel, the Morning *Twilight* begins, or the Evening ends, in whatever Parallel of the Æquator the *Sun* is.

IN

In the Figure let HQO be the Horizon, V *a* X the Circle parallel to it, terminating the *Twilight*, the Circle HZO the Meridian, ÆQ*a* the Æquator. It is manifest that the more oblique the Æquator is to the Horizon, so much the greater are the Arches of the Æquator and its Parallels, intercepted between the Horizon and the terminating Circle V *a* X. The Arches QR, *d a*, C*e*, G*h*, K*l*, are called the Arches of the *Twilights*, because they determine their Duration: And as each Arch has a bigger or less Proportion to its Circle, so will the *Twilight*, when the *Sun* is in that Parallel, be longer or shorter. In the Circle bounding the *Crepuscles* take any Point *a*, thro' which passes a Parallel to the Æquator *d a*; and thro' *a* imagine a great Circle to be drawn, as M *a* N, touching the Circle of perpetual Apparition: And since the Horizon likewise touches the same Circle, these two Circles will make equal Angles with the Æquator and its Parallels; for the Measure of each Angle is the Distance of the Parallel from its great Circle. So likewise all the Arches of the Æquator and its Parallels, between the Horizon and the Circle M *a* N are similar, by *Prop.* 13, Book II. *Theodosius*'s Sphericks. This Circle M *a* N will either cut the bounding Circle V *a* X in two Points, or touch it in one. Let it first cut it in two Points *a* and *h*; and therefore the Arches of the Parallel *d a*, G *h* are similar: Wherefore when the *Sun* by its diurnal Motion describes these two Parallels, the *Twilights* are equal; but while he describes any intermediate Parallel as C *e*, the Time of the *Twilight* is shorter; for in this Case C*m* the Arch of *Twilight* is less than C *e*, which is similar to the Arch *d a* or G *h*, and C *e* and *d a* are described by the *Sun* in equal Times. But when the *Sun* is in Parallels that are at a greater Distance from the Æquator than G *h*, the *Twilights* last longer; for the *Twilight* Arch *l* K is greater than *q*K, which is described by the *Sun* in the same Time as the Arch of the *Crepuscle* G *h*.

Lecture XX.

The Arches of the Crepuscles *or* Twilight.

Plate XVIII. Fig. 2.

WHILE

Lecture XX.

The different Duration of the Twilights.

WHILE the *Sun* is in Parallels that are towards the elevated Pole, the *Twilights* do constantly grow longer, according as those Parallels approach the Poles: For the *Twilight* Arch *o p* is longer in being described than Q R, and Y U the same way is longer than *o p*. But if the *Sun* describe the Parallel S *t*, it never will meet with the bounding Circle, and then the *Twilight* lasts the whole Night long.

HENCE arises a great Difference between the Increase of the *Twilight* and its Decrease, and the Increase and Decrease of Days and Nights. For while the *Sun* moves from the Beginning of ♋ to the first of *Capricorn*, all that Time the Days constantly decrease, and the Nights increase: But in the *Twilight* it is otherwise; for though the *Twilight* and Days are at the longest when the *Sun* is in the first Degree of ♋, and then they both decrease together; yet the Times of *Twilight* do not continually decrease till the *Sun* comes to ♑, but there is a certain Point between ♋ and ♑, to which when the *Sun* arrives, we have the shortest *Twilight*. From thence the *Twilights* will begin to increase again, and there will be no Arch of *Twilight*, similar to that when the *Sun* is in the Æquator, before he reaches ♑: And if the *Sun* should go further South even beyond the Tropick, the *Twilights* would still increase, although the Days decreased. And although the Days from the Beginning of the *Sun*'s Entry into ♑ do constantly increase, yet the *Twilights* grow shorter till the *Sun* comes to a Point between ♑ and ♈, in which again we have the shortest *Twilight*: This appears plain by what we are here to demonstrate in the next Place.

Plate XVIII. Fig, 3.

2*dly*, LET the Circle M *a* N touch the bounding Circle in one Point, which suppose to be *a*, through which draw the Parallel to the Æquator *d a*; I say, that when the *Sun* is in this Parallel, the *Twilight* will be the shortest of all. For because the Arches of the Parallels intercepted between the Horizon and the Circle M *a* N are all similar,

they

they will be described by the *Sun* in equal Times: But because the *Twilight* Arches *c e* and *g h* are greater than *c m* or *g i*, the *Sun* will be longer in moving through the Arch *c e* than *c m*, and through the Arch *g h* than *g i*; that is longer than in describing the Arch *d a*, which Arch therefore is the shortest *Twilight*.

Lecture XX.

The Time of the shortest Twilight *investigated.*

THE Distance of that Parallel from the Æquator, in which is the shortest *Twilight*, is thus investigated. Because the Circle M *a* N and the Horizon HO touch the same Parallel, which is the Circle of perpetual Apparition, they will both be equally inclined to the Æquator: And therefore the Angle *a n* T of the Æquator, and the Circle M *a* N, is equal to the Angle FQ*d* of the Æquator and the Horizon. Through the Zenith Z and the Point *a*, draw the vertical Circle ZY *a*, cutting the Æquator in the Point T. The Spherical Triangles *a n* T, TQY, are mutually equiangular to each other, because the Angles at *a* and Y are right; and we have shewed that the Angles at Q and *n* are equal; also the Angles at T are equal, being vertical to each other: These Triangles then being equiangular, are also equilateral; and therefore T *a* will be equal to TY, or to half the Distance of the bounding Circle from the Horizon: Moreover, *a n* is equal to Q*d*, by 13 *Prop.* Book II. *Theod.* for FR and *d a* are parallel, and therefore *d* Q is equal to QY.

IN the Spherical Triangle TQY rectangular at Y, we have the Side TY half the Distance of the bounding Circle from the Horizon; as also the Angle YQT equal to FQ*d*, which measures the Complement of the Latitude of the Place; wherefore we can find QY, and Q*d*, which is equal to it. From the Point *d* to the Æquator draw the Circle of Declination *d* F; and in the Spherical Triangle *d* QF, we have *d* Q and the Angle Q, by which we can find the Arch *d* F, the Declination of the Parallel of the least *Twilight* from the Æquator, which was to be found.

THIS

Lecture XX.

The same Problem *solved by a single Analogy.*

THIS *Problem* might have been solved by one single Analogy. For in the Triangle TQY, the Radius : Tang. TY :: Co-tang. Q : Sin. QY or to the Sin. of dQ: But the Sin. of Q : Co-sin. of Q :: Rad. : Co-tang. Q. Therefore by the Rules of the 5*th* Element, the Rad. multiplied by the Sin. of Q, will be to the Tang. of TY into the Co-sin. of Q, as the Radius is to the Sin of Qd: But in the right-angled Triangle QdF, Radius is to the Sine of Qd as the Sine of the Angle Q to the Sine dF; wherefore Rad. × Sine Q will be to Tang. of TY × Co-sin. of Q, as the Sine of Q to Sine of dF; and thence, *ex æquo*, it will be as Radius to Tang. of TY, so Co-sine of Q or the Sine of the Latitude, to the Sine of the Distance of the Parallel from the Æquator.

The Duration of Twilight *determined.*

Having the Declination of the *Sun*, the Time of the Beginning of the Morning *Twilight*, which we call Break of Day, the End of the Evening *Twilight*, is thus to be found. Let op be the Parallel of the *Sun*, meeting with the bounding Circle in p; and draw through the Pole the Circle of Declination Pp. In the Spherical Triangle PZp we have all the Sides, for ZP is the Complement of the Latitude, Pp the Complement of the *Sun's* Declination, and Zq equal to the Sum of a Quadrant, and the Distance of the bounding Circle from the Horizon $= Zl + lp$. From which we can find the Angle ZPp, and its Complement to two Rights pPV: And the Arch of the Æquator, measuring this Angle, being converted into Time, will shew the Beginning or End of *Twilight*.

Refraction by the Atmosphere.

THE *TERRESTRIAL ATMOSPHERE*, by reflecting the *Sun's* Beams, not only produces the Morning and Evening *Twilight*, but it also bends and refracts the Rays of the *Sun*, and all the *Stars* which fall on it, and changes their Directions by propagating the Light in other Lines, making the apparent Places of the *Stars* different from their true Places.

By manifold Experiments, we find that the Rays of a luminous Body, even of any visible Object,

ject, when they fall upon a *Medium* or Diaphanous Body, as Air or Water, of a different Density from that from whence they first proceeded, do not afterwards go directly in the same strait Lines, but are broken or bended, and propagated as if they had proceeded from another Point than they really did. And if the *Medium* on which the Rays fall be denser than the first, they are bent towards a Line perpendicular to that Surface whereon they fall, at the Point of Incidence; but if it be a *Rarer Medium*, in their bending they recede from the Perpendicular. Lecture XX.

Various Effects of Refraction.

We observe in Nature many Effects of Refraction. A Staff, whose one Part is immerged in Water and the other in the Air, appears broken; and that Part which is in Water appears higher than it really is. All the *Stars* by Refraction appear higher or nearer to our Vertex than they would be, were there no Air; so that the Light might arrive to us without Refraction.

Plate XVIII. Fig. 4.

Let ZV be a Quadrant of a Vertical Circle in the Heavens, described from the Center of the *Earth* T; under which is AB, a Quadrant of a Circle on the Surface of the *Earth*, and GH a Quadrant on the Surface of the *Atmosphere*: And let S be any *Star* from which proceeds the Beam of Light SE, falling on the Surface of the Air in E. Now, since this Ray comes from the Etherial Air much rarer than ours, or rather from a perfect Void, and falls on our *Atmosphere*, which is dense in Comparison of it; in E it will be refracted towards the Perpendicular: And because the upper Air is rarer than that which is nearer the *Earth*, and grows still denser the nearer it is to us, this Ray of Light, as it proceeds, will constantly be refracted and bended; so that it will arrive at our Eye in the curve Line EA: Suppose the right Line AF to touch this Circle in A; according to the Direction AF, the Ray of Light will enter the Eye at A. And because all Objects are seen in the Line according to whose Direction the Rays enter the Eye, and strike upon the *Sensorium*,

 the

Lecture XX.

the Object will appear in the Line AF, that is, in the Heavens at Q; which is nearer to our Vertex than the *Star* really is. And it may even happen, that a *Star* which is below the Horizon may be seen above it. This Refraction is also the Cause why the two great Luminaries the *Sun* and *Moon*, when one of them is above the Horizon and the other below it, both may appear above the Horizon; so that the *Moon* has been observed eclipsed, when she was below the Horizon and the *Sun* above it.

By Refraction an Eclipse of the Moon *may be seen when she is under the Horizon.*

A Star in the Vertex or Zenith has no Refraction, for a perpendicular Ray goes strait on; but the more obliquely the Ray falls upon the Surface of the Air, so much the greater is the Refraction; so that the Horizontal Refraction is the greatest of all. And a *Star* that is above 50 Degrees high has scarcely a sensible Refraction. In equal Altitudes the Refractions are equal: And therefore the *Sun*, *Moon*, and *fixed Stars*, at the same apparent Height, have all the same Degree of Refraction; though the noble *Tycho Brahe*, the Restorer of *Astronomy*, and first Observer of Refractions, thought otherwise. Hence, if the Refractions of the *fixed Stars* are known, we shall know likewise the Refractions of the *Sun*, *Moon*, and *Planets*: And it is easier to find by Observation the Refraction of a *fixed Star* than that of the *Sun* and *Moon*; for the Parallaxes of these Bodies not being exactly known, the Observations about their Refractions will be doubtful; but the *fixed Stars* having no Parallax, all the Difference between their true and observed Places is wholly owing to Refraction.

Where the Refraction is greatest; where the least.

All the Stars *at equal Heights have equal Refractions.*

Those *fixed Stars*, that rise higher above the Horizon than 50 Degrees, have their Declinations, Right Ascensions, Longitudes and Latitudes accurately enough known; for in so great an Altitude, the Refraction is next to nothing. Now these being known, we find the Refractions near the Horizon by the following Method. Let OPZH be the Meridian, HO the Horizon, ÆQ the Æquator, P the

The Method of observing the Refraction. Plate XVIII. Fig 5.

the Pole, and the Vertex Z. Let A be a *Star* whose Refraction is to be found, and let ZD be a vertical Circle passing thro' the *Star*, whose apparent Place suppose to be C; the Arch AC is the Refraction. Let the apparent Distance of the *Star* from the Vertex be observed, that is the Arch ZC: And at the Time of the Observation take the Altitude of another *Star*, which is so high that it is not liable to Refraction, with which find out the Moment of Time the Observation was made; which may also be known by a good Pendulum Clock: By this Time, and the Right Ascension of the *Sun*, we shall find the Point of the Æquator, which then culminates, or is in the Meridian, that is, the Point Æ. But we have also the Right Ascension of the *Star*, that is, the Point B, where the Circle of Declination, passing through the *Star*, meets the Æquator, and consequently the Arch ÆB will be known; which is the Measure of the Angle ZPA. Therefore, in the spherical Triangle ZPA, having ZP, the Distance of the Pole from the Vertex, and PA, the Complement of the *Star*'s Declination, as also the Angle ZPA, we find out by *Trigonometry* the Side ZA, the true Distance of the *Star* from the Vertex; from which subtract ZC, the apparent Distance known by the Observation, and there will remain AC, the Refraction of the *Star*, which was to be found.

Lecture XX.

Another Method for finding the Refraction.

THE Refraction may likewise be found by observing the Azimuth of a *Star*, or the Arch of the Horizon between the Meridian and the vertical Circle passing through the *Star*, that is, the Arch DO; for that Arch measures the Angle PZA, from which, and the Sides PZ, PA, we may find ZA, the true Distance of the *Star* from the Vertex; from which subtract ZC, the observed Distance, and we shall have CA, the Refraction required.

The Method of observing the Azimuth of a Star. Plate XVIII. Fig. 6.

THE Azimuth of any *Star* is best observed, by drawing on the Plane of the Horizon the Meridian Line AE; upon which erect the Perpendicular CA, which is easily performed by a Line and a Plummet: Then

Lecture XX.

Then take another Thread with a Weight, as BD, and hang it so that the Body of the *Star* may be covered by the two Threads CA, DB, and then the *Star* will be in the Plane of a vertical Circle, in which Plane the Threads do likewise stand: Mark then the Point B in the Plane of the Horizon, and in the Meridian Line, the Point A, upon which is erected the Thread AC: And taking in the Meridian Line any Point E, draw AB, BE; then, by the Help of a Scale of equal Parts, measure the three Sides of the Triangle BAE, from which by *Trigonometry* we shall find the Angle BAE, which is the Azimuth that was to be found.

FROM Refraction, the Reason is plain why the *Sun* and *Moon* near the Horizon appear of an oval Figure; for their inferior Limbs are more refracted, and raised higher than the superior Limbs are; and therefore these two Limbs will seem nearer to each other, and the Breadth of the Bodies contracted, while both Ends of the horizontal Diameter being equally refracted and raised, keep the same Distance from one another, and its apparent Magnitude remains the same.

When the Sun *is in the Horizon its Rays pass thorough a larger Space of Air than when he is in the Zenith.*

Plate XVIII. Fig. 7.

THE Rays of the *Sun*, when he is in or near the Horizon, pass thorough a larger Body of Air, than when he is nearer the Vertex. For let ABD be the *Earth*, ECF the Orb of Air which surrounds it, whose Altitude is commonly reckoned to be 50 Miles. Let CA be an horizontal Ray, EA a vertical Ray: It is manifest that CA is longer than EA; and the Proportion of these Lines may be thus found out. Suppose the Semidiameter of the *Earth* in round Numbers to be 4000 Miles, and EA 50; then is TE = CT 4050, whose Square is equal to the Squares of CA and AT: And therefore, if from the Square of CT we take away the Square of TA, there will remain the Square CA; that is, if from 16402500 we subtract 16000000, there will remain 402500, which is the Square of CA, whose Root is 634: And therefore CA : AE :: 634 : 50, which

is

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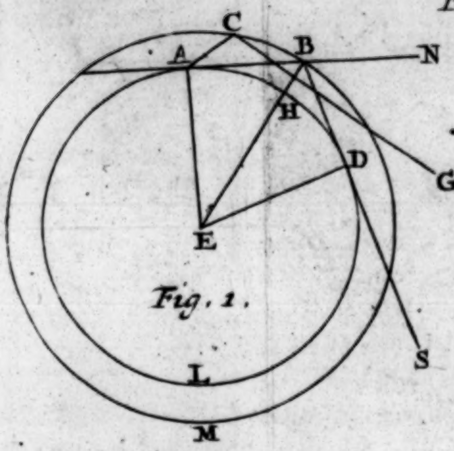


Fig. 1.

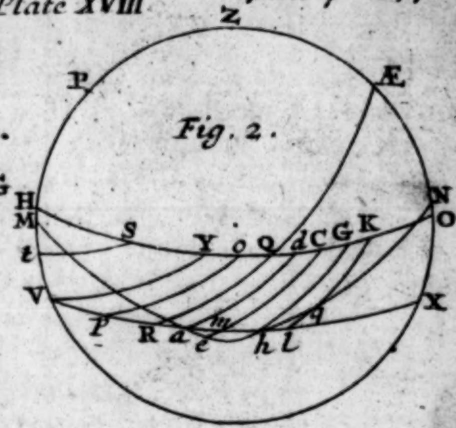


Fig. 2.

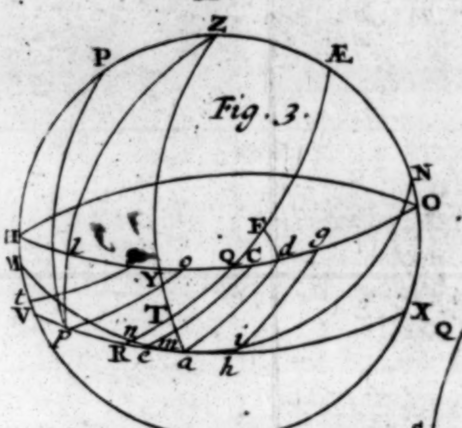


Fig. 3.

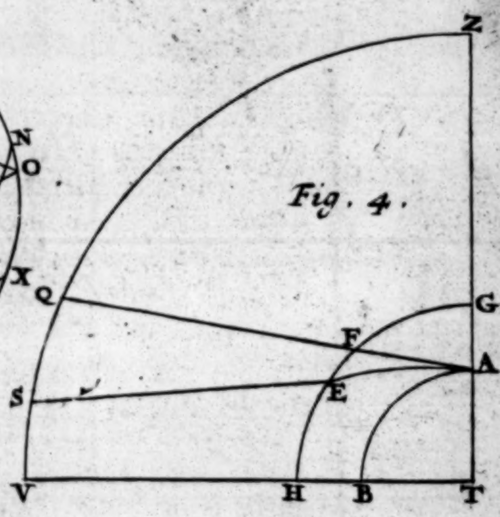


Fig. 4.

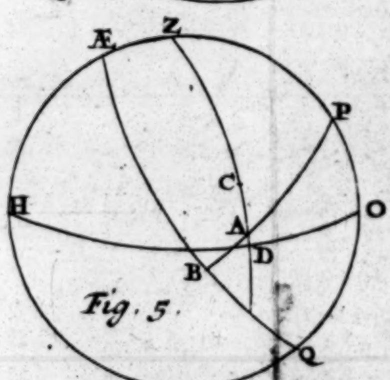


Fig. 5.

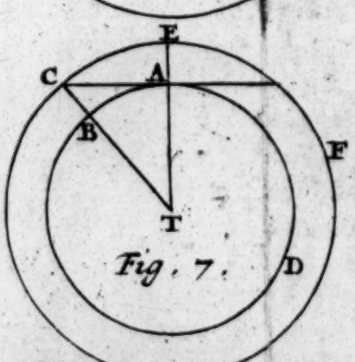


Fig. 7.

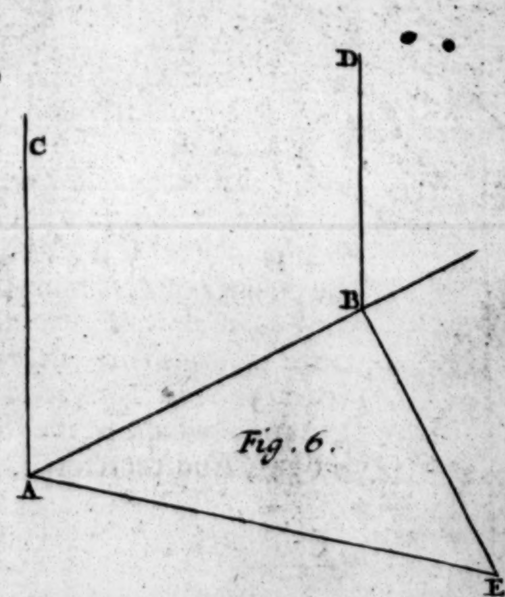


Fig. 6.

is greater than the Proportion of 12 to 1. Hence we see the Reason why, without hurting our Eyes, we can look upon the *Sun* at rising and setting: But in the Meridian he is not to be looked upon, without the Danger of hurting our Sight. For the Rays, of the *Sun* in the Horizon penetrating so large a Body of Air, hit against an infinite Number of Particles swimming in it; and being reflected or absorbed, the Force of the Light is thereby much weakened. Since therefore Light is so much weakened in passing through so small a Space as our *Atmosphere*; if this *Atmosphere* were so large as to reach the *Moon*, and were of the same Density, neither *Sun*, *Moon*, nor *Stars* could then be seen.

Lecture XXI.

LECTURE XXI.

Of the Parallaxes of the STARS.

SINCE all the apparent diurnal Motions are performed uniformly round the Axis of the *Earth*, and not round the Place of the Spectator, who lives upon the *Earth*'s Surface; he who observes the Motion of the *Stars* from this Surface, must find, that they appear to move with a Motion that is not equal. For, if a Body by its Motion describes equally the Periphery of a Circle, the Equality of Motion can be seen from no other Points than those in the Axis of this Circle. And therefore any *Star* or *Phænomenon*, seen from the Center of the *Earth*, will appear in a different Place from what it does when observed from the Surface; and this Difference of Place of the same *Star*, seen from the *Earth*'s Center and viewed from its Surface, is called the Parallax of that *Star*.

The Equality of circular Motion observable only from the Axis.

Lecture XXI.

Plate XIX. Fig. 1.

The Parallax what.

LET A B be a Quadrant of a great Circle on the *Earth's* Surface, where A is the Place of the Spectator, and the Point V in the Heavens his Vertex or Zenith. Let V N H represent the Starry Firmament, the Line A D the sensible Horizon, in which suppose the *Star* C to be seen, whose Distance from the Center of the *Earth* is T C. If this *Star* were observed from the Center T, it would appear in the Firmament in E, and elevated above the Horizon by the Arch D E. This Point E is called the true Place of the *Phænomenon* or *Star*: But an observer viewing it from the Surface of the *Earth* at A, will observe its Place in the Horizon at D, which is called the visible or apparent Place of the *Star*: And the Arch D E, the Distance between the true and visible Place, is named the *Parallax of the Star*.

IF the *Star* rise higher above the Horizon to M, its true Place visible from the Center is P; but its visible Place from the Surface is N; and its Parallax is the Arch P N, which is less than the Arch D E. And therefore, the horizontal Parallax is greatest of all Parallaxes; and the higher the *Star* rises, the less is its Parallax: And if it should come to the Vertex, it would have no Parallax at all. For when it is in Q, it is seen both from T and A in the same Line T A V; and there is no Difference between its true and visible Place. The further a *Star* is distant from the *Earth*, so much the less is its Parallax: So the Parallax of the *Star* F is G D, which is less than the Parallax of the nearer *Star* C. Hence it is plain that the Parallax is the Difference of the Distances of a *Star* from the Vertex, when seen from the Center, and from the Surface of the *Earth*. For the true Distance of the *Star* M from the Vertex is the Arch V P; but when observed from A, its visible Distance is V N.

The greater the Distance the less the Parallax.

THESE Distances are measured by the Angles V T M and V A M, contained between the Line V T drawn to the Vertex, and the Right Lines T M and A M, drawn from the Center and the Surface

Surface of the *Earth* to the *Star* M: But the Difference of these two Angles is TMA. For the external Angle VAM is equal to the two inward and opposite Angles ATM and AMT: And therefore ATM is the Difference of the two Angles VAM and ATM or VTM. This Angle AMT does therefore measure the Parallax; and upon that Account itself is frequently called the Parallax: And this is always the Angle under which the Semidiameter of the *Earth* AT, appears to an Eye placed in the *Star:* And therefore, where this Semidiameter is seen directly, there the Parallax is greatest, that is, in the Horizon. When the *Star* rises higher, the Parallax is diminished in the Proportion we shall shew in the following *Theorem:*

Lecture XXI.

The Parallax when greatest.

THE *Sine of the Parallax is to the Sine of the* Star's *Distance from the Vertex, in a constant and given Proportion; which is, as the Semidiameter of the* Earth *to the Distance of the* Star *from the* Earth's *Center.*

FOR, by a well known Theorem in *Trigonometry*, in the Triangle ATM, the Sine of the Angle AMT is to the Sine of the Angle TAM or VAM, as AT is to TM; that is, in the constant Proportion of the Semidiameter of the *Earth* to the *Star's* Distance. And therefore, the Sine of the Parallax in C is to the Sine of the Parallax in M, as the Sine of the Angle VAC is to the Sine of the Angle VAM: And therefore, if the Parallax of a *Star* be known when it is at any one Distance from the Vertex, we can find its Parallax at any other Distance from the Vertex. If any *Star* or *Phænomenon* be further distant from the *Earth* than 15000 Semidiameters of the *Earth*, its Parallax will be so small, that it will be insensible, and cannot be observed: For, since TF is to TA as 15000 to 1; and, as TF is to TA, so is the Radius to the Sine of the Angle TFA. Hence we shall find the Angle TFA less than 14 Seconds; which Angle is so small, that it cannot be observed by any Instrument.

Lecture XXI.

If we have the Diſtance of a *Star* from the *Earth*, we can eaſily find its Parallax; for, in the Triangle TAC rectangular at A, having TA, the Semidiameter, of the *Earth*, and TC, the Diſtance of the *Star*, the Angle ACT, which is the horizontal Parallax, is found by *Trigonometry:* And again, if we have this Parallax, we can find the Diſtance of the *Star*; for, in the ſame Triangle, having AT and the Angle ACT, we may find out the Diſtance TC.

If a *Star* has no proper Motion of its own, its true Diſtance from any *fixed Star*, meaſured by an Arch of a Circle, always remains unchangeably the ſame: But, if it have a ſenſible Parallax, its apparent Diſtance will be thereby changed; and, if the *fixed Star* be in the ſame vertical with the *Phænomenon*, but higher than it, the Diſtance will appear to grow leſs as it riſes higher. If the *Star* be lower, as they aſcend, the Diſtance will increaſe; yet ſeen from the Center, they will appear conſtantly to keep the ſame Diſtance from each other: And therefore, the viſible Diſtances of a *Phænomenon* from a *fixed Star* which is near it, are not the real, but apparent ones.

By the Parallaxes, the Diſtance of a Phænomenon *from the next* fixed Star *is conſtantly variable.*

Let there be a *Phænomenon* or *Star* appearing in the Horizon in C: If it were obſerved from the Center, it would be ſeen in Conjunction with the *fixed Star* E. But the Spectator in A will ſee it in the ſame Line with the *Star* D, and will be diſtant from the *Star* E by the Arch DE. But as it riſes higher into M, it will ſtill appear, from the Center of the *Earth*, in Conjunction with the ſame *Star* E, which then will appear in P. But from the Surface of the *Earth* in A, it will appear in N, nearer to the *Star* E than it was when in the Horizon; and therefore will not appear in Conjunction with the ſame *Star* D, as it did before; but will be diſtant from it by the Arch N *d*, making the Arch P *d* equal to ED. Hence it follows, that if any *Phænomenon* always keeps the ſame Poſition in reſpect of the *fixed Stars*, and changes not its arcual Diſtances from them, it has

no

no ſenſible Parallax. But even likewiſe, if its Diſtance from the *Stars* be changed, yet if that Change be only ſo much as ariſes from its proper Motion, in that Caſe likewiſe it will have no ſenſible Parallax. But if any *Phænomenon* departs further from a *fixed Star*, or comes nearer to it, than what it would do by its proper Motion, this Difference of Acceſs or Receſs is the Effect of a Parallax.

Lecture XXI.

THE Parallax of a *Star* in a vertical Circle changes its Place, in regard to the other Circles of the Sphere; and makes its viſible Longitude, Latitude and Right Aſcenſion, to be different from the true ones, which are ſeen from the Center: And from hence ariſe four other Kinds of Parallaxes.

The Kinds of Parallaxes.

LET HO be the Horizon, whoſe Pole is V, EQ the Ecliptick, and its Pole P, VA a vertical Circle paſſing thro' the *Star* whoſe true Place is C, but apparent Place D, in the ſame Vertical but nearer to the Horizon; ſo that the Parallax of Altitude is DC. Thro' the Pole of the Ecliptick and the *Star* draw the Circle of Latitude PCG, end G will be the true Place of the *Star* reduced to the Ecliptick. But a Circle of Latitude, thro' the apparent Place D, will meet with the Ecliptick in H, which will be the viſible or apparent Place of the *Star* in the Ecliptick: The Arch of the Ecliptick GH, intercepted between the two Circles of Latitude paſſing thro' the true and apparent Place, is called the Parallax of Longitude, and CN the Parallax of Latitude.

PlateXIX. Fig. 2.

The Parallax of Longitude.

IF the *Star* be in the vertical Circle, which cuts the Ecliptick in the 90th Degree from the Horizon; *i. e.* in that Vertical which cuts the Ecliptick at Right Angles; as for Example, in the Point *c* of the Circle VE, the Parallax of Longitude will be nothing. For becauſe the vertical Circle VE, is in this Caſe, perpendicular to the Ecliptick, it will paſs thro' its Poles, and will be the ſame Circle of Latitude in which is the true and apparent Place of the *Star*; and both theſe Places reduced to the Ecliptick will coincide in the ſame Point: And here

Lecture XXI.

here the Parallax of Latitude will be the same with the Parallax of Altitude.

The Parallax of Latitude.

The Eastern Quadrant of the Ecliptick is that, which lies between the 90th Degree, and the Point of it that rises: The Western Quadrant lies between the 90th Degree, and the setting Point thereof. A *Star*, that is in the Eastern Quadrant, has its apparent Longitude greater than its true Longitude; for, while the *Star* rises, the Parallax depresses it towards the *East*. So in the *Fig.* the visible Place of the *Star* in the Ecliptick is the Point H, which is more Easterly than the true Place G: But if the *Star* be in the Western Quadrant, its visible Longitude is less than the true, because the Parallax thrusts it Westward.

Let the Circle EQ, which before represented the Ecliptick, be now in the Place of the Æquator, and P its Pole, PVH the Meridian, VCA a vertical Circle passing through the *Star*; in which let C be its true Place, and D its apparent, PCG, PDH Secondaries of the Æquator or Circles of Declination passing through the true and apparent Places of the *Star*, meeting with the Æquator in G and H. The Point G shews the true Right Ascension of the *Star*, H its apparent, and the Distance GH is called the Parallax of Right Ascension. The true Declination of the *Star* is GC, and its visible is HD, and their Difference NC is the Parallax of Declination. If a *Star* be to the *East* of the Meridian, the visible Right Ascension is greater than the true; if to the *West* of it, it is less: And when the *Star* culminates, the Parallax of Right Ascension is nothing; because the same Circle of Declination does there pass through both the apparent and true Place.

The Parallax of Right Ascension; Parallax of Declination.

The *Astronomers* have invented several Methods for finding the Parallaxes of *Stars*, that from thence their Distances from the *Earth* may be known; for if we knew this, we could make some Estimate of the Largeness and Amplitude of the Universe. Let us

us now give some of the Methods which the *Astronomers* have contrived for searching out the the Parallaxes. Lecture XXI.

The first way of finding the Parallax. PlateXIX. Fig. 3.

FIRST, observe the *Star*, when it is in the same vertical Circle with two other *fixed Stars*. Let V B be the vertical Circle, in which are seen the *fixed Stars* C and D, and the *Phænomenon* or *Star* S, whose apparent Place will be likewise in the same Vertical, which suppose to be E; and if the *Star* have no proper Motion of its own, it will constantly be in the same Line with the two *Stars*. After some Time, again observe the Position of the *Phænomenon* with the same *Stars*, when it is not in a vertical Circle with them, but rather in a Circle Parallel to the Horizon; *i. e.* Suppose the *fixed Stars* in *c* and *d*, and let the visible Place of the *Star* be *e*; but its true Place is in the Line *c d*, which joins the two Fixed. It is also in the Vertical V *e*; and therefore it must be in the Point where these Lines cut one another; that is, in S. Observe the Distances of the *fixed Stars d* and *c*, and of the *Star e* from the Vertex V: Measure likewise with an Instrument the Arches *d e*, *c e* and *d c*: And, because *e* is the apparent Place of the *Star*, and S its true Place, the Arch *e* S is its Parallax. In the Triangle *d* V *c* we have all the Sides; wherefore we can find the Angle V *d c*. Again, in the Triangle V *d e*, we have all the Sides; therefore we can find the Angle *d* V *e*. Lastly, in the Triangle *d* V S we have the Sides *d* V, and the Angles *d* V S and V *d* S, which we found before; therefore we can find the Side V S; which being subtracted from V *e*, known by Observation, leaves S *e* for the Parallax, which was to be found out.

A Second Method. PlateXIX. Fig. 4.

THE Parallax of a *Star* may be likewise easily found this Way. Observe when the *Phænomenon* is in any Vertical with a *fixed Star* which is near it, and then measure its apparent Distance from this *Star*: Then observe again, when the *Phænomenon* and *fixed Star* are in equal Altitudes from the Horizon;

and

Lecture XXI.

and then again measure their Distance. The Difference of these Distances will be very near the Parallax of the *Star*. For let HO be the Horizon, V the Vertex, VE a vertical Circle passing through the *Star* in E, and the *fixed Star* in D. Let the true Place of the *Phænomenon* be S, so that SE is the Parallax of Altitude, and the Difference of the *Star's* and the *Phænomenon's* Height is, in this Case, their visible Distance. Afterwards, observe when the *Star* and the *Phænomenon* comes to be equally distant from the Horizon, and then measure their Distance by an Instrument: This visible Distance is nearly equal to their true Distance. For let the true Place of the *Phænomenon* be *s*; the Parallax *s e* is very small in Comparison of the Arch V*e*: And therefore *d s* and *d e* will be very near equal; for if the Parallax *s e* were a whole Degree, yet even then *d s* and *d e* would not differ above one Minute: Therefore the Distance *d e* being measured, we shall have their true Distance *d s* or DS, which is greater than DE. And if from DS we subtract DE, known by the first Observation, there will remain SE, the Parallax, which was to be found.

The Third Method.

THE Parallax of a *Phænomenon* may likewise be obtained by an Observation of its Azimuth and Altitude, and by marking the Time between the Observation and its Arrival at the Meridian. Let HVPO be the Meridian, V the Vertex, P the Pole, HO the Horizon, and VB a vertical Circle, passing thro' the true and apparent Place; thro' which draw also Circles of Declination PSC, PE. The Arch of the Horizon BO is the Azimuth of the *Star*; which must be observed, in the Manner we shewed in our last *Lecture*. Observe likewise the Arch VE, the Distance of the *Phænomenon* from the Vertex, and mark the Moment of Time when these Observations are made. Then stay till the *Phænomenon* or *Star* comes to the Meridian, and note the Moment of its Arrival there; which may be either done by a Pendulum Clock, or by an Observation of a *Star*.

Plate XIX. Fig. 5.

Let

Let the Diſtance of the Time between the firſt Obſervation and the ſecond of the *Star*'s being in the Meridian, be converted into Degrees and Minutes of the Æquator, and we ſhall have the Arch Æ C, which meaſures the Angle V P S. Therefore, in the Triangle V P S, we have the Side V P, the Diſtance of the Vertex from the Pole, and the Angles P V S and V P S; whereby we can find the Arch V S, the true Diſtance of the *Phænomenon* from the Vertex: This being ſubtracted from the obſerved Diſtance V E, there will remain the Parallax S E, which was to be found. Lecture XXI.

It is here to be noted, that to reduce the Time into Degrees and Minutes of the Æquator, the Time muſt be firſt reduced into Hours and Minutes of the *Primum Mobile*, or to the Time of the Revolution of the Heavens, which Hours are ſomewhat ſhorter than the Solar Hours: Or if you keep the Solar Hours, you muſt reckon for each of them 15 Degrees, 2 Minutes, 27 Seconds, and 51 Thirds: And ſo proportionably for the reſt of the Particles of Time.

A Fourth Method.

Suppose H O an Arch of the Horizon, A M the Meridian, in which P is the Pole, and V the Vertex of the Place. Suppoſe E the apparent Place of the *Star*; before the *Star* comes to the Meridian, obſerve the Arch V E, its Diſtance from the Vertex, and its Azimuth E V M: Let the true Place of the *Star* be S, its Parallax is S E: Mark the Time of the Obſervation. Again, after the *Star* has paſſed the Meridian, obſerve when it is exactly at the ſame Diſtances from the Vertex, ſo that V *e* may be equal to V E: And here, ſince the viſible Diſtances of the *Star* from the Vertex are equal, the real Diſtances will be likewiſe equal, *i. e.* V S = V *s*. Take the Time between the firſt and ſecond Obſervation, and turn it into Degrees and Minutes of the Æquator, and we ſhall have the Angle S P *s*, the Half of which is the Angle S P V. Therefore, in the Triangle S V P, we have the Angle S P V, and the Angle S V P, which is the Complement of the Azi- Plate XIX. Fig. 6.

Lecture XXI.

muth to two Right Angles; alſo the Side V P, the Diſtance of the Pole and Vertex; from them we ſhall know V S, the true Diſtance of the *Star* from the Vertex; which being ſubtracted from V E, leaves S E for the Parallax.

The Fifth Method.

THESE Practices depend upon Obſervations of the Azimuths; but without obſerving them the Parallax may be known, by finding out the apparent and true Right Aſcenſions; and from them, by Calculation, finding the Azimuth: For by obſerving the Diſtance of a *Phænomenon* from two known *fixed Stars*, we can compute its apparent Right Aſcenſion, according to the Method explained in *Lecture* XIX: Then again, when the *Star* comes to the Meridian, by the ſame Method find its Right Aſcenſion; which is the true Right Aſcenſion, or the Point where the Circle of Declination, paſſing thro' the true Place of the *Star*, cuts the Æquator. Knowing then the apparent Right Aſcenſion of the *Star* in the Vertical V B, and the Point of the Æquator which at the ſame Time culminates, we ſhall likewiſe know the Angle V P E: Therefore, in the Triangle V P E, having the Sides V P, V E, and the Angle V P E, we can find the Angle P V E, which determines the Azimuth. But having the true Right Aſcenſion of the *Star* as was obſerved in the Meridian, and the Point of the Æquator culminating at the firſt Obſervation, the Diſtance between them will give us the Angle V P S. Therefore, in the Triangle V P S, having the two Angles V P S and S V P, as alſo the Side V P, we can find the Side V S, the real Diſtance of the *Star* from the Vertex; which ſubtracted from V E, leaves S E, the Parallax, which was to be found.

Plate XX. Fig. 1.

IN determining the Right Aſcenſions of the *Stars*, we are not to rely too much, in ſo nice an Affair as the Parallax is, on a Pendulum Clock for determining the Time; for there the Error of one Second in numbering being made, will produce an Error of Right Aſcenſion of 15 Seconds.

FOR,

For, to obferve the Right-Afcenfion of a *Star*, there is no need of ftaying till it come to a Meridian; but it is more eafily and certainly had by two Obfervations, one made in the Eaftern Quadrant, and the other in the Weftern Side of the Heavens; but both muft be made when the *Star* is at the fame Height above the Horizon: For if we take the Diftance of the *Phænomenon* from two known *fixed Stars*, when it is in the Eaftern Region, we fhall by that Means find its apparent Right Afcenfion, which is greater than the true, becaufe the Parallax depreffes a *Star* more Eaftern. Again, when the *Star* defcends on the Weftern Side, and comes to the fame Height, let its Diftance be likewife obferved from two *fixed Stars*, and get from them its apparent Right Afcenfion, which is juft as much lefs than the true, as the former exceeded it. And therefore, if the Differences between the two apparent Right Afcenfions be halved, and this Half be added to the leaft or fubtracted from the greateft, we fhall have the true Right Afcenfion, or the Point in the Æquator where it meets with the Circle of Declination paffing through the *Star*, that is, the Point C. But from the Time of the Obfervation, we have the Point of the Æquator which culminated at that Moment; and confequently we have the Arch ÆC, and the Angle ÆPC meafured by it: Therefore, in the Triangle VPS, having the Side VP, and the Angles VPS and PVS, we can find the Side VS, the true Diftance of the *Phænomenon* from the Vertex; which fubducted from the apparent Diftance, there will remain SE, the Parallax required.

Lecture XXI.

Table XIX. Fig. 5.

The Sixth Method.

The eafieft and beft Way of finding the Parallax of Right Afcenfion is by a Telefcope, in whofe *Focus* are four Threads, croffing one another at half Right Angles, as we fhewed in our XIXth *Lecture*. Directing this Telefcope to the *Star*, turn it conftantly round, till its apparent diurnal Motion appear to be along the Thread AB; in which Pofition, the Thread will reprefent a Portion of the Parallel which the *Phænomenon* defcribes; and the Thread CD, cutting

Table XX. Fig. 2.

Lecture XXI.

cutting it at Right Angles, will represent a Horary Circle. Observe therefore the Time when the *Phænomenon* is seen in the horary Circle. The Telescope remaining thus fixed and unmoved, observe the Time when any other *Star*, whose Right Ascension is known, comes to the same Horary Circle: The Distance of Time between the Appulse of the *Phænomenon* to the Horary Circle, and of the *fixed Star* to the same Circle, being turned into Degrees and Minutes of the Æquator, will shew the Difference of Right Ascensions of the *Star* and *Phænomenon*. Again, when the *Star* comes to the Meridian observe it with the Telescope, and by the same Method find out its Right Ascension, which will be the true one; and by it we shall have the Point of the Æquator, where the Circle of Declination, passing thro' the true Place of the *Star*, does cut the Æquator. Having therefore the true Right Ascension and the apparent, we have their Difference, or the Parallax of Right Ascension. And because we have the apparent Right Ascension and the Point of the Æquator then culminating, we have the Arch of the Æquator, intercepted between them, which is the Measure of the Angle V P E. Therefore, in the Triangle V P E, we have the Sides V P, P E, and the Angle V P E, whence we find the Angle P V E. From the Angle V P E take the Angle S P E, the Parallax of Right Ascension, and we shall have the Angle V P S. Lastly in the Triangle V P S, having the Angles V P S and P V S, together with the Side V P, we can from them find the Side V S, the true Distance of the *Star* from the Vertex; which being subducted from the apparent Distance, leaves the Parallax that was to be found.

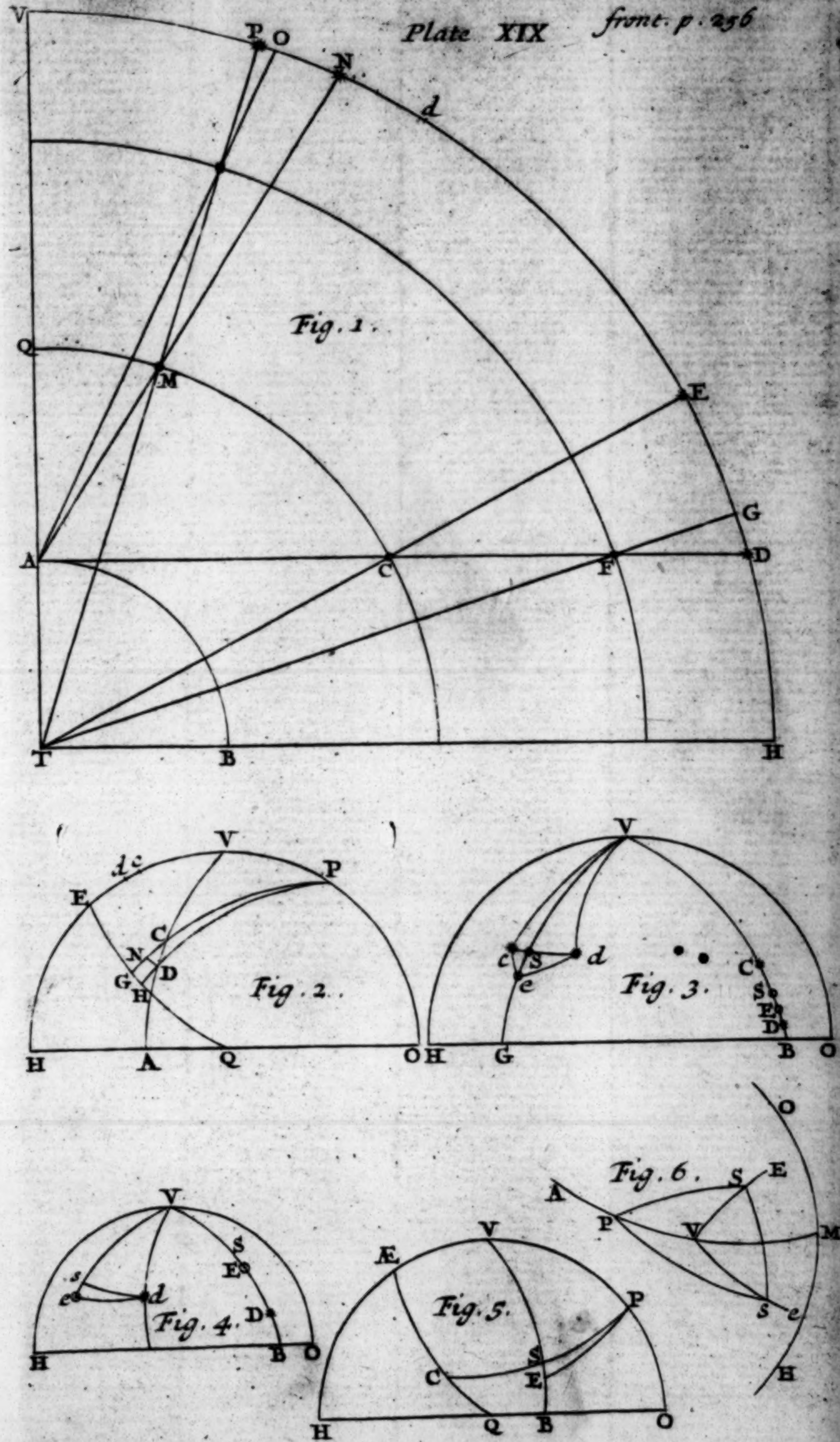
TableXX. Fig. 1.

The Method of finding the Parallax when the Star *has a proper Motion of its own.*

If the *Phænomenon* have a proper Motion of its own, its Right Ascension will constantly be changed by this Motion, unless it should happen to move in some Circle of Declination: And therefore, Care must be taken to determine the Change of Right Ascension, that arises by the Motion of the *Star*; which is done, by observing the

21

Plate XIX front. p. 256



the right Aſcenſion of the *Star*, when it is in the Meridian; and the next Day let its Right Aſcenſion be in the ſame Manner obſerved. The Difference of theſe Right Aſcenſions will ſhew the Change that the Right Aſcenſion has, for the Time between the two Obſervations: And from them we can find the Change of Right Aſcenſion, or the Motion of the *Phænomenon* along the Æquator in a Day: From this diurnal Motion, we can find by proportioning the Motion for any given Time, for Example, if the diurnal Motion according to the Æquator be 30 Min. that is, ſuppoſe the *Phænomenon* advanced according to the Æquator every Day 30 Minutes; and ſuppoſe the Time between the Obſervation on the Eaſtern Side of the Heaven, and that in the Meridian be ſix Hours, the Motion according to the Æquator in that Time is $7\frac{1}{2}$ Minutes; let the Difference of Right Aſcenſion, obſerved in the Vertical and in the Meridian, be 20 Minutes, ſeven and a half of thoſe Minutes are owing to the proper Motion of the Body; wherefore the Parallax of Right Aſcenſion is $12\frac{1}{2}$ Minutes.

Lecture XXI.

AFTER the ſame Manner, by the Apparent, and real Longitude of a *Phænomenon*, the Parallaxes may be inveſtigated. The apparent Longitude is found, by obſerving the Diſtance of a *Phænomenon* from two fixed *Stars*, whoſe Longitudes and Latitudes are known. And the true Longitude is had, by making the ſame Obſervations, when the *Star* is in the 90th Degree of the Ecliptick, where the apparent and true Longitudes coincide.

BY theſe, and the like Methods, if any *Phænomenon* has a Parallax not leſs than one Minute, it may be found out. In the *Moon* we find the Parallax very conſiderable, which in the Horizon amounts to about a Degree or more. But there are ſome particular Methods, only applicable to the *Moon*, by which its Parallax is known.

A Method for obſerving the Parallax of the Moon.

IN an Eclipſe of the *Moon*, obſerve when both its Horns are in the ſame vertical Circle, and then in that Moment take the Altitudes of both Horns:

Lecture XXI. The Difference of these two Altitudes being halved and added to the least, or subtracted from the greatest, does give nearly the visible Altitude of the *Moon's* Center: But the true Altitude is nearly equal to the Altitude of the Center of the Shadow at that Time. Now we know the Altitude of the Center of the Shadow, because we know the Place of the *Sun* in the Ecliptick, and its Depression under the Horizon which is equal to the Altitude of the opposite Point of the Ecliptick, in which is the Center of the Shadow. And therefore we have the true Altitude of the *Moon*, and the apparent Altitude, whose Difference is the Parallax, which will therefore be known.

Tables of the Moon's *Parallaxes.*

As the Distance of the *Moon* grows less, according as it recedes from its *Apogæum*, her Parallax must in the same Proportion be increased constantly, the nearer she comes to us. Therefore the Artists make Tables, which shew the horizontal Parallax for every Degree of its Anomaly.

The Parallax of the Sun *not to be found by these Methods.*

THE Methods we have given for finding the Parallaxes, shew, that the *Moon* has a great Parallax, and is very near us; but none of them is sufficient for finding out the Parallax of the *Sun*: For that is so small, that the Observations requisite cannot be made accurately enough to determine it; for an Error in observing can scarcely be avoided, which is not equal or greater than the *Sun's* Parallax. This Defect of Observations put the antient *Astronomers* on the Search of other Methods peculiar to the *Sun*, for finding out its Parallax: But even those Methods, though they make manifest the Acuteness and Sagacity of the Antients, yet are not sufficient in so nice and subtle a Disquisition. However, they are useful to shew, that the Distance of the *Sun* from the *Earth* is very great in Comparison of the *Moon's* Distance from the same: And therefore it will not be unfitting to explain them in this Place.

Hipparchus's *Method for the Parallax of the* Sun.

THE first Method was invented by *Hipparchus*, and has been made use of by *Ptolemy* and his Followers, and many other *Astronomers*. It depends on

an Observation of an Eclipse of the *Moon*: And the Principles on which it is founded are, *First*, In a Lunar Eclipse the horizontal Parallax of the *Sun* is equal to the Difference between the apparent Semidiameter of the *Sun*, and half the Angle of the Conical Shadow; which is easily made out in this Manner. Let the Circle A F G represent the *Sun*, and D H E the *Earth*; let D H M be the Shadow, and D M C the half Angle of the Cone. Draw from the Center of the *Sun* the right Line S D touching the *Earth*, and the Angle D S C is the apparent Semidiameter of the *Earth*, seen from the *Sun*, which is equal to the horizontal Parallax of the *Sun*; and the Angle A D S is the apparent Semidiameter of the *Sun*, seen from the *Earth*: The external Angle A D S is equal to the two Internals D M S and D S M, by the 32d *Prod*. Elem. I. And therefore the Angle D S M, or D S C, is equal to the Difference of the Angles A D S and D M S. 2*dly*, Half the Angle of the Cone is equal to the Difference of the horizontal Parallax of the *Moon*, and the apparent Semidiameter of the Shadow, seen from the *Earth* at the Distance of the *Moon*. For let C D E be the *Earth*, C M E the Shadow, which at the Distance of the *Moon* being cut by a Plane, the Section will be the Circle F L H, whose Semidiameter is F G, and is seen from the Center of the *Earth* under the Angle F T G. But by the 32d *Prop*. Elem. I. the Angle C F T is equal to the two Internals F M T and F T M. Wherefore the Angle F M T is the Difference of the two Angles C F T and G T F: But the Angle C F T is the Angle under which the Semidiameter of the *Earth* is seen from the *Moon*, and this is equal to the horizontal Parallax of the *Moon*; and the Angle G T F is the apparent Semidiameter of the Shadow seen from the *Earth*'s Center. It is therefore evident that the half Angle of the Cone is equal to the Difference of the horizontal Parallax of the *Moon*, and the apparent Semidiameter of the Shadow seen from the *Earth*. Wherefore, if to the apparent Semidiameter of the *Sun* there be

Lecture XXI.

Plate XX. Fig. 3.

Plate XX. Fig. 4.

 added

Lecture XXI.

added the apparent Semidiameter of the Shadow, and from the Sum you take away the horizontal Parallax of the *Moon*, there will remain the horizontal Parallax of the *Sun*; which therefore, if these were accurately known, would be likewise known accurately: But none of them can be so exactly and nicely obtained, as to be sufficient for determining the Parallax of the *Sun*; for very small Errors, which cannot be easily avoided in measuring these Angles, will produce very great Errors in the Parallax; and there will be a prodigious Difference in the Distances of the *Sun*, when drawn from these Parallaxes. For Example: Suppose the horizontal Parallax of the *Moon* to be 60′ 15″, the Semidiameter of the *Sun* 16 and the Semidiameter of the Shadow 44′ 30″; we should conclude from thence, that the Parallax of the *Sun* was 15″, and his Distance from the *Earth* about 13700 Semidiameters of the *Earth*. But if there be an Error committed, in determining the Semidiameter of the Shadow, of 12″ in Defect, (and certainly the Semidiameter of the Shadow cannot be had so percisely, as not to be liable to such an Error) that is, if instead of 44′ 30″ we put 44′ 18″ for the apparent Diameter of the Shadow, all the others remaining as before, we shall have the Parallax of the *Sun* 3″, and its Distance from the *Earth* almost 70000 Semidiameters of the *Earth*, which is five Times more than what it was by the first Position. But if the Fault were in Excess, or the Diameter of the Shadow exceeded the true by 12″; so that we should put in 44′ 42″, the Parallax would arise to 27″, and the Distance of the *Sun* only 7700 of the *Earth*'s Semidiameters; which is nine Times less than what it comes to by a like Error in Defect. If an Error in Defect was committed of 15″, which is still but a small Mistake, the *Sun*'s Parallax would be equal to nothing, and his Distance infinite. Wherefore, since from so small Mistakes the Parallax and Distance of the *Sun* vary so much, it is plain that the Distance of the *Sun* cannot be obtained by this Method.

Hipparchus's *Method insufficient.*

SINCE

SINCE therefore the Angle that the *Earth's* Semidiameter subtends at the *Sun*, is so small that it cannot be determined by any Observation; *Aristarchus Samius*, an ancient and great *Philosopher* and *Astronomer*, contrived a very ingenious Way for finding the Angle which the Semidiameter of the *Moon's* Orbit subtends when seen from the *Sun*: This Angle is about sixty times bigger than the former, subtended only by the *Earth's* Semidiameter. To find this Angle he lays down the following Principles.

Lecture XXI.

Aristarchus's *Method for the* Sun's *Distance*,

IN that *Lecture* where we explained the *Phases* of the *Moon*, we shewed that if a Plane passed through the *Moon's* Center, to which the Line joining the *Sun* and *Moon's* Center was perpendicular, this Plane would divide the illuminated Hemisphere of the *Moon* from the dark one: And therefore, if this Plane should likewise pass through the Eye of a Spectator on the *Earth*, the *Moon* would appear bisected, or like a half Circle; and a right Line, drawn from the *Earth* to the Center of the *Moon*, would be in the Plane of Illumination, and consequently would be perpendicular to the right Line which joins the Centers of the *Sun* and *Moon*. Let S be the *Sun*, and T the *Earth*, A L *q* a Quadrant of the *Moon's* Orbit; and let the Line S L, drawn from the *Sun*, touch the Orbit of the *Moon* in L; the Angle T L S will be a right Angle: And therefore when the *Moon* is seen in L, it will appear bisected, or just half a Circle. At the same Time take the Angle L T S the Elongation of the *Moon* from the *Sun*, and then we shall have the Angle L S T its Complement to a right Angle. But we have the Side T L, by which we can find the Side S T, the Distance of the *Sun* from the *Earth*.

TableXX. Fig. 5.

BUT the Difficult Point is to determine exactly the Moment of Time when the *Moon* is bisected, or in its true *Dichotomy*; for there is a considerable Space of Time both before and after the *Dichotomy*, nay, even in the Quadrature, when the *Moon* will appear bisected, or half a Circle; so that

Aristarchus's *Method insufficient*.

Lecture XXI. that the exact Moment of Bisection cannot be known by Observation, as Experience tells us: This can be also made out by the following Reason. *Pag*. 95. in the *Lecture* concerning the *Phases* of the *Moon*, it was demonstrated, that the Diameter of the *Moon* was to the Portion of it illustrated by the *Sun*, and seen by us, as the Diameter of a Circle is to the *Versed Sine* of the Elongation of the *Moon* from the *Sun*, nearly: But accurately, it is as the Diameter of a Circle to the *Versed Sine* of the exterior Angle at the *Moon*, of the Triangle formed by Lines joining the Centers of the *Sun*, *Earth*, and *Moon*, as we shewed in the *Lecture*, *pag*. 163. concerning the *Phases* of *Venus*. Let us suppose, in the Time of true *Dichotomy* or Bisection, that the Angle L S T is 15′, and that the Semidiameter of the Lunar Orbit were 60 Semidiameters of the *Earth*; the Distance of the *Sun* would in that Case be 13758 of the *Earth*'s Semidiameters. This being supposed, let us imagine the *Moon* to be in her Quadrature at *q*; that is, let the Angle *q* T S be a right Angle, the exterior Angle of the Triangle *q* T S at *q* would be 90°. 15′, whose *Versed Sine* is equal to the Radius and the right Sine of 15′ together: Therefore, as the Diameter of a Circle is to the Radius + Sine 15′; so is the Diameter of the *Moon* to that Part of it which is illustrated by the *Sun*, and seen from the *Earth*. Wherefore, taking half the Antecedents, and by Division of Ratio, the Radius will be to the right Sine of 15′, as is the Semidiameter of the *Moon* to that Excess, wherewith the illuminated Part seen from the *Earth* is greater than half the *Moon*'s Diameter. Now the Sine of 15′ is 436, of such Parts as the Radius is 100000, and the apparent Semidiameter of the *Moon* is about 15′: Say therefore, as 100000 is to 436, so is 15′ to a Fourth, which is less than 4″; but this is so small a Quantity, that it is not in the least to be perceived by our Senses: And therefore the *Moon*, even in the Quadratures, has its Illumination exceeding the bisected Illumination by such a Quantity as is altogether

gether imperceptible: But if the real *Dichotomy* or Bisection were in the Quadrature, the Distance of the *Sun* would be Infinite; for in that Case the Angles ST*q* and S*q*T being right, the Lines ST and S*q* would be parallel, and would not meet but at an infinite Distance.

2*dly*, SUPPOSE the Elongation of the *Moon* from the *Sun* 89°, 30′: In that Case the exterior Angle at the *Moon* is 89°. 45′, and its *Versed Sine* equal to the Radius bating the right Sine of 15′. And because as the Radius is to the *Versed Sine* of the exterior Angle, that is, to the Radius diminished by the Sine of 15′; so is the Semidiameter of the *Moon* to that Part of it which is illustrated and seen by us; then, by Division of *Ratio*, the Radius will be to the Sine of 15′; as is the *Moon*'s Semidiameter to that Part whereby the Semidiameter of the *Moon* is greater than the illuminated Part thereof which is seen by us; which therefore (as in the former Case) will be scarce 4″: Now the *Moon* wanting but so small a Portion to be intirely bisected, will appear to us as if she were really bisected; so that its *Phasis* can in no wise be distinguished from the true *Phasis* of a *Dichotomy*: And therefore, if this apparent *Phasis* should be taken for the true *Phasis* of the *Dichotomy*, which is half a Degree distant from the Quadrature, we should find the Distance of the *Sun* from us to be 6876 Semidiameters of the *Earth*.

OBSERVATIONS inform us, that when the *Moon* is 30 Min. distant from the Quadratures it appears bisected; and in the Quadrature its *Phasis* cannot be perceived to be different from a bisected *Phasis*: Nay, the *Moon*, observed with the best Telescopes, after it has past the Quadratures, appears bisected, as *Ricciolus* himself acknowledges in his *Almægest*, p. 734. And therefore the *Moon*, at least for the Space of one Hour, appears to be bisected, in which Time any Moment may be taken for the true Point of the *Dichotomy*, as well as any other: And for the infinite Number of Moments of Time, we shall have an infinite

Lecture XXI.

infinite Diverſity of Diſtances of the *Sun* from the *Earth:* And conſequently, the true Diſtance of the *Sun* from the *Earth* cannot be obtained by this Method.

SINCE the Moment in which the true *Dichotomy* happens is uncertain, but it is certain that it happens before the Quadrature; *Ricciolus* takes that Point of Time which is in the Middle, between the Time that the *Phaſis* begins to be doubtful whether it be biſected or not, and the Time of Quadrature: But he had done righter, if he had taken the middle Point between the Time when it becomes doubtful whether the *Moon*'s Side is concave or ſtraight, and the Time again when it is doubtful whether it is ſtraight or convex; which Point of Time is after the Quadrature: And if he had done this, he would have found the *Sun*'s Diſtance a great deal bigger than he has made it.

There is no need to confine this Method to the Phaſis *of a Biſection.*

THERE is no need to confine this Method to the *Phaſis* of a *Dichotomy* or Biſection, for it can be as well perform'd when the *Moon* has any other *Phaſis* bigger or leſs than a *Dichotomy:* For obſerve by a very good Teleſcope, with a Micrometer, the *Phaſis* of the *Moon*, that is, the Proportion of the illuminated Part of the Diameter to the Whole; and at the ſame Moment of Time take her Elongation from the *Sun:* The illuſtrated Part of the Diameter, if it be leſs than the Semidiameter, is to be ſubducted from the Semidiameter; but if it be greater, the Semidiameter is to be ſubducted from it, and mark the Reſidue: Then ſay, as the Semidiameter of the *Moon* is to the Reſidue, ſo is the Radius to the Sine of an Angle, which is therefore found: This Angle added to, or ſubtracted from a right Angle, gives the exterior Angle of the Triangle at the *Moon:* But we have the Angle at the *Earth*, which is the Elongation obſerved; which therefore being ſubducted from the exterior Angle, leaves the Angle at the *Sun*. And in the Triangle S L T, having all the Angles and one Side L T, we can find the other Side S T, the Diſtance of the *Sun* from

from the *Earth*. But it is almoſt impoſſible to determine accurately the Quantity of the Lunar *Phaſis*, ſo that there may not be an Error of a few Seconds committed; and conſequently, we cannot by this Method find preciſely enough the true Diſtance of the *Sun*. However, from ſuch Obſervations, we are ſure, that the *Sun* is above 7000 Semidiameters of the *Earth* diſtant from us. Since therefore the true Diſtance of the *Sun* can neither be found by Eclipſes, nor by the *Phaſes* of the *Moon*, the *Aſtronomers* are forced to have Recourſe to the Parallaxes of the Planets that are next to us, as *Mars* and *Venus*, that are ſometimes much nearer to us than the *Sun* is: Their Parallaxes they endeavour to find by ſome of the Methods above explained: And if theſe Parallaxes were known, then the Parallax and Diſtance of the *Sun*, which cannot directly by any Obſervations be attained, would eaſily be deduced from them. For from the Theory of the Motions of the *Earth* and Planets, we know at any Time the Proportion of the Diſtances of the *Sun* and Planets from us; and the horizontal Parallaxes are in a reciprocal Proportion to theſe Diſtances. Wherefore, knowing the Parallax of a Planet, we may from thence find the Parallax of the *Sun*.

Lecture XXI.

The Sun's *Diſtance and Parallax may be deduced from the Parallax of the Planets.*

MARS, when he is in an Acronychal Poſition, that is, oppoſite to the *Sun*, is twice as near to us as the *Sun* is; and therefore his Parallax will be twice as great. But *Venus*, when ſhe is in her inferior Conjunction with the *Sun*, is four Times nearer to us than he is, and her Parallax is greater in the ſame Proportion: Therefore, tho' the extreme Smallneſs of the *Sun*'s Parallax renders it unobſervable by our Senſes, yet the Parallaxes of *Mars* or *Venus*, which are twice or four Times greater, may become ſenſible. The Aſtronomers have beſtowed much Pains in finding out the Parallax of *Mars*; but of late *Mars* was in his Oppoſition to the *Sun*, and alſo in his *Perihelion*, and conſequently, in his neareſt Approach to the *Earth*: And then he was moſt accu-

Particularly by Mars *in an Acronychal Poſition.*

rately

Lecture XXI.

rately observed by two of the most eminent *Astronomers* of our Age, who have determined his Parallax to have been scarce 30 Seconds; from whence we can easily collect, that the Parallax of the *Sun* is scarce 11 Seconds, and his Distance about 19000 Semidiameters of the *Earth*.

The Parallax of the Sun *found by observing* Venus *in the Body of the* Sun.

By an Observation of the Body of *Venus*, seen passing over the Body of the *Sun*, which will happen in *May* 1761, Dr. *Halley* has shewed a Method of finding the Parallax and Distance of the *Sun* to a great Nicety, *viz.* within a five hundredth Part of the whole; and we must wait till then, before it can be determin'd to so great an Exactness.

Because the Method whereby the *Astronomers* foretel Eclipses of the *Sun*, requires that the *Moon's* Parallax both as to Longitude and Latitude should be known by Calculation: And also, as often as the *Moon's* Place in the Heavens is to be observed, that it may be compared to the Place found by *Astronomical* Tables, in order to establish her Theory; it will be necessary to reduce the true Place found by the Tables to her apparent Place, which cannot be done without the Calculation of the Parallax: It will be convenient to explain the Method by which the *Moon's* Parallax for any Point of Time is to be calculated.

How the Moon's *Parallax is to be found for any Time by Calculation.* Table XX. Fig. 6.

First, by *Astronomical* Tables compute the Place of the *Moon* in the Ecliptick and her Latitude, for the given Time. In the Figure suppose HO the Horizon, HZO the Meridian, Z the Vertex, EC the Ecliptick, in which let L be the Place of the *Moon*, found by the Tables. And first, let us suppose the *Moon* to be without Latitude. From the Vertex Z let fall upon the Ecliptick the Perpendicular Z *n* A, which will be therefore a Circle of Latitude, and the Point *n* will be the 90th Degree of the Ecliptick. From the Time given we have the Right Ascension of the *Sun*, and his Equatorial Distance from the Meridian: From thence we shall find the Point of the Æquator culminating, which

is

is the Right Aſcenſion of Mid-Heaven, or of that Point of the Ecliptick which culminates: And therefore we know that Point which is then in the Meridian, as alſo the Angle Z E *n* of the Ecliptick and the Meridian. This is either found by the Calculation we explained in the ſpherical Doctrine, or by Tables of *Aſtronomy:* By this Means we find the Arch E L; but we have the Arch E Æ the Declination of the Point E, and conſequently the Arch Z E will be known. Therefore in the right-angled Triangle Z *n* E, we have the Side Z E, and the Angle Z E *n*. Hence we can find E *n* and the Point *n* or the Point of the 90th Degree, and the Arch Z *n*, its Diſtance from the Vertex; whoſe Complement *n* A is the Meaſure of the Angle that the Horizon and the Ecliptick make: And becauſe we have the Place of the *Moon*, we muſt have the Arch *n* L. Therefore in the right angled Triangle Z *n* L, having the Sides Z *n* and *n* L, we ſhall have from them the Angle Z L *n*, which is called the *Parallactick Angle*, as likewiſe the Side Z L, the Diſtance of the *Moon* from the Vertex. Let the Radius be to the Sine of the Arch Z L, as the horizontal Parallax of the *Moon* taken from the Tables to its Parallax in L, which therefore is found. Let it be *o* L. From *o* on the Ecliptick let fall the Perpendicular *o m*. And in the Triangle *o* L *m* (which being very ſmall, may be taken for a rectilinear one) we have beſides the Right Angle, the Side L *o* and the Angle *o* L M equal to Z L *n*; wherefore we ſhall find out the Arch L *m*, the Parallax of Longitude, and *o m*, the Parallax of Latitude, which were to be found.

Lecture XXI.

The Parallactick Angle.

SUPPOSE now the *Moon* has ſome Latitude, and its Place in the Ecliptick be the Point L, but let it be placed in the Circle of Latitude L P at P. And becauſe the Angle *n* L P is right, and we have the Angle *n* L Z, and its Complement Z L P; in the Triangle Z L P we have two Sides, Z L, which was found before, and L P, the *Moon*'s Latitude, and the

Lecture XXI.

the Angle ZLP, whereby we can find out the Side ZP, and the Angle ZPL. Let the Radius be to the Sine of the Arch ZP, as the horizontal Parallax of the *Moon* to a Fourth, which will be P*q*: This will be the Parallax of the *Moon* in the Circle of Altitude. Let *qd* be an Arch parallel to the Ecliptick; and in the ſmall Triangle P*dq*, which may be taken as a right-angled Triangle, we have the Angle *d*P*q*, which is the Complement of the Angle ZPL to two right Angles, and the Side P*q*: Therefore we ſhall have P*d* the Parallax of Latitude, and *qd* the Parallax of Longitude: For becauſe the Latitude of the *Moon* is but ſmall, the Arch of the Parallel *dq* is nearly equal to the Arch of the Ecliptick which is correſpondent to it.

LECTURE XXII.

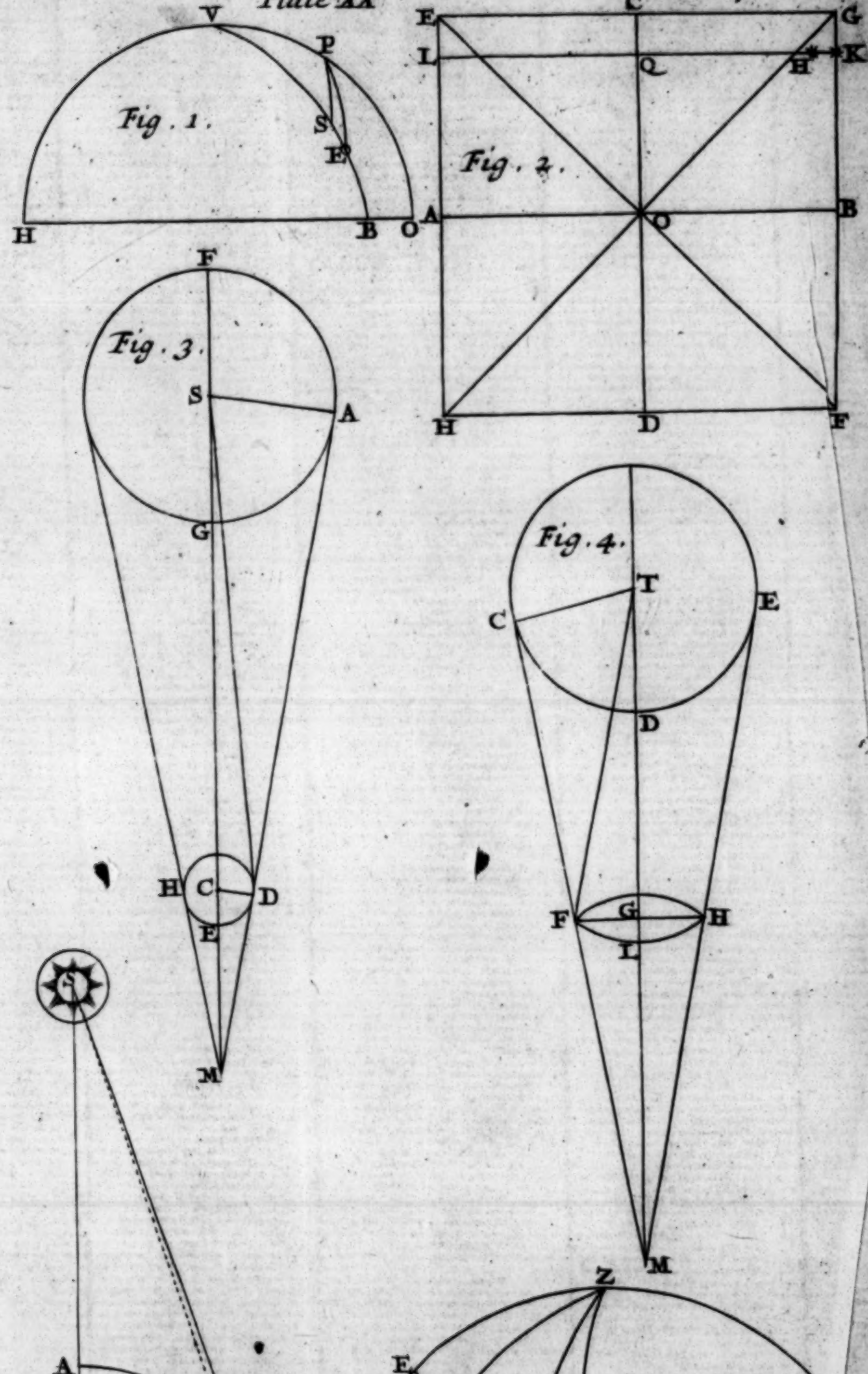
The Theory of the Annual Motion of the EARTH.

The Theories of the Planets founded on the Motion of the Earth.

HITHERTO we have given an Account of the general Affections of the Planets Motions, and have explained the Appearances which ariſe from their Motions and the Motions of the *Earth* together. We will now come to their particular Theories, in which the Period of each, its Diſtance from the *Sun*, the Form of its Orbit, and its Poſition are determined; which

22

Plate XX front. p. 268



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which being once known, the Place of any Planet in the *Zodiack* may be computed for any given Time. And because the Theories of the Planets are founded on the Motion of the *Earth*, and are investigated by this Motion, it will be necessary to begin with the Theory of the *Earth*.

Lecture XXII.

In our VIIth *Lecture* we shewed how the Motion of the *Earth* round the *Sun* was the Cause of the Appearance of the annual Motion of the *Sun* in the Ecliptick; and that the *Sun*, observed from the *Earth* seemed to describe the same Circle in the Heavens, that a Spectator in the *Sun* would see the *Earth* really to move in. But the Place of the *Earth* seen out of the *Sun* is always diametrically opposite to that Point of the Ecliptick in which we on the *Earth* observe the *Sun* to be placed: And therefore when the *Sun* appears to us in ♈, the *Earth* is really in ♎; when he is in ♋, the *Earth* has its Mansion in ♑. And therefore, from the apparent Places of the *Sun*, which we find out by Observation, we can easily determine the Place of the *Earth* in its proper Orbit.

The Motion of the Earth *found by observing the Motion of the* Sun.

Since the Ecliptick cuts the Æquator in two opposite Points, the *Sun* will twice every Year appear in the Æquator or Equinoctial Circle; which happens, when by his apparent Motion, he arrives at the Intersections of these two Circles: All the rest of the Year he will seem to decline either to the *North* or *South* from the Æquator; and he is at his farthest Distance from the Æquator, when he is just in the Middle between the two Sections, that is, 90 Degrees removed from either, and there the *Sun* does not seem to alter his Declination for some Time; and then the Days keep the same Length: And therefore these Points which are the first of ♋ and ♑ are called the *Solstitial Points*, as the Intersection of the Æquator and Ecliptick, are called the *Equinoctial Points*, because when the *Sun* is seen in them, the Days and Nights are equal.

The Sun *is twice every Year in the Equinoctial Circle.*

Since

Lecture XXII.

Days and Nights not equal but when the Sun *enters the Æquator at Mid-day.*

SINCE the *Sun* is seen continually moving in the Ecliptick, and every Day seems to advance a Degree Eastward, he makes no Stay in the Equinoctial Points; but in passing on, in the same Moment he arrives there, he leaves them: And therefore, though the Day the *Sun* enters the Equinoctial Point is called the *Equinox*, because it is reputed equal to the Night; yet it is not precisely so unless the *Sun* enters the Æquator at Mid-day. For if the rising *Sun* should enter the Vernal Equinox, at Setting he will have departed from it, and decline Northwards about the Space of 12′; and therefore, that Day will be somewhat longer than 12 Hours, and the Night shorter; but the Difference is so small that it may be neglected in this Matter.

The Investigation of the Sun's *entring the Æquator.*

THE Moment of Time, in which the *Sun* enters the Æquator, is found out by Observation, and from the Latitude of the Place of the Observer. For in the Equinoctial Day, or near it, with an Instrument exactly divided into Degrees, Minutes, and Parts of Minutes, take the Meridian Altitude of the *Sun:* If it be equal to the Altitude of the Æquator, or to the Complement of the Latitude, the *Sun* is in that very Moment in the Æquator; but if it is not equal, take the Difference and mark it, for it will be the Declination of the *Sun*. Then the next Day again observe the Meridian Altitude of the *Sun*, and gather from thence his Declination. If these two Declinations be of different Kinds, as the one *South* and the other *North*, the Æquinox happens sometime between the two Observations, or if they be both of the same Sort, the *Sun* has either not entered the Æquinoctial, or has past it. And from these two Observations of the *Sun*'s Declination, the Moment of the Equinox is thus investigated.

Table XXI. Fig. 1.

LET C A B be a Portion of the Ecliptick, Æ A Q an Arch of the Æquator, and let their Intersection be in A. Let C Æ be the Declination

tion of the *Sun* at the Time of the firſt Obſerva- tion, ED his Declination in the ſecond Obſerva- tion, the Arch CE will be the Motion of the *Sun* in the Eclíptick for one Day. In the ſpherical Tri- angle AÆC right-angled at Æ, we have the Angle A, which the Æquator and the Ecliptick make, which we ſhewed how to find out in *Lecture* XIX; as alſo CÆ, the Declination of the *Sun*, known by Obſervation, by which we ſhall find the Arch CA. And in the ſame Manner in the Triangle AED the Side AE is found, and thence the Arch CE, which is the Sum or Difference of the Arches CA, AE. Say then, As CE is to CA, ſo is twenty-four Hours to the Time between the firſt Obſervation and the Moment of the Ingreſs of the *Sun* to the Æquinox.

Lecture XXII.

The Quantity of the Tropical Year.

If again, the next Year, the Time of the *Sun*'s Entry into the Æquator be obſerved in the ſame Manner, the Time elapſed between the two In- greſſes is the Space of a *Tropical Year*, or the Time wherein the *Sun*, or rather the *Earth*, compleats its Courſe in the Ecliptick; which is called the *Tropical Year*, becauſe after it is finiſhed, all the Seaſons re- turn again in the ſame Order. But by Obſervations that are made at the Diſtance of a Year, we cannot ſafely rely upon the true Quantity of the Year col- lected from them; for a ſmall Error of one Minute, being conſtantly increaſed and multiplied by the Number of Years, in Proceſs of Time would amount to a prodigious Miſtake in the Place of the *Sun*. Therefore the *Aſtronomers* more accurately determine the Quantity of the Year, by taking the Obſerva- tions of two Equinocties at many Years Diſtance from one another; and dividing the Time between the Obſervations, by the Number of Revolutions the *Sun* has made, the Quotient will ſhew the Time of one Revolution, or nearly the Period of the *Earth* in her Orbit. For by this Means, if there be any Miſtake made in the Obſervation, it will be divided into ſo many Parts, according to the Number of Years,

Lecture XXII.

Years, that it will be insensible for the Space of one Year.

The Anomalistical Year.

The Space of Time belonging to the *Tropical Year*, is by this Means found to consist of 365 Days, 5 Hours, 48 Minutes, and 57 Seconds. This Time is somewhat less than the periodical Time of the *Earth* in her Orbit, which is called the *Anomalistical* or *Periodical Year*. For by reason of the Procession of the Equinocties, which was explained by us in the VIIIth *Lecture*, by which the Points of Intersection do constantly every Year move back 50 Seconds, and as it were, meet the *Sun*; the *Sun* will arrive at the Intersection before he has compleated his Course. Now the Time of the *Earth*'s Period or *Anomalistical Year* is 365 Days, 6 Hours, 9 Minutes and 14 Seconds.

The Motion of the Sun *in the Ecliptick unequal.*

If the Motion of the *Earth* round the *Sun* were equal, that is, if it described equal Angles round the *Sun* in equal Times, the apparent Motion of the *Sun* in the Ecliptick would always be equal, and would proceed each Day in the Ecliptick $59' \, 8''$: And therefore the Place of the *Sun* would easily be computed for any Time. But we are sure by Observation, that the apparent Motion of the *Sun* is not equal, and that he goes thro' some Parts of the Ecliptick quicker than thro' others; and particularly in going thro' the Northern Semicircle of the Ecliptick, he spends near eight Days more than in passing over the Southern Semicircle; which ought to be performed precisely in the same Time, if the apparent Motion of the *Sun* were equal. Moreover, if we make Observations, and from them find out the Motion of the *Sun* in the Ecliptick for each Day; in some Days he will be found to move thro' the Space of 61 Minutes in a Day; at other Times he will scarcely be seen to have compleated 57 Minutes.

The daily Motion of the Sun.

The daily Motion of the *Sun* in the Ecliptick is observed in this Manner. Let CB represent the Ecliptick, ÆQ the Æquator, whose common Intersection is in A: Take with an Instrument the Meridian Altitude of the *Sun*; the Altitude of the

the Æquator in the Place of Obſervation is likewiſe to be known: The Difference of theſe two Arches is the Declination of the *Sun*, which will therefore be known. Let G be the Place of the *Sun* in the Ecliptick, and F G his Declination. In the right-angled Triangle G F A, having the Side F G and the Angle A, we ſhall find the Arch A G, the Diſtance of the *Sun* from the Equinoctial Point, or his Longitude and Place in the Ecliptick at the Time of the Obſervation. The next Day again obſerve the Meridian Altitude of the *Sun*, and find from thence his Declination, which ſuppoſe to be M L, from which, and the Angle A, by the ſame Method we ſhall find out the Arch M A; from which ſubtract the Arch A G, and we ſhall have the Arch G M, which is paſſed through by the *Sun* in one Day. The Bigneſs of this Arch is mutable, according to the Place the *Earth* has in its Orbit. Lecture XXII.

The circular Hypotheſis of the Antients by which they explained the Appearances.

THE ancient *Aſtronomers*, who allowed no Motions in the Heavens but what were circular and equal, that they might give an Account of this apparent Inequality of the *Sun*'s Motion, imagined that the *Sun* moved round the *Earth*, or the *Earth* round the *Sun*, (for it is the ſame thing which we ſuppoſe, to move or ſtand ſtill) in a circular Orbit, but Eccentrical; that is, whoſe Center was at ſome Diſtance from the Center of the Ecliptick, in which they placed either the *Sun* or the *Earth*; and this circular Orbit, they ſuppoſed, was deſcribed by an equal Motion: And therefore, becauſe the Center of the Ecliptick was at ſome Diſtance from the Center of equal Motion, the Motion of either the *Sun* or *Earth*, ſeen from the Center of the Ecliptick, would appear unequal.

Table XXI. Fig. 2.

Eccentricity what.

LET the Circle ♈ ♋ ♎ ♑ repreſent the Ecliptick, in whoſe Center is the *Sun*. M P N A the Orbit of the *Earth*, whoſe Center is C, diſtant from the Center of the Ecliptick by the Line C S, which is called the Eccentricity. They ſuppoſed the *Earth* to move in this Circle with an equal Motion: And

 therefore

Lecture XXII.

therefore all the Angles described round the Point C would be proportional to the Times; and the *Earth* seen from C would not appear to move slower in A than in P; but viewed from the Center of the Ecliptick, because at A it is farther distant than in P, it would appear to describe less Arches in equal Times: And therefore when the *Earth* is in A, and a Spectator in it, observing the *Sun* in ♋, he will find that he moves slower than when the *Earth* is in P, and the *Sun* is seen in ♑. And because the Arch of the Circle MAN is greater than a Semicircle, and NPM less than one; it is evident that there is more Time required to describe the Arch NAM than NPM. But in the Time the *Earth* is carried through the Periphery NAM, the *Sun* seems to describe the Northern Semicircle of the Ecliptick, *viz.* ♈, ♋, ♎; and while the *Earth* is moving through the Arch MPN, the *Sun* will seem to have gone through the other, or the Southern Semicircle ♎, ♑, ♈. From hence the Reason is plain why the *Sun* stays longer in the Northern Signs than he does in the Southern.

The Determinations of the Eccentricity and Position of the Apsides *on this Supposition.*

Upon these Suppositions they thus determined the Eccentricity and Position of the *Apsis.* In the same Year observe the Moments of Time wherein the *Sun* enters both the Equinoctial Points, *viz.* the Vernal and Autumnal; as also the Place of the *Sun* in the Ecliptick in any other intermediate Time; which suppose to be in ♌, the *Earth* being really in ♒. When the *Earth* is in the Point of its Orbit N, the *Sun* is seen in the Point ♈; then the *Earth* coming to L, the *Sun* appears in ♌; and when it has arrived in M, the *Sun* will be observed in ♎. Draw to the Place of the *Earth* in L the right Lines SL, CL. Let likewise CM, MN, and CN be joined; and let CM, and SL, cut one another in O. By Observations of the Places of the *Sun*, we have the Angle ♈ S ♌, as likewise its Complement to two right Angles ♒ S ♈. Also by the Distance of the Time between the Observations we have the Arch LM, or the Angle LCM, as also the Arch NAM,

these being proportional to the Times; therefore we have likewise the Arch MPN, and the Angle MCN. In the Isosceles Triangle MCN, having the Angle MCN, we have likewise the two Angles at the Base M and N: But in the Triangle MOS, we have the Angle MSO, and the Angle M: Therefore we have also the Angle MOS, and LOC which is vertical and equal to it. Suppose LC, the Radius of the Eccentrick, to consists of 100000 equal Parts, then in the Triangle LOC, having all the Angles, and the Side LC, we can find the Side OC. But we know MC, which is equal to LC; therefore we have MO. In the Triangle MOS we have all the Angles, and one Side MO, and therefore we shall have OS. Lastly, In the Triangle SOC, having SO and OC, and the Angle SOC, which is the Complement of the Angle SOM to two right Angles, we shall find SC the Eccentricity, and the Angle OSC; to which add the Angle MSO, and we shall have the Angle MSA, or the Arch ♈ ♑; which shews the Position of the *Aphelion*, or its Distance from the Equinoctial Point ♈. Lecture XXII.

By this Method the Antients found the Eccentricity to be 3450 of such Parts as the Radius of the Eccentrick was 100000; from which they easily calculated the Motion and Place of the *Sun* for any given Time, in the Manner following. In the Orbit of the *Earth*, let AP be the Line of the *Apsides*, and suppose the *Earth* at L describing its circular Orbit; the Arch AL, or the Angle ACL proportional to the Time, will be the *Earth*'s mean *Anomaly*, as the Arch of the Ecliptick ♑ ♒, or the Angle ♑ S ♒ is the true *Anomaly*. Having now the mean *Anomaly*, we have its Sine LQ, and its Co-sine CQ, to which add the known Eccentricity, and we shall have SQ. Say, as SQ is to LQ, so is the Radius to the Tangent of an Angle, which is QSL, which therefore will be known. Or thus: In the Triangle SCL we have the Sides SC and CL,

Lecture XXII.

and the Angle SCL, the Complement of the mean *Anomaly* to two right Angles: Therefore we can find the Angle LSC, or LSA, the true *Anomaly:* for as CL+CS: CL—CS :: Tangent of half the Angle LCA to a Fourth, which will be the Tangent of half the Difference of the Angles CSL and CLS: And because SC and CL are given and constant Quantities, the Difference of the Logarithms of CL+CS and CL—CS will be a constant Quantity; and if it be always subtracted from the Log. Tangent of half the Angle LCA, we shall have the Log. Tangent of half the Difference of the Angles CLS and CSL. But we have their Sum, and consequently the Angle LSA will be known; which shews the Place of the *Earth* in the Ecliptick, seen from the *Sun*; and the Point opposite is the Place of the *Sun*, seen from the *Earth*. In the first half Circle of *Anomaly* ALP, the mean *Anomaly* ACL is greater than the true *Anomaly* ASL. For the external Angle ACL is greater than the internal and opposite ASL: And if from the mean *Anomaly* ACL you take away the Angle CLS, there will remain the Angle LSC, the true *Anomaly*. In the second Semicircle of *Anomaly*, the mean *Anomaly* is less than the true. For suppose the *Earth* in R, the mean *Anomaly* is the Arch APR; or, casting away the Semicircle, the Arch PR, or the Angle PCR: But the true *Anomaly* rejecting the Semicircle is PSR, which is equal to PCR and CRS. Therefore if to the mean Anomaly we add the Angle CRS, we shall have the true *Anomaly* PSR, and the Place of the *Earth* in the Ecliptick. The Angle CLS or CRS is called the *Equation* or *Prosthaphæresis*, because sometimes it is to be added, sometimes to be subtracted from the mean Motion, that we may have the true Motion or Place of the *Earth*.

Equation *and* Prosthaphæresis.

THIS Theory of the Antients answered well enough to the apparent Motions of the *Sun*, which were founded on Observations that were not very

accurately

accurately made. But it was evident, from Obſer- vations of the other Planets, that their Motions could not be accounted for by ſuch a Theory. And even in the *Sun* itſelf there is a *Phænomenon*, which is not to be explained by the Theory of the Ancients, but clearly overturns that Theory, and proves it to be falſe, *viz.* By the moſt accurate Obſervations we find, that the apparent Diameter of the *Sun*, when he is in his *Apogæon*, is 31′ 29″; in his *Perigæon* it is 32′ 33″, but the apparent Diameters are reciprocally as the *Sun*'s Diſtances. From whence we find, that the true Diſtance in the *Apogæon* is to the Diſtance in the *Perigæon*, as 1953 is to 1889, or as 101661 is to 98339; ſo that the Eccentricity is but 1661 of ſuch Parts, whereof the Radius of the Eccentrick is 100000. The Theory of the Ancients makes the Eccentricity above double of this. And therefore that Theory muſt be falſe, which ſuppoſes ſo great an Eccentricity: For if we ſhould allow but one half for the Eccentricity, that would better anſwer to the apparent Diameters of the *Sun*, when they are nicely obſerved: But then, on the other Hand, ſo ſmall an Eccentricity would not account for the Inequalities of the *Sun*'s Motion, making the Center of the Eccentrick the Center of the middle Motion; for by computing we find the *Equations* or *Proſthaphæreſes* twice as great as what they would amount to with half only of the Eccentricity of the Ancients: And therefore it is plain, that this Theory of the Ancients muſt be falſe.

Lecture XXII.

The Theory of the Ancients is not true.

THE ſagacious *Kepler* obſerving this, ſhewed that the Eccentricity was indeed to be biſected; but ſo, that the Center of the Eccentrick was in D, in the middle Point between the *Sun* and the Point C; from which C, if the Motion of the *Earth* were viewed, it would appear equal. This Point C, which was diſtant from the Center of the Eccentrick by half the Eccentricity of the Ancients, was called the Center of the middle Motion; becauſe from it the Motion of the *Earth* would always be ſeen in a

Kepler's *Correction of it.*

Lecture XXII.

mean Motion between its quick and slow Progress in the Ecliptick.

'Tis true, *Copernicus*, and many other *Astronomers*, thought it absurd to suppose the *Earth* carried in a Circle, whose Center was not the Center of the equal Motion; for then the *Earth's* Motion must not only be in Appearance, but really in itself unequal; and in some Parts of the Periphery of its Orbit it would move faster, in other slower, contrary to their established Maxim, of having all the Motions perfectly uniform.

Kepler's *Elliptick Theory.*

But *Kepler*, when he had demonstrated that *Mars*, and the other Planets, were not carried round the *Sun* in circular, but in elliptical Orbits; and that the *Sun* was in one of the *Foci* of those Ellipses; and that the Planets, in moving round him, did so regulate their Motions, that a Line or Ray drawn from the *Sun* to the Planet, did sweep an *Elliptick Area* or Space, always proportional to the Time the Planet moved, he thought it but reasonable to suppose the *Earth*, in turning round the *Sun*, should observe the same Law, and be carried likewise in an elliptick Orbit. This Theory answers exactly to all Appearances; but it follows from it, that there are no Centers of equal or middle Motion, from which the Planets can be seen to describe Angles proportional to the Times. And therefore many *Astronomers*, still adhering to the Opinion that there were Centers of equal Motion, rejected this Theory of *Kepler's*, but for all that they retained the elliptick Form of the planetary Orbits. And because in the Axis of an Ellipse there are two Points equally distant from the Center, which are called the *Foci*, in one of which, they, with *Kepler*, placed the *Sun*; the other, which was distant from the *Sun* the double of the Eccentricity, they imagined to be the Center of equal Motion, and round it they supposed the Planets to describe Angles proportional to the Times; which, indeed, in Ellipses, that are not very eccentrical, is nearly true, as *Kepler* himself acknowledges, and we shall

There are no Centers of middle Motion.

shall hereafter demonstrate. This Hypothesis they liked the better, because there was no direct or geometrical Method in the Theory of *Kepler*, to find out the true *Anomaly* from the Mean; which, by the other Theory, they could easily find. Upon the Account of this Deficiency of Method, many *Astronomers* objected to *Kepler* ἀγεωμετρησίαν or want of *Geometry* in his Theory; and rejecting it, went upon other Hypotheses, which did not so well agree with the true Laws of Nature, and they feigned, that in each Orbit there was a certain Point for the Center of equal Motion, round which the Planets described Angles proportional to the Times: But since the Theory of *Kepler* is that which does really obtain, and only has Place in Nature; and all Observations declare, that the Planets do really regulate their Motions by its Laws; it is not to be rejected upon the Account of a Want in *Geometry*, nor is the Fault to be laid upon the Theory, which is rather to be imputed to the Unskilfulness of the *Astronomers* in *Geometry*. We therefore, that we may remove this Blemish of want of *Geometry* for the future, in the following *Lecture* will shew a direct Method of finding the true *Anomaly* of a Planet, from its mean *Anomaly* given.

Lecture XXII.

Kepler's *Theory esteemed ungeometrical.*

LECTURE XXIII.

Of the Motion of a Planet in an Ellipse, and the Solution of Kepler's *Problem about the cutting of the* Elliptick Area.

THE Great *Kepler* was the first who demonstrated, that the Planets did not move in circular Orbits, but that they were carried round the *Sun* in elliptical ones; all which had one common *Focus*, in which the *Sun* resided: And that the Planets in their Motions constantly observed a certain Law, *viz.* that a Ray or Line, reaching from the *Sun* to the Planet, did sweep elliptical Spaces that were proportional to the Times.

By Kepler's *Theory Sir* Isaac Newton *found out the physical Causes of the Planets Motions.*

THIS admirable and divine Invention of the sagacious *Kepler*, was owing to the exact Observations of *Tycho Brahe*; and is so much more to be valued, for that by the Help of it, the most incomparable *Philosopher* Sir *Isaac Newton* discovered the universal Laws of Motion, the System of the Universe, and the whole Body of the Celestial *Philosophy*, which was intirely unknown before. *Kepler* also demonstrated, from Observations of the Motions, that in all the Planets their periodical Times were in a sesquiplicate Proportion of their mean Distances from the *Sun*, or of the greater Axis of the Ellipses, which are equal to twice the mean Distances; that is, the Squares of the periodical Times are constantly as the Cubes of the greater Axes: And therefore, if in two different Ellipses the greater Axes be called A, *a*, their

The Proportion of the Planets periodical Times to their Distances found out by Kepler.

their periodical Times T and t; then we shall have their Analogy $T^2 : t^2 :: A^3 : a^3$, and $T : t :: A^{\frac{3}{2}} : a^{\frac{3}{2}}$. Lecture XXIII.

HENCE it follows, that in different Ellipses, the Areas, described by two Planets in the same Time, are in subduplicate Proportion of the Parameters or *Latera recta* of the Ellipses; which I thus prove. It is known from the Property of the Ellipse, that its Area is as the Rectangle under the two Axes of it; that is, if the two Axes of the greater Ellipse be called A and M, and the two Axes of the smaller Ellipse be called a and m; the Area of the greater Ellipse will be to the Area of the lesser, as a $A \times M$ is to $a \times m$. And therefore, when we are speaking of the Proportion of the Areas, we may put these Rectangles instead of the Areas. In the greater Ellipse call the Area described in a given Time X, the Area described in the lesser Ellipse in the same Time x, and the Time given in which they are described y; the *Latera recta* of the Ellipses call L and l, the periodical Times T and t. From the Theory above explained it follows, that $X : A \times M :: y : T$, also that $a \times m : x :: t : y$. And therefore, by Equality of Propotion, 'twill be $X \times a \times m : x \times A \times M :: t : T :: a^{\frac{3}{2}} : A^{\frac{3}{2}}$. But since the lesser Axis is a mean Proportional between the greater Axis and the *Latus rectum*, M will be $= A^{\frac{1}{2}} \times L^{\frac{1}{2}}$, and $m = a^{\frac{1}{2}} \times l^{\frac{1}{2}}$: And therefore $X \times a^{\frac{3}{2}} l^{\frac{1}{2}} : x \times A^{\frac{3}{2}} L^{\frac{1}{2}} :: a^{\frac{3}{2}} : A^{\frac{3}{2}}$. And therefore $X \times l^{\frac{1}{2}} : x \times L^{\frac{1}{2}} :: 1 : 1$; that is in Proportion of Equality: And therefore $X : x :: L^{\frac{1}{2}} : l^{\frac{1}{2}}$. And therefore, in different Ellipses, the Areas described in the same Time, are as the square Roots of their *Latera recta*.

The Areas described in the same Time in a subduplicate Proportion of the Latera recta *of the Ellipses.*

SINCE therefore the Law, by which the Planets regulate their Motions, is the equal or uniform De-

Lecture XXIII.

scription of the Areas, it is impossible that the Planets can every where move with the same uniform Velocity, but it must constantly be changed: So that going from the *Perihelion* to the *Aphelion*, they must constantly slacken their Pace; but as they descend from the *Aphelion* to the *Perihelion* they must again quicken their Motions: And in the *Aphelion* they have the slowest, in the *Perihelion* the quickest Motion: And the Velocity will be every where reciprocally as a Perpendicular that falls upon a Right Line passing through the Planet and touching the Orbit.

The Velocity every where reciprocal to the Squares of their Distances.

Plate XXI. Fig. 3.

Let D A F be an Ellipse, whose *Focus* is S; and suppose the Arches A B, *a b* thereof to be gone over by the Planet in equal Times that are exceeding small, the Triangles S A B and S *a b* will be equal; for they are the Areas that the carrying Ray describes in equal Times. From the *Focus* S let fall on the Tangents the Perpendiculars S P, S *p*, and the Triangle S A B will be equal to $\frac{1}{2}$ S P × A B: So likewise the Triangle S *a b* will be equal to $\frac{1}{2}$ S *p* × *a b*: And therefore S P : S *p* : : *a b* : A B: But A B and *a b*, since they are Lines described in the same Time, are as the Velocities: Wherefore the Velocity in A is to the Velocity in *a*, as S *p* is to to S P, the Perpendicular. Mr. *De Moivre*, in the *Philosophical Transactions*, N° 352, has likewise demonstrated the two following *Theorems* concerning the Elliptick Motion.

THEOREM I.

A Theorem *to determine the Proportion of the Velocity.* Table XXV. Fig. 7.

LET *A P B be the elliptick Orbit, in which suppose a Planet to move round the* Sun *in the* Focus *S. Let C be the Center of the Ellipse, C B half the greater Axis, C D half the lesser, and F the other* Focus. *The Planet being in P, draw the right Lines SP, FP; then the Velocity of the Planet in P, will be to the Velocity in its mean Distance S D, in a subduplicate Proportion of its Distance F P from the* Focus *F, to its Distance S P from the* Focus *S.*

LET the right Line E P G touch the Ellipse in P, and from each of the *Foci* on the Tangent let fall the

the Perpendiculars SE, FG; and let SH be a Perpendicular on the Tangent DH. The Velocity in P is, as we have shewed, to the Velocity in D, as SH is to SE: And therefore the Square of the Velocity in P is to the Square of the Velocity in D, as SH square or CD square to SE square; that is by the Nature of the Ellipse (because CD square is equal to $SE \times FG$) as $SE \times FG$ is to SE Square, or as FG to SE. But because of the equiangular Triangles FG is to SE, as FP is to SP: Wherefore the Square of the Velocity in P, is to the Square of the Velocity in D, as FP is to SP: And consequently, the Velocity in P, is to the Velocity in D, as $\sqrt{FP}$ is to $\sqrt{SP}$, which was to be demonstrated.

Lecture XXIII.

THEOREM II.

THE Radius is to the Sine of the Angle SPE as $\sqrt{SP \times FP}$ to CD. For SP Square is to $SP \times FP :: SP : FP :: SE : FG ::$ SE square : $SE \times FG ::$ SE square : CD square. And by Alternation of Proportion SP square : SE square $:: SP \times FP :$ CD square: And therefore $SP : SE :: \sqrt{SP \times FP} : CD$: But $SP : SE ::$ Radius to the Sine of the Angle SPE. Therefore as the Radius is to the Sine of the Angle SPE, so is $\sqrt{SP \times FP}$ to CD, which was to be demonstrated.

The angular Velocity at the Sun *is as the Square of their Distances reciprocally.* PlateXXI. Fig. 3.

WE have already shewed the Proportion by which the absolute Velocity increases or decreases: But we have another *Theorem* for determining the angular Velocity, or the Angle which a Planet, seen from the *Sun*, will appear to describe in a small Particle of Time: For it is every where reciprocally in a duplicate Proportion of the Distance from the *Sun*; which I thus demonstrate. At the Center S, at the Distances SB, S*b* describe the small Arches BE, *be*, where AB, *ab* are the small elliptick Arches described

Lecture XXIII.

described in equal Times: In SB take S*m* equal to S*b*, and draw the small Arch *mn:* And the angular Velocity in *b* is to the angular Velocity in B, as the Arch *be* is to the Arch *mn:* But the Proportion of *be* to *mn* is compounded of the Proportion of *be* to BE, and of BE to *mn*. And because the Triangles BSA and *b*S*a* are equal, *be* will be to BE as SB is to S*b:* And because the Arches BE and *mn* are similar, BE is to *mn* likewise, as SB to S*m*, or as SB to S*b:* Wherefore the Proportion of *be* to *mn* is compounded of the Proportion of SB to S*b*, and again of SB to S*b*; that is, the angular Velocity at *b* is to the angular Velocity at B, as the Square of SB is to the Square of S*b*, that is, reciprocally as the Squares of the Distances.

The Angular Velocity of a Planet compared with an equal Velocity of a Body moving in a Circle. Plate XXI. Fig. 4.

But to explain more clearly the Inequality of the Planets Motions and the various Increase and Decrease of their angular Velocities, it will be requisite to compare their Motions in different Points of their Orbit, with an equal and uniform Motion of a Body moving in a Circle. Let therefore the Ellipse AE BF be the Orbit of a Planet in whose *Focus* is the *Sun* S. Its greater Axis AB, and lesser OQ. At the Center S and Distance SE, which is a mean Proportional between AK and OK, the two Semiaxes, describe the Circle CE GF. The Area of this Circle will be equal to the Area of the Ellipse, as it is easily to be demonstrated from the Nature of the Ellipse: And let us suppose a Point to move with an uniform or equal Motion thro' the Periphery CE GF, in the same Time that the Planet described the Ellipse: And when the Planet is in its *Aphelion* A, let the circulating Point be in C, in the Line of the *Apsides*. The Motion of this Point will represent the equal or middle Motion of the Planet, and the Point will describe round S Areas or Sectors of Circles, which are proportional to the Times, and equal to the elliptick Areas the Planet at the same Time describes. Let now the equal Motion or the Angle round S proportional to the

Time be CSM; and take the Area ASP, equal to the Sector CSM; and then the Place of the Planet in its own Orbit will be P, and the Angle MSD, the Difference between the true Motion of the Planet and its mean Motion, is the Equation or *Prosthaphæresis*: And the Area ACDP will be equal to the Sector DSM, and consequently, proportional to the *Prosthaphæresis*; and consequently, where this Area is biggest, there the *Prosthaphæresis*, or the Equation, will be biggest: But the Area is biggest in the Point E, where the Circle and the Ellipse cut each other. For when the Planet descends further to R, the Equation becomes proportional to the Difference of the Areas ACE and *m*ER, or to the Area GBR*m*. For when the Planet is in R, let V be the Place of the Point moving uniformly in the Circle, the Sector CSV will be equal to the Elliptick Area ASR: And taking away the common Spaces, the Area ACE less the Area RE*m* is equal to the Sector VS*m*, or to the Equation.

Lecture XXIII.

The Area proportional to the Prosthaphæresis. *Where the Equation is greatest.*

In the *Perihelion* the equal Motion and the true Motion of the Planet coincide; for the Semicircle CEG is equal to the Semi-ellipse AEB. But after the Planet departs from the *Perihelion* B, its Motion is constantly quicker, and it goes before the Point moving equally with the mean Motion. For let the Angle GSZ be proportional to the Time, take the Area BSY, equal to the Sector GSZ, and Y will be the Place of the Planet in its Orbit; and the Angle BSY will be greater than the Angle GSZ; and the Area GBYL will be equal to the Sector ZSL, whose Angle is the Equation: And where the Area GBYL is greatest, there the Equation is the greatest; that is, in the Point F, where the Circle and Ellipse cut one another. In A the Velocity of the Planet is the least of all, because the Distance SA is the greatest; from thence the Planet descending to its *Perihelion*, its Velocity will constantly increase; but it will still be less than the mean Velocity, till it comes to E, the Intersection of

Where the Velocity of a Planet is least.

Lecture XXIII.

of the Ellipse and Circle: And there its Velocity becomes just equal to the mean; which I thus prove. When the Planet is in E, let the Point going with the equal or mean Velocity be in *m*, and the Areas described round S, in the same infinitely small Time, be *n* S E, and the Sector *i* S *m*, which will be equal. And therefore *h* E × E S is equal to *i m* × *m s*: And therefore, because S *m* and S E are equal, *h* E and *i m* must be equal; and the Angle *n* S E will be equal to the Angle *i* S *m*. At the Point therefore E, the angular Velocity of the Planet is equal to its mean Velocity; the Planet going from E, and approaching still nearer to its *Perihelion*, the Velocity grows bigger than the mean Velocity; and its Distance from the *Sun* being constantly decreasing, the Velocity will continually increase, till it comes to the *Perihelion*, where it is the greatest of all, because the Distance SB is the least of all. The Planet departing from thence, and ascending to the *Aphelion*, it leaves the Point, which proceeds constantly with the middle Motion, behind it; but as it goes farther off from the *Sun*, its Velocity decreases, but is still bigger than the mean Velocity, till it comes to F, the Point of Intersection of the Ellipse and the Circle, where the Planet's angular Velocity is again equal to the mean angular Velocity; and when it has passed that Point, its Velocity becomes less than the mean, and constantly diminishes till it arrives at the *Aphelion*, where it is the least of all; its Distance from the *Sun* being then greatest of all.

Where its Velocity is equal to the mean.

Where the Velocity is greatest.

SINCE therefore each Planet in different Parts of its Orbit has different Degrees of Velocities, and the only Equality which is observed in its Circulation round the *Sun*, is the equal Description or Increase of the elliptick Area, which grows bigger uniformly with the Time; to determine the Place of a Planet in its Orbit at any given Time, we must take an elliptick Area that is proportional to the Time. And to do this, it is necessary to solve the following *Problem*.

KE-

KEPLER's PROBLEM.

To find the Position of a Right Line, which passing through one of the Foci *of an Ellipse, shall cut off an Area described by its Motion, which shall be to the whole Area of the Ellipse in a given Proportion.*

Tab. XXI. Fig. 5.

LET the Ellipse be APB, whose *Focus* is S. I must find the Position of the Right Line SP, which cuts off the trilineal Area ASP, to which the whole Area of the Ellipse has the same Proportion that the periodical Time of the Planet has to any other given Time; which Position being found, we shall have the Place the Planet is in at the given Point of Time. Or let AQB be a Semicircle, described on the greatest Axis of the Ellipses; we must draw from S the Line SQ, which shall cut off the Area ASQ, to which the Area of the whole Circle is in the above-mentioned Proportion. For by such a Section of a Circle, the Section of the Ellipse is easily found out, by letting fall from the Point Q a Perpendicular on the Axis AB; which will cut the Ellipse in the Point P required, to which draw the Line SP, and it will be the Right Line which divides the Area of the Ellipse in the given Proportion; so that P will be the Place of the Planet: For the elliptick Segment APH is to the circular Segment AQH, as HP is to HQ; that is, as the Area of the whole Ellipse is to the Area of the whole Circle, as is known from the Nature of the Ellipse: But the Triangle SPH is to the Triangle SQH, in the very same Proportion by *Prop.* 1. *El.* VI. And therefore, by *Prop.* 12. *El.* V. the elliptick Area ASP is to the circular Area ASQ, as the Area of the whole Ellipse is to the whole Circle:

And,

Lecture XXIII.

And, by Alternation, the Area A S P is to the whole Ellipsis, as the Area A S Q is to the Circle. Hence, if we have a Method of drawing thro' S a Line which will cut the Area of the Circle in a given Proportion, it will be easy to cut the Area of the Ellipse in the same Proportion.

The Anomaly of the Eccentrick.

KEPLER, who first proposed the *Problem*, knew no direct Method of computing the Planet's Place from the Time; and expresly tells us, that there was no direct Way of finding, from the Time given, the true *Anomaly* of the Planet, or its Place in its Orbit. And therefore he found it necessary to go thro' every Degree of the Semicircle A Q B; and to search from the Arch A Q, which he called the *Anomaly of the Eccentrick*, the Time which was expressed by the Area A S Q, which is proportional to the mean *Anomaly*; as also the Angle A S P, which gives the true Place and true *Anomaly* of the Planet, which he computed by Calculation. And therefore, because he could not directly and geometrically solve the *Problem*, some *Astronomers* objected to him an ἀγεωμετρησία or want of *Geometry*; and that he was so fond of physical Causes, that he had departed from *Geometry*; and they blamed his *Astronomy*, as not being geometrical, since it was founded on such a Theory. And therefore that they might escape the committing of such a Fault, they went upon other *Hypotheses*, and feigned a Point round which the Planet's Motion should be equal, or the Angles proportional to the Times: And from thence, the mean *Anomaly* being given, they calculated the true Place of the Planet. But the Calculations founded on these *Hypotheses* were found not to answer Observations: For there is really no fixed Point which is the Center of equal Motion, round which the Planets describe Angles proportional to the Times: and the only Theory, that answers all Observations, is that above explained of *Kepler*. And therefore, the *Astronomers* must now for ever embrace this Theory of *Kepler*, since it

not

not only agrees perfectly with the Motions of the Heavens, but also lays most elegantly open the Cause and Source of all those Motions. *Kepler* himself valued this Theory so much, that he chose rather to take up with an indirect Method of Calculation, than contrive another *Hypothesis* that was not agreeable to the Nature of Things; and for this the ablest Judges were not displeased with him. Therefore to take away this Blemish of want of *Geometry* out of our *Astronomy*, we will here shew a direct Method by which the Area of an Ellipse, or of a Circle, which is equivalent, may be cut in a given Proportion. Lecture XXIII.

Let AQB be a Semicircle whose Diameter is the greater Axis of the Ellipse, its Center C, and S the *Focus* of the Ellipse in which the *Sun* is placed. Thro' the Place of the Planet imagine a Perpendicular QH to be drawn to the Axis, meeting with the Circle in Q: Then the Area ASQ will be to the whole Circle as the given Time is to the periodical Time of the Planet. Draw CQ, and from S let fall upon it, produced if required, the Perpendicular SF: The Area ASQ is equal to the Sector ACQ and the Triangle QSC; that is, equal $\frac{1}{2}$ QC × AQ + $\frac{1}{2}$ QC × SF. And therefore, because $\frac{1}{2}$ QC is a constant Quantity, the Area ASQ will be always proportionable to the Arch AQ +, the right Line SF, when the Motion is from the *Aphelion* to the *Perihelion*: But when the Planet ascends from the *Perihelion* to the *Aphelion*, the Area BS*q* is equal to the Sector BC*q*—Triangle CS*q*: And therefore it will be proportional to the Arch B*q*—the right Line S*f*. Hence, if we take the Arch AN or B*n* proportional to the Time, AQ + SF will be equal to AN, or B*q* — S*f* = B*n*; for then AN and B*n* will be proportional to the Areas ASQ and BS*q*. Table XXI. Fig. 6.

Hence, if we have the Arch AQ, and add to it the Arch QN, which is equal to the right Line SF; the Arch AN will be proportional to the Time, or equal to the mean *Anomaly* of the Planet; and therefore, if we have the true *Anomaly* of a Planet,

Lecture XXIII.

we may easily find the Mean, or the Time. For let QC be to SC as 57,29578 (which Number expresses the Length of an Arch in Degrees and Parts of a Degree that is equal to the Radius) to a fourth Number; and we shall have an Arch equal to SC in Degrees and decimal Parts. Call this Arch B; and because SC is to SF, as the Radius is to the Sine of the Angle SCF or ACQ, say, As the Radius is to the Sine of ACQ, so is the Arch B to a Fourth; and then we shall have, in Degrees and decimal Parts, an Arch in the Periphery AQB, which is equal to the right Line SF: And because SF is equal to QN, we have the Arch QN, and also the Arch AN, which is proportional to the Time.

Let us explain this by Examples in the Orbit of *Mars*. The Eccentricity of this Orbit is to its mean Distance as 14100 is to 152369. And therefore the Logarithm of the Arch B, which is equal to SC is 0,7244446: And therefore, if we would have the mean *Anomaly* when the *Anomaly* of the Eccentrick is one Degree, add the Log. Sine of 1 Degree to the Log. of B, the Sum is 8,9662999. This being the Log. of the Number 0,092533, expresses the Length of the Arch QN in decimal Parts of a Degree. And therefore the Arch AN, or the mean *Anomaly*, is 1,092533 or $1^\circ\ 5'\ 33''$. In like Manner, if the *Anomaly* of the Eccentrick be 30°, to its Log. Sine add the constant Log. of B; and the Sum will be 0,4234146, which is the Log. of the Number 2, 651; and therefore the mean *Anomaly* AN answering to 30 Degrees of the Eccentrick *Anomaly*, is 32,651, or $32^\circ\ 39'\ 3''$. This Method is much quicker and easier than that which *Kepler* gives; where, by an indirect Method, and the Rule of false Position, he shews how to compute the true *Anomaly* from the Mean.

Let us now come to the Method I promised, of directly finding the true *Anomaly* from the Mean. In the Figure, let the Arch AN be the mean *Anomaly*, or proportional to the Time;

AQ

AQ the *Anomaly* of the Eccentrick which is to be found. Call the Arch NQ y, and the Sine of AN call e, and Co-sine f; and let the Eccentricity SC be g. The Sine of the Arch AQ is equal to the Sine of the Arch AN — NQ, equal to the Sine of the Arch AN — y. But we have demonstrated in the Elements of *Trigonometry* that if the Sine of the Arch AN be e, the Sine of the Arch AN — y, or of AQ, will be $e - \frac{fy}{1} - \frac{ey^2}{1.2} + \frac{fy^3}{1.2.3} + \frac{ey^4}{1.2.3.4}$ &c. But the Radius which is, 1, is to the Sine of the Arch AQ, as SC or g is to SF or NQ; that is to y: And therefore $y = ge - \frac{gfy}{1} - \frac{gey^2}{1.2} + \frac{gfy^3}{1.2.3} + \frac{gey^4}{1.2.3.4}$ &c. And therefore we have $ge = y + \frac{gfy}{1} + \frac{gey^2}{1.2} - \frac{gfy^3}{1.2.3} - \frac{gey^4}{1.2.3.4}$ &c. Let $ge = z$, and $1 + fg$ call a, $\frac{ge}{2} = b$, $\frac{gf}{1.2.3} = c$; and $\frac{ge}{1.2.3.4} = d$. And the Equation will be in this Form $z = ay + by^2 - cy^3 + dy^4$ &c. And therefore, by the Method of Reversion of Series, invented by Sir *ISAAC NEWTON*, we have $y = \frac{z}{a} - \frac{bz^2}{a^3} + \frac{2b^2 + ac}{a^5} z^3 - \frac{5abc - 5b^3 + a^2 d}{a^7} \times z^4$, &c. But because $b = \frac{ge}{2} = \frac{z}{2}$ and $d = \frac{z}{24}$, we shall have $y = \frac{z}{a} - \frac{z^3}{2a^3} + \frac{cz^3}{a^4} - \frac{5cz^5}{2a^6}$ &c. But if the Arch AN be greater than 90 Degrees, and less than 270, then ge or

Lecture XXIII.

or $z = y = gfy + \frac{gey^2}{2} + \frac{gfy^3}{1.2.3} - \frac{gey^4}{1.2.3.4}$. And

then $a = 1 - gf$, and $y = \frac{z}{a} - \frac{z^3}{2a^3} - \frac{cz^3}{a^4}$ &c. This Series expresses the Arch QN, the Parts whereof the Radius is 100000: But to have it in Degrees, and Parts of a Degree, say, as the Radius is to this Series, so is 57.29578, which are the Degrees of an Arch equal to the Radius, to a Fourth; consequently, since the Radius is Unity, if we multiply the Series by 57.29578, which Number call R, we shall have the Arch y in Degrees and decimal Parts $= \frac{Rz}{a} - \frac{Rz^3}{2a^3} + \frac{Rcz^3}{a^4}$, &c. The very first Term of this Series $\frac{Rz}{a}$ is sufficient to determine the *Anomaly* of the Eccentrick in almost all the Planets, nearly enough. For in *Mars*, the Error seldom exceeds the 200th Part of a Degree; in the *Earth* it is less than the 10000th Part of a Degree. But it will be best to shew the Use of this Method by Examples.

In the *Earth*'s Orbit the Eccentricity is 0,01691, when the mean Distance CQ is 1. Suppose we are to find the *Anomaly* of the Eccentrick, and the equated *Anomaly* when the mean *Anomaly* is 30 Degrees.

The Log. of Eccentricity is —	8,2281436
The Log. Sine of 30° ——	9,6989700
The Log. of R —— ——	1,7581226
The Log. of R z ———	9,6852362
The Log. of a subtract ——	0,0063137
The Log. of the Arch y ——	9,6789225

To which answers the Number 0,47744, or in sexagesimal Numbers 28′ 38″: The rest of the Terms don't amount to the 10000th Part of a Degree, and may therefore be neglected. If therefore from 30 Degrees we deduct 28′ 38″, we shall have

have the Arch A Q 28° 31′ 22″. In the Triangle QCS, we have the Sides QC, CS, and the Angle SCQ: Wherefore we shall have the Angle QSC. The Analogy is QC + CS, or AS: QC — CS, or BS : : Tangent $\frac{CSQ + CQS}{2}$: Tangent of $\frac{CSQ - CQS}{2}$. Therefore, if from the Tangent of half the Angle ACQ we subtract a constant Log. 0,0146893, we shall have the Tangent of an Angle, which, added to half the Angle ACQ, gives the Angle CSQ, or ASQ, which in the present Case, is 29°. 3′. 7″. But to find the Angle ASP, we must diminish the Tangent of the Angle ASQ, in the Proportion of the bigger Axis of the Ellipse, to its lesser. Therefore, from the logarithmick Tangent of ASQ, take away the constant Log. 0,0000622, which is the Log. of the *Ratio* of the greater Axis to the less, and we shall have the Log. Tangent of the Angle ASP; which Angle is equal to 29°. 2′. 54″. And this is the coequated *Anomaly*.

Lecture XXIII.

Plate XXI. Fig. 5.

IN the Orbit of *Mars*, the Eccentricity is 14100 of such Parts as the mean Distance is 152369. And therefore, the Log. of the *Ratio* of SC to CQ, is 8,9663226 = Log. of *g*. Let us find the *Anomaly* of the Eccentrick, when the mean *Anomaly* is 1 Degree.

The Log. of Eccentricity — —	8,9663226
The Log. Sine of 1 Deg. ——	8,2418453
The Log. of R —— ——	1,7581220
The Log. of R z ——	8,9662899
The Log. of *a* subtract ——	0,0384299
The Log of $\frac{Rz}{a}$ ——	8,9278600

First, THE Number answering this Log. is 0,084697, and gives the Bigness of the Arch NQ, and the Error is less than the 30000th Part of a Degree.

Lecture XXIII.

Secondly, SUPPOSE the mean *Anomaly* is 45 Degrees, and I am to find the *Anomaly* of the Eccentrick.

The Log. of Eccentricity	——	8,9663226
The Log. Sine of 45°	——	9,8494850
The Log. of R	——	1,7581220
The Log. of R z	——	0,5739296
The Log. of a subtract	——	0,0275249
The Log. of $\frac{Rz}{a}$	——	0,5464047

To which answers the Number 3,5189, which is more than the Truth, by about 150th Part of a Degree: And to correct this Error, take the second Term of the Series $-\frac{\overline{Ra+2Rc} \times z^2}{2a^4}$, which will be found equal to the Fraction 0,0065, and subtract this from the first, there will remain 3,5124, which expresses the Arch N Q true to the 1000000th Part of a Degree.

Thirdly, LET us find out the *Anomaly* of the Eccentrick, when the mean Motion is 100 Degrees: In this Case $a = 1 - gf = 0,983930$.

The Log. Eccentricity, or of g		8,9663226
The Log. Sine of 100°, or of 80°		9,9933515
The Log. of R	——	1,7581220
The Log. of R z	——	0,7177961
The Log. of a subtract	——	9,9929598
The Log. of $\frac{Rz}{a}$	——	0,7248363

The Number answering to this Log. is 5,3068, which is greater than the Truth, by about the 50th Part of a Degree; and therefore, to correct this Error, double the Log. of $\frac{z}{a}$, and to the Product add the Log. of $\frac{Rz}{a}$, and we shall have the Log. of

of $\frac{Rz^3}{a^3}$, and the Number anſwering to it, is 0,04552, whoſe Half 0,02276 is equal to $\frac{Rz^3}{2a^3}$: This being ſubtracted from the former, there will remain 5,2841 for the Arch NQ; which is not the 10000th Part of a Degree diſtant from the Truth. 'Tis here to be obſerved, that though the ſecond Term of the Series be $-\frac{\overline{Ra+2Rc}\times z^3}{2a^4}$ yet the Part of it $-\frac{Rz^3}{2a^3}$ is ſufficient to determine AQ truly to the 10000th Part of a Degree.

HAVING found the Arch AQ, or the Angle ACQ, we compute the Angle ASQ by the Reſolution of the Triangle QCS, whoſe Sides QC and CS are given, with the Angle contained between them; and then the Logarithm Tangent of the Angle ASQ is to be diminiſhed, by taking from it the Logarithm of the *Ratio* of the greater Axis to the leſs: And then there will remain the logarithmick Tangent of the Angle ASP, which is the true or coequated *Anomaly*.

LECTURE XXIV.

Sir ISAAC NEWTON'*s Solution of* Kepler'*s Problem; and* Ward'*s Elliptick Hypothesis explained.*

OUR Method of Solution, explained in the preceding *Lecture*, and that of Sir ISAAC NEWTON, delivered by him in his Principles, *pag.* 101, are built upon the same Foundation; which is, that the right Line SF is equal in Length to the Arch QN: But the *Newtonian* Method is not unlike to that used by the *Analysts*, when they extract the Roots of affected Equations: And it is so much more to be valued, that it not only gives easily the Planets Places, whose Orbits are nearly circular, but almost with the same Ease it may be used to determine the Comets Places, who have very eccentrick Orbits. And this may likewise be performed by our Method, if instead of the Arch AN, we take another Arch A, more nearly equal to AQ, whose Sine is e; and instead of making $z = ge$, suppose $z = ge + A - AN$. And finding the Sine of the Arch $A + y$ we shall come to an Equation of the same Form with the former, where z and y are much less, and consequently the Series will converge much faster.

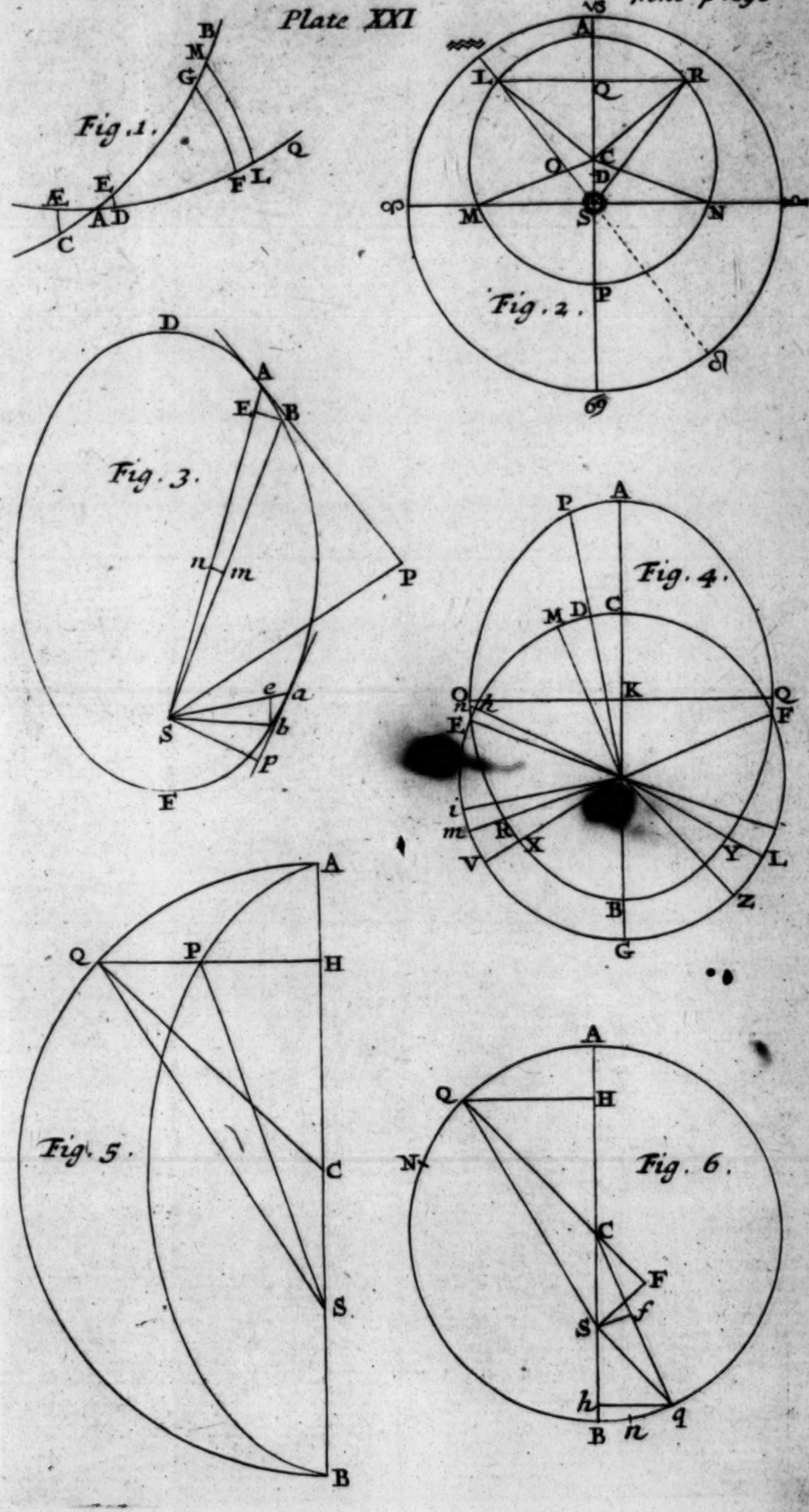
I will here explain the *Newtonian* Method, since it is of great Use and Expedition, for the Sake of those who are willing to calculate Tables upon Principles grounded on the true Laws of Motion, and not upon absurd Hypotheses.

WE have already shewed that if the Arch AQ be the *Anomaly* of the Eccentrick, that, together with

23

Plate XXI

front. p. 296



with the right Line SF let fall from S upon the Radius C Q perpendicularly, will be proportional to the Time when the Planet descends from the *Aphelion* to the *Perihelion*; and the Difference between the Arch B Q and SF proportional to the Time, when it ascends from the *Perihelion* to the *Aphelion*: And therefore, if we take the Arch A N or B N proportional to the Time, the Arch Q N will be equal to S F. Therefore, to find in Degrees and decimal Parts the Measure of an Arch in the Periphery which is equal to S F: Say, as C Q is to C S, so is 57,29578 Degrees, which is equal to the Radius, to a Fourth: This Number will express the Bigness of an Arch in the Periphery A Q B which is equal to C S: The Log. of this Arch call B. And because S C is to S F, as the Radius is to the Sine of the Angle A C Q; say, as the Radius is to this Sine, so is the Arch whose Log. is B, to another, which call D; then this Arch D, will be equal to S F. And therefore, if at the given Time, the Arch A N and the Area A S Q are proportional each to the Time, and I take N P equal to D, the Point P will fall on Q: But if the Area A S Q be not exactly proportional to the Time, the Point P will either fall above or below Q, according as the Area A S Q is bigger or less than the Truth. Let the true Area be A S q, and upon C q let fall the Perpendicular S E; which by what we have already shewed, is equal to N q: And therefore S E — S F or S F — S E, that is, nearly the Line L E, is equal to q P = Q P — Q q or Q q — Q p. Now if the Angle Q C q be small, we have C E : C q : : L E : Q q : : Q P — Q q : Q q. And therefore C E + C q : C q : : Q P : Q q. After the same way, when B q is less than a Quadrant, C q — C E : C q : : Q P : Q q. When the Planet is near the *Aphelion* or *Perihelion*, C E becomes nearly equal to C S; and C Q + C E is almost the same with A S: And therefore Q P : Q q : : A S : A C, when the

Lecture XXIV.

Table XXII. Fig. 1.

Lecture XXIV.

the Arch $A\,q$ is less than a Quadrant, but when $B\,q$ is less than a Quadrant, then as $SB : CB :: QP : Qq$. Say, as $CS : CQ :: R$ the Radius to a Line L, and then $CQ = \frac{CS \times L}{R}$. Also the Radius is to the Co-sine of ACQ as $SC : CF$ or CE, which are nearly equal: Wherefore $CE = \frac{SC \times \textit{Co-sine}\ AQ}{R}$, whence we have the Analogy $QP : Qq :: \frac{SC \times L + SC \times \textit{Co-s.}\ AQ}{R} : \frac{CS \times L}{R} :: L + \text{Co-sine}\ AQ : L$; when AQ is less than a Quadrant: But if it be greater than a Quadrant, QP will be to $Qq :: L - \text{Co-sine}\ AQ, L$. And in this Manner, if there be an Arch taken as $A\,q$, which is either a little less or bigger than the Truth, we shall find the Arch AQ, which is to be added or subtracted; so that the Area ASQ may be nearly proportional to the Time. And, if instead of AQ we take another Arch AQ, and argue as in the former Arch, we shall have a new $A\,q$, nearer to the Truth: And by this Means we shall constantly approach to the true Arch, so that the Difference may be less than any given Quantity.

This Method illustrated by Examples.

THERE is no need of explaining this Method any farther, we will only illustrate it with Examples, in the Motions of the Planet *Mars*. In *Mars*'s Orbit the Logarithm B, is 0,7244446; and the Longitude L, is 1080631 of such Parts as the Radius is 100000.

1st Example.

LET us find the Angle ACQ, when the mean *Anomaly* or Arch proportional to the Time is only one Degree: Because CS is almost the tenth Part of CA. I suppose AQ to be 0,9 Deg. that is a tenth Part less than the mean *Anomaly*. Add the Log. Sine of 0,9 to the Log. B, and the Sum is 8,9205466 = to the Log. of the Number 0,083281; this Number expresses the Bigness of an

2

an Arch equal to SF $=$ NP: And if the Arch AQ had been rightly assumed, AN $-$ NP had been equal to AQ, and QP $=$ 0. But in the present Case, QP is equal 0,01671; from which, if I take away its tenth Part, because AS is greater than AC, by about a tenth Part, we shall have Qq $=$ 0,01504, which being added to AQ, gives Aq $=$ 0,91504. Lecture XXIV.

2*dly*, LET the Arch AN, or the mean *Anomaly* be 2 Degrees; I suppose the Arch AQ to be 1,83, almost double of the former; and adding to its Log. Sine the Log. B, the Sum is 9,2286992, the Log. of the Number 0,16931. And then QP $=$ 0,00069; from which, subtracting a tenth Part, Qq is nearly $=$ 0,00063, and Aq 1,83063, which is not the 100000th Part of a Degree different from the Truth. *2d Example.*

3*dly*, SUPPOSE the Arch proportional to the Time, to be three Degrees, I take AQ to be 2,745 $=$ 1,83$+$0,915; and to its Log. Sine, adding the Log. B, we have the Log. of the Number 0,25392 $=$ NP, and AN $-$ NP $=$ 2,74638, therefore Qq $=$ 0,001 nearly, and Aq $=$ 2,746: So that by one Addition of two Logarithms, we have the Arch Aq, true to the thousandth Part of a Degree. *3d Example.*

4*thly*, NOW if we should not proceed by single Degrees, but were to find the Angle ACQ, when the mean *Anomaly* is much larger; for Example, 45°: I make my Supposition, that the Arch AQ is 40 Degrees, and to its Log. Sine add the Log. B; the Sum is 0,5320121, equal to the Log. of the Number 3,4081, which Number subtracted from 45, leaves AN $-$ NP $=$ 41,5919, which exceeds the Arch AQ by 1,5919. And therefore, if we take as L$+$Co-sine AQ to L, so 1,5919 to a Fourth, we shall have the Arch Qq 1,4865. And therefore Aq $=$ 41,4865, which does not differ much above the thousandth Part of a Degree from the Truth. But without this Proportion we might have found Aq, by taking an Arch which is somewhat less than AN $-$ NP, but nearly equal to it. Suppose we had *4th Example.*

Lecture XXIV. had made A Q = 41,5, and adding its Log. Sine to the Log. B we shall have another Arch NP = 3,5132, which subtracted from A N gives 41, 4868, for a new A *q*. And this Arch is easier found, than by the former Proportion; and besides, comes nearer to the Truth, than the last A *q* was.

5th Example. 5*thly*, AFTER having found A *q*, or the *Anomaly* of the Eccentrick, that answers to the mean *Anomaly* of 45 Degrees; if we should again proceed by single Degrees, by only one Addition of two Logarithms, we may find the *Anomaly* of the Eccentrick to all the following Degrees of the Semicircle, *viz.* when the mean *Anomaly* is 46 Degrees, I make A Q to be 42,4, and adding its Log. Sine to the Log. B, I find A N — N P 42,4249; to which, if I make the new A Q equal, I shall have an A *q*, which is not 1000th Part of a Degree distant from the true *Anomaly* of the Eccentrick. So likewise, when the mean *Anomaly* is 47 Degrees, I take A Q 43,36 = to the former A *q* + the Increment that accrues to it, by adding a Degree: And adding the Log. B to the Log. Sine of 43,36, the Sum is the Log. of the Number 3,6402, which subtracted from A N, leaves A N — N P = 43,3598, equal to a new A *q*: And this Arch is about the 10000th Part of a Degree, different from the true *Anomaly* of the Eccentrick.

6th Example. 6*thly*, IF again, passing over the intermediate Degrees, I would find the Arch A *q*, when the mean *Anomaly* is 100 Degrees; I make A Q equal 96 Degrees, and adding its Log. Sine to the constant Log. B, the Sum is the Log. of the Number 5,273, and A N — N P = 94,727. Therefore I put again A Q = 94,72, and adding together its Log. Sine, and the Log. B, I have the Log. of the Number 5,285, which subtracted from A N, leaves A *q* = to 94,715. In like manner, when the mean *Anomaly*, is 101 Degrees, I make A Q 95,71; and I find N P to be 5,2756, which subtracted from 101, leaves A *q* 95,7244. And here again; if we proceed by Degrees, we shall have the eccentrick *Anomaly*,

maly, constantly by the Addition of two Logarithms, one of which being constantly the same, may be set once down in a Piece of Paper by itself, and the Labour saved of frequently transcribing it. Lecture XXIV.

Let us now pass to another Sort of Orbit, whose Eccentricity bears a great Proportion to its mean Distance. For Example, let us suppose the *Aphelion* Distance to be to the *Perihelion* Distance, as 70 is to 1; and such is nearly the Orbit of that Comet, which Dr. *Halley* first found to compleat its Period in $75\frac{1}{2}$ Years. Here A C or C Q, the mean Distance, is 35,5, and C S is 34,5, of such Parts as SB is 1. And the constant Log. B is 1,7457133. We must find the Arch Bq, when the mean *Anomaly* computed from the *Perihelion* is $\frac{1}{100}$ of a Degree. I suppose B Q to be 0, 35: To its Log. Sine, add the Log of B, and the Sum is the Log. of the Number 0,34013, which added to the Arch A N, makes ,35013. If this Arch had been only 35, BQ had been rightly taken; but the Difference is 0,00013. And therefore because C B is to S B, as 35,5 to 1; multiply the Difference, ,00013 by 35,5, and we shall have Qq = ,004615, and the Arch Bq = 0,354615, and the Error less than ,0003 of a Degree. Again, let the mean Motion be ,02, I put B Q = 0,71, and adding its Log. Sine and B together, I have the Log. of the Number 0,68998, and B N + N P = ,70998: The Difference is 0,00002; which multiply by 35,5, and the Product subtracted from B Q, leaves B q = 0,7092, and the Error is not greater than the $\frac{1}{10000}$ of a Degree. If the mean *Anomaly* be 0,03, I make B Q = 1,06, and adding its Log. Sine to B, I have the Log. of the Number 1,03008, to which add the Arch BN; the Sum is 1,060088, which is greater than B Q: Wherefore, if the Difference ,00008 be multiplied by 35,5, and the Product added to B Q, we shall have Bq = 1,06284. In the same manner when the mean *Anomaly* is ,04, I suppose B Q, 1,4, and I find P N = 1,3604, to which adding ,04, the

Example in the Orbit of a Comet.

Table XXI. Fig. 6.

Lecture XXIV. the Sum is 1,4004, which exceeds 1,4 by ,0004; multiply this Difference by 35,5, and the Product will be ,0142 = Q*q*, and therefore B*q* = 1,4142. In all these Examples the Errors are very small, seldom exceeding the 1000th Part of a Degree.

Let us now find the *Anomaly* of the Eccentrick, when the mean *Anomaly* is one Degree. Here I make B*q* = 20 Degrees, and adding its Log. Sine to B, I have the Log. of the Number 19,045, to which adding 1 the Sum is 20,045, and is greater than 20 by ,045. And because in this Example, L — Co-sine BQ is to L, as 1 is to 11,5 nearly: I multiply the Difference ,045 by 11,5, and the Product 0,5175, added to BQ, makes 20,5175. Therefore 2*dly*, I make BQ 20,51, and NP will be 19,5092; to which, adding BN, the Sum is 20,5092. And therefore if the Difference ,0008 be multiplied by 11,5, and the Product ,0092, subtracted from BQ, there will remain B*q* = 20,5008.

Lastly, Let the mean *Anomaly* be 2 Degrees; I put BQ = 30 Degrees, and then I find NP 27,84; to which adding 2, and the Sum is 29,84, which is less than 30 Degrees: Multiply the Difference, 16 by 6,3 (for L — Cosine BQ is to L, as 6,3 to 1,) and we have 1,008 = Q*q*: And therefore, this Arch subtracted from BQ, gives B*q* = 28,982. Therefore to correct the Error, I put again BQ = 29 Degrees; and by a like Process I find B*q* = 28,9672.

Table XXI. Fig. 5, 6. Having found the Angle ACQ, the Angle ASQ is easily found. For in the Triangle QCS, we have the Sides QC, CS, and the Angle QCS: Therefore we shall find the Angle ASQ, and the Side SQ; then say, as the greater Axis of the Ellipse is to its less, so the Tangent of the Angle ASQ is to the Tangent of ASP, which will thereby be found; and it is the coequated or true *Anomaly*. Again, say, as the Secant of the Angle ASQ is to the Secant of ASP, so is SQ to SP, the Distance of the Planet from the *Sun*. Or perhaps these Things may be easier computed in this Manner. Having the

the Arch AQ, we have its Sine QH, and its Co-sine HC: But we have SC in such Parts as the Radius, or CQ is 100000; therefore we have HS. Say, as the greater Axis of the Ellipse is to its less, so is QH to PH, which will therefore be given. In the rectangle Triangle PHS, we have also the Sides PH and HS, wherefore we can find PSH the true *Anomaly*, and PS the Distance of the Planet from the *Sun*. Lecture XXIV.

BECAUSE in the *Aphelions* and *Perihelions*, the Points Q and N, or the mean and true Place of a Planet coincide; and in the first Semicircle of *Anomaly*, the mean Place is before the true Place, in the second, the mean Place is behind the true; if we have determined the Position of the *Apsides* of the *Earth's* Orbit, we shall know the Time when the true and mean Place coincide. For when the *Sun* is observed in that Point of the Ecliptick where the *Perihelion* is, then the *Earth* is in the *Aphelion*. And having this Moment of Time, by *Astronomical* Tables, we shall have the mean *Anomaly* for any other Time, as likewise the Arch AN. For these Arches are computed according to the Proportion of the Times, and are placed orderly in the Tables. Now having for any Point of Time the Arch AN, we have shewed how from thence we may compute the true *Anomaly*, and the Place of the *Earth* in the Ecliptick, to which the Place of the *Sun* is always opposite.

BESIDES the Theory of *Kepler*, according to which the Planets do really regulate their Motions, there is another elliptick Hypothesis, which has been chiefly improved by two most celebrated Astronomers, *Ismael Bullialdus*, and Dr. *Seth Ward*, formerly Professor in this Chair, and afterwards Bishop of *Salisbury*, by whose Pains *Astronomy* has been much advanced. And since this Hypothesis does not want Elegance, and a Neatness in Geometry; and besides, it admits of an Easiness in computing, we will here briefly explain it. In this Hypothesis, with *Kepler*, it is supposed that the Planets Orbits are *Ellipses*, Ward's *Theory*.

Lecture XXIV.

Ellipses, and that they have all one common Focus, in which the *Sun* resides. Moreover, they suppose, that each Planet does move in the Periphery of its Orbit, in such a Manner, that drawing Rays or Lines to the other Focus, they describe Angles proportional to the Times. These Things being supposed, Dr. *Ward* shews an elegant Method of finding the true *Anomaly* from the Mean, having determined the Species of the Planet's Orbit: And it is as followeth.

Table XXII. Fig. 2. Ward's *Method*.

LET A P B be the Ellipse which the Planet describes, A P the Line of the *Apsides*, S the Focus in which the *Sun* is placed, F the other, or the upper Focus, which is the Center of equal Motion. Let the Angle A F L be proportional to the Time, or be the mean *Anomaly*, then L will be the Place of the Planet in its Orbit, and the Angle A S L, the coequated or true *Anomaly*. Produce F L to E, so that F E may be equal to A P the greater Axis of the Ellipse; and therefore, since by the Nature of the Ellipse, F L and L S are equal to the same A P, then L E must be equal to L S, and the Triangle L S E will be an Isosceles Triangle; and therefore the Angles E and E S L are equal; and the exterior Angle F L S, being the Sum of both, will be the double of each, or double of the Angle LES. Therefore in the Triangle E F S, having F E and F S, and the Angle E F S, which is the Complement of L F A to two Rights, we can find the Angle E, whose double is equal to the Angle FLS, which therefore will be known: But the Angle A F L is equal to the two Angles F S L and FLS, and therefore the Angle F L S is the *Prosthaphæresis* or the Equation, which is to be subtracted from the mean *Anomaly*, or added to it, to have the true *Anomaly*.

IN resolving the Triangle E F S, having E F and F S, the Analogy is $\frac{1}{2}$ E F + $\frac{1}{2}$ F S : $\frac{1}{2}$ E F — $\frac{1}{2}$ F S; that is, as A S to S P : : Tang. $\frac{1}{2}$ A F E : Tangent of $\frac{1}{2}$ Difference of the Angles E and F S E; but be-

because the Angle E is = LSE, FSL is the Difference of the Angles E and FSE; wherefore the Angle, found by the Analogy, being doubled, gives the Angle FSL, which is the true *Anomaly*. Now the Practice here is extremely easy; for because AS and SP are constant Quantities, the Difference of their Logarithms is a constant Quantity; wherefore a given Number is to be added to Log. Tangent of half the mean *Anomaly*, and then we shall have the Tangent of half the true *Anomaly*. Moreover, in the Triangle LFS, having all the Angles, and the Side SF, we can find SL the Distance of the Planet from the *Sun*. Lecture XXIV.

THIS Hypothesis of Dr. *Ward*'s is a very useful Approximation, and serves to shorten the Calculation, and make it easy: But yet it is still only an Approximation, and does not come up to the Truth: We will here shew the Reason of it. Let APB be the Orbit of the Planet, AQB the Circle circumscribed; the Arch AQ the *Anomaly* of the Eccentrick, and AN the mean *Anomaly*. From the Center C draw NC, and QG parallel to it; the Angle QGA is equal to NCA, or the mean *Anomaly*; and CG will be nearly equal to CS, but a little less than it. For from the *Focus* S, on QC let fall the Perpendicular SF, which we shewed before to be equal to the Arch QN: But because the Arch QN is small, its Sine will be almost equal to the Arch; and therefore GO, a Perpendicular on NC, will be nearly equal to SF, but somewhat less. But the Triangles GOC, and SFC, are nearly equiangular, (for NCQ, the Difference of the Angles GCO, and SCF, is very small) and therefore, because OG is almost equal to SF, but a little less than it, CG will be almost equal to CS, but somewhat less. The other *Focus* of the Ellipse then, must be a little above the Point G, but very near it; and if we draw from the Planet's Place the Line PL, parallel to QG, the Point L will be likewise above the Point G, but yet not far distant from it; and therefore, the Point L, and the other *Focus* of the

Ward's Hypothesis an Approximation only; the Reason of it. Plate XXII. Fig. 3.

Lecture XXIV.

Ellipse, do nearly coincide. But the Angle P L A is equal to N C A, the mean *Anomaly*; and the Point L, nearly coinciding with the other *Focus*, the Line drawn from P to the other *Focus* will make an Angle with the Axis, nearly equal to the Angle P L A or N C A, that is, to the mean *Anomaly*. And therefore the Angles, at the superior *Focus*, are nearly proportional to the Times.

Where the Angles N C A and Q C A, or S C F, differ but little from one another, that is, where the Angle N C Q, and the Eccentricity are but small, the Points G and L, are nearly coincident with the superior *Focus*. And therefore this Theory is accurate enough to answer to the Motion of the *Earth*, whose Orbit is nearly circular: But in the other Planets, and particularly *Mars* and *Mercury*, it does not do so well. And therefore, *Bullialdus*, from four Places of *Mars*, observed by *Tycho*, in the first and third Quadrant of *Anomaly*, found that *Mars* was further advanced in his Orbit, than he ought to be by this Theory. But in the second and fourth Quadrants, *Mars*'s true *Anomaly* was found to be less than it should be, according to this Hypothesis; and therefore *Bullialdus* gave it the following Correction. Upon the Diameter A P, which is the greater Axis of the Ellipsis, describe the Circle A D P. Let A F L be the mean *Anomaly*; through L draw the Line Q L G perpendicular to the Axis, meeting with the Circle in Q. Join F Q, which cuts the Ellipse in ♈, and ♈ will be the Place of the Planet in its Orbit, answering to the mean *Anomaly* A F L. Now the Angle A F Q, answering to the mean *Anomaly* A F L, is easily found by taking an Angle whose Tangent is to the Tangent of A F L, as the greater Axis of the Ellipse is to the lesser. And having the Angle A F Q, or A F ♈, the Angle A S ♈ is found in the same Manner as before the Angle A S L was found.

Bullialdus's *Correction of this Hypothesis.*

Plate XXII. Fig. 2.

The Calculations we have here explained, suppose that the Species, or Forms of the Orbits are given, as likewise their Positions. We shall afterwards

wards shew a Way by which the Orbits of the other Planets are determined: But the Form and Position of the *Earth*'s Orbit is to be found by the following Methods. First, observe the apparent Diameter of the *Sun*, as likewise his Motion; for when the *Earth* is in its *Aphelion*, the *Sun*'s Diameter is the least of all, and his Motion slowest; the *Earth* being there at the greatest Distance from the *Sun*. In the *Perihelion*, it coming nearest the *Sun*, we shall observe his Diameter to appear biggest. Let any right Line SP, represent the *Perihelion* Distance of the *Sun*: Say, as the apparent Diameter of the *Sun* in the *Aphelion* is to its apparent Diameter in the *Perihelion*, so is SP to a Fourth. In SP produced, take SD equal to this Fourth, and it will be the *Aphelion* Distance; bisect PD in C, and CS will be the Eccentricity, and C the Center. Describe an Ellipse, whose *Focus* is S, and greatest Axis PD; that Ellipse will be of the same Form with the *Earth*'s Orbit; and the Points of the Ecliptick, where the *Sun*'s Diameter appears the biggest and the least, shew the Position of the *Apsides*, or the *Aphelion* and *Perihelion*. But because the Diameter of the *Sun*, in the *Aphelion* and *Perihelion* is scarcely seen to alter its Bigness for some Days, it will be very difficult to determine the Position of the *Apsides*, by Observations made on the apparent Diameter of the *Sun* only; and therefore, it will be better to find out the *Aphelion*, and *Perihelion* Distances and Positions, by observing the *Sun*'s Motion: For the angular Velocity of the *Earth* and the apparent Motion of the *Sun*, which is equal to it, is always reciprocally as the Square of the Distance, as we have above demonstrated.

Lecture XXIV.

The For of the Earth'*s Orbit.*

Plate XXII. Fig. 4.

THEREFORE, to determine the Species of the Ellipse, in which the *Earth* moves, we must observe the apparent Velocities of the *Sun* when it is greatest and least. Call the least, A, and the greatest B; and let any right Line SP, represent the *Perihelion* Distance: Say, as A is to B, so is SP to another Line C. Produce SP to D, so that SD may be a mean Proportional between SP and C; this Line SD will

Lecture XXIV.

represent the *Aphelion* Distance: And therefore, if the Ellipsis be described, whose *Focus* is S, and its greater Axis PD, that Ellipse will be of the same Form with the *Earth*'s Orbit. For because SP, SD, and C, are continually proportional, PS square will be to DS square, as SP is to C, or as A to B, that is, as the Velocities. Moreover, if the Places of the Ecliptick be diligently marked, where the Velocities are greatest and least, in those Points will the *Apsides* be situated. Lastly, If there be two Places of the Ecliptick observed, where the *Sun*'s apparent Velocities are equal, and the Arch of the Ecliptick between the two Places be bisected, the Point of Bisection, and its opposite, will shew the Places of the *Apsides*. But these Methods require Observations that are very nice and accurate, such as can scarcely be made.

Ward'*s Theory best explains the Orbit and Position of the Apsides.* Plate XXII. Fig. 5.

From the Theory of Dr. *Ward*, we have a more certain Method of finding the Form of the Orbit, by three Observations of the *Sun*, and marking the Time between them; which does likewise determine the Position of the *Apsides*. Let ABPDC be the Orbit of the *Earth*, S the *Focus* in which the *Sun* is placed, F the other *Focus*: The *Apsides* A and P. Let BC, and D, be there Places of the *Earth* in the Ecliptick, which are found by observing three Places of the *Sun*, to which they are opposite. At the Center F, and Distance FM, equal to the greatest Axis of the Ellipse, describe the Circle MHEL; and let the Lines FB, FC, FD produced, meet with the Circle in the Points G, H, E. Draw likewise from the *Focus* S, the Lines SB, SC, SD, as also SG, SH, and SE. We have the Angles BSC, BSD, and CSD, for they are measured by the Arches of the Ecliptick, intercepted between the Points observed. But, according to this Theory, the *Earth* moves in the Perimeter of an Ellipse in such a Manner, that it describes Angles about the *Focus* F, that are proportional to the Times: And therefore we shall have the Angles BFC, BFD, and CFD, taking each of them such that they may have the same

ſame Proportion to four right Angles, as the Times Lecture between the Obſervations have to the whole periodical XXIV. Time. Moreover, twice the Angle F G S, that is the Angle F B S, is the Difference of the Angles A F B, and A S B: This we ſhewed before. And the Double of the Angle F H S is the Difference of the Angles A F C, and A S C; the Difference of the Angles B F C, and B S C, will therefore be equal to 2 F G S + 2 F H S. But becauſe we know the Angles B F C, and B S C, we know likewiſe their Difference, therefore, we have the Sum of the Angles F G S, and F H S. But the Angle F G S is the Difference of the Angles B F A, and G S A; and the Angle F H S, is the Difference of the Angles H F A, and H S A: Whence both the Angles F G S, and F H S, will be equal to the Difference of the Angles B F C, and G S H. But we have the Angle B F C, and the Sum of the Angles F G S, and F H S; and therefore we have the Angle G S H. In the ſame Manner we can find the Angle G S E. Alſo, in the ſame Manner, the Angle F E S doubled, is the Difference of the Angles D F A, and D S A; alſo the Double of the Angle F H S is the Difference of the Angles C F A, and C S A. And therefore, twice the Angle F E S—twice the Angle FHS will be equal to the Difference of the Angles C F D, and C S D; but we have the Angles C F D, and C S D: And therefore, we have half their Difference, that is, F E S—F H S. But the Angle F E S—F H S, is the Difference of the Angles C F D, and H S E; and we have the Angle C F D; wherefore, we have the Angle H S E. We have therefore, all the Angles at F, *viz.* B F C, B F D, C F D, and all the Angles at S, *viz.* B S C, B S D, C S D; as alſo G S H, G S E, H S E. Theſe Things being laid down,

Express the Line S H by any Number, *viz.* 100000, and produce E S, till it meets with the Periphery in L, join H L, L G, and H G. In the Triangle H S L we have the Angle H S L, the Complement of H S E to two Rights: And the Angle HLS, equal to half the Angle H F E, by the 20th

Lecture XXIV. *Prop. El.* III. and the Side HS 100000; wherefore, we shall find the Side SL: Then in the Triangle SLG, we have the Angle LSG, the Complement of the known Angle ESG to two Rights; and the Angle SLG, being half the Angle EFG, by the 20th *Prop. El.* III. and the Side SL, therefore we shall find SG. And again, in the Triangle SHG, we have the Sides SH and SG, and the Angle HSG; and consequently we shall find the Side HG, and the Angle SHG. In the Isosceles Triangle HFG, we have the Angle HFG, and the Base HG; wherefore, we shall find the Side HF, which is equal to the greater Axis of the Ellipse; as also the Angle GHF, which being subtracted from the known Angle GHS, leaves the Angle FHS known. Lastly, In the Triangle FHS, having FH, and HS, and the Angle FHS, we shall find the Side SF, and the Angle HSF; from which subtracting the Angle HSC=FHS, there will remain the Angle CSF, which shews the Position of the *Apsides*.

THIS Method does suppose, indeed, that the Angles at the superior *Focus* be always proportional to the Times, which is not true. But in the Orbit of the *Earth*, whose Eccentricity is small, the Angles that are really described at that *Focus* differ so little from the Angles that are proportional to the Time, that no sensible Error can arise from thence, in determining the Species and Position of the Orbit.

THE most celebrated *Astronomer* Dr. *Edmund Halley*, from whose Labours *Astronomy* has received great Improvement, hath contrived a Method which depends on no Theory of the *Earth*'s Motion: From which, by Observation alone, the Form and Position of the Orbit are to be determined.

Table XXII. Fig. 6. SUPPOSE the *Sun* at S, ABCD the Orbit of the *Earth*, P the Planet *Mars*, who for this Purpose is to be preferr'd, or chosen before the others. First, Observe the true Time and Place, when *Mars* is in Opposition to the *Sun*; for then the *Sun*, the *Earth*, and *Mars*, are in one right Line; or if it happens (as it often does) that *Mars* has any Latitude, the *Sun*,

Sun, the *Earth*, and *Mars* his Place reduced to the Ecliptick, are in a right Line. Let the *Sun*, the *Earth*, and *Mars*, have their Places in the Points S, A, and P, in the right Line SP. Since *Mars* his Period consists of 687 Days, after that Time, *Mars* will return to the Point P; and seen from the *Sun*, he will appear in the same Place as before, in which he was seen also from the *Earth*: But the *Earth* does not return to A, till after 730½ Days; and therefore, when *Mars* is in P, it will be in B, and will observe the *Sun* in the Line SB, and *Mars* in the Line BP. By observing the Places of the *Sun* and *Mars*, we have all the Angles of the Triangle PBS. And supposing PS to consist of 100000 Parts, we can find the Distance SB, in those Parts, as likewise its Position. After the same Manner, when *Mars* has finished another Period, the *Earth* will be in C, and we can find the Length of the Line SC, and its Position; and likewise, by the same Method, another Line SD, and its Position may be obtained. And by this Means we are come to this *Geometrical* Problem: Having three Lines meeting in the *Focus* of an Ellipse, all given in Length and Position, to find the Length of the transverse Axis, its Position and the Distance of the *Foci*: Which Problem, the *Geometers* shew how to construct; and we, in the following Lectures, will likewise give its Solution.

LECTURE XXV.

Of the Equation of TIME.

Motion, the Measure of Time.

ALTHOUGH *Time* be in its own Nature, a real Quantity, as being endowed with the chief Properties of Quantity, Equality, Inequality, and Proportion; yet to measure this Quantity, we must have the Aid and Assistance of Motion, as a Measure to estimate and compare the Quantities of *Times*; and therefore Time when it is considered as measurable, marks out some Motion: For if all Things were at rest, we could by no means know the Flux or Quantity of *Time*, and the Duration of all Things would go on without Perception.

Uniform Motion the proper Measure.

BUT because *Time* constantly flows equally, and in the same Manner, to measure it, we must make Use of such a Motion, as is in itself simple, uniform, and always going on at the same Rate; so that the Body which has this Motion, at least as to its Periods, may always keep the same Force, and yet go through equal Spaces in equal *Times*.

The Motions of the Sun *and* Moon, *the fittest Measures.*

FOR common Use, we must take that Motion which is most remarkable, evident to every Body, and plain to common Sense; such is the Motion of the *Stars*, and chiefly of the *Sun* and *Moon*, which not only, by the common Consent of all Mankind, are agreed upon for this Effect, but by the Almighty and wise Creator of the Universe, are established for this Purpose: For in the

24

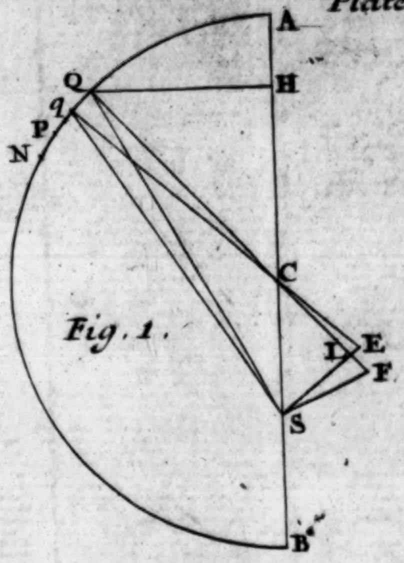


Fig. 1.

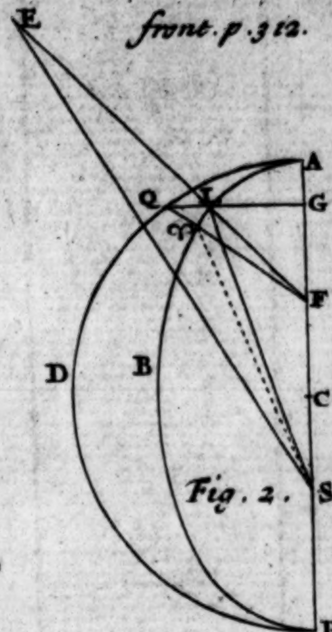


Fig. 2.

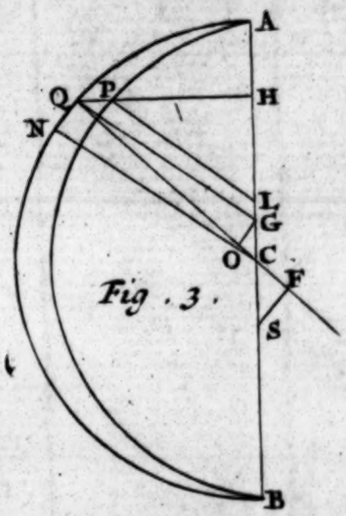


Fig. 3.

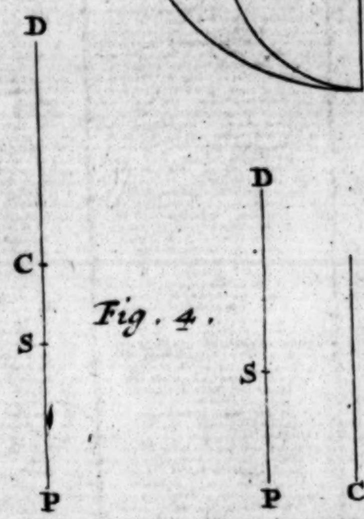


Fig. 4.

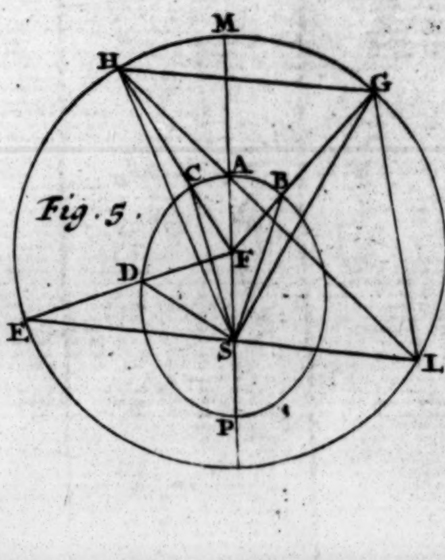


Fig. 5.

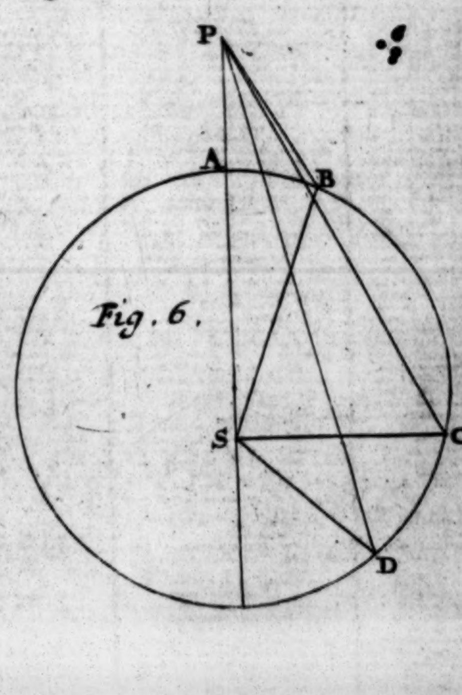


Fig. 6.

the Scriptures we read, that God said, *Let there be Lights in the Firmament of the Heaven, to divide the Day from the Night; and let them be for Signs and for Seasons, and for Days and for Years.* And therefore by the celestial Motions, and chiefly by that of the *Sun*, are the Times rightly distinguished and marked out. Who therefore dare say that the *Sun* will not tell us Truth! The *Astronomers* are the bold Men who tell us so; for they, by their nice Search into Things, have found that the *Sun*'s apparent Motion is no ways equal; they observe, that he now and then slackens his Pace, and afterwards quickens it again: And therefore *Equal Time*, which goes on always at the same Rate, cannot truly be measured by the *Sun*'s Motion.

Lecture XXV.

The Distinction between the True and apparent Time.

HENCE the *Time* which the *Sun*'s Motion shews, and which is called the *Apparent Time*, is different from that Time which flows uniformly and always at the same Rate, which is called by the *Astronomers* the *True and Equal Time*; according to which all the celestial Motions are to be estimated, regulated and settled. For, upon the Account of the unequal Motion of the *Sun*, and the Obliquity of the Ecliptick to the Æquator, we have neither Days nor Hours perfectly equal, as we shall here shew.

THE solar Day is that Space of *Time* which passes while the Plane of the Meridian of any Place, passing through the Center of the *Sun* by the *Earth*'s Revolution turning round its own Axis, returns again to the *Sun*'s Center; or it is the *Time* between one Mid-day, and the next which comes after. Now if the *Earth* had no other Motion but that round its Axis, all the Days would be exactly equal to one another, and to the Time of the Revolution round the said Axis. But because while the *Earth* is whirling round its Axis, it is also going forward in its proper Motion Eastward; when any Meridian has compleated its Revolution, after having passed the *Sun*'s Center, its Plane will not have then passed through the *Sun*, as is plain by the Figure. For let the

Lecture XXV.

Table XXIII. Fig. 1.

the *Sun* be S, AB a Portion of the Ecliptick; let the Line MD represent any Meridian, whose Plane produced, passes through the *Sun* when the *Earth* is in A: Let the *Earth* proceed in its Orbit, and come to B, while it has compleated a Revolution round its Axis; and then the Meridian MD will be in the Position *md*, parallel to the former MD; and consequently will not as yet have passed through the *Sun*, nor will the Inhabitants under that Meridian have had their Mid-day. But the Meridian *d m*, with its angular Motion, must still go on, before its Plane can pass through the *Sun*, and must describe the Angle *d*B*f*. And therefore all the solar Days are longer than the *Time* of one Revolution round the *Earth*'s Axis. If the Planes of all the Meridians were perpendicular to the Plane of the *Earth*'s Orbit, and the *Earth* described this Orbit with an equal Motion, after any Meridian has compleated its Revolution, because MD and *md* are parallel, the Angle *d*B*f* would be equal to the Angle BSA, and the Arches *df* and AB similar: And because the *Times* are always equal, the Arch AB, and the Angle *d*B*f* would constantly be of the same Quantity, and all the solar Days would be equal to one another; and then the apparent and equal *Time* would agree. But neither of these two Cases have Place in Nature, for the *Earth* does not proceed in its Orbit with an equal Motion; but in its *Aphelion* it describes a less Arch, in its *Perihelion* a greater. And moreover the Planes of the Meridians are not perpendicular to the Ecliptick, but to the Æquator: And therefore the *Time* of the angular Motion *d*B*f*, which, besides the intire Revolution, is to be added to the Space of a solar Day to compleat it, is not always of the same Quantity; whence the solar Days will not be equal.

It is proved that the solar Days are unequal.

The same is made plain by the apparent Motion of the Sun.

BUT perhaps this may appear plainer if we pass from the real Motion of the *Earth*, and consider the apparent Motion of the *Sun*; for it is by his apparent Motion, that we measure the *Apparent Time*.

And

And therefore you must observe, that the natural solar Day is that Space of *Time*, in which, by the Revolution of the whole Heavens, which is called the Revolution of the *Primum Mobile*, or of the first moveable Orb, the whole Circumference of the Æquator passes through the Meridian; and also so much more of the same Circle, as answers to the apparent Motion of the *Sun* to the East, in the mean while. Lecture XXV.

But the Arch of the Æquator passing through the Meridian, is not always equal to the correspondent Arch of the Ecliptick, which passes through the same in the same Time; but is sometimes bigger, and sometimes less than it, even though the *Sun*'s Motion were equal in the Ecliptick: The Difference between them arises from the oblique Position of the Ecliptick to the Æquator, as is plain by the Figure. Suppose ♈ ♋ a Quadrant of the Ecliptick, and ♈ E a Quadrant of the Æquator. Suppose the Arch ♈ A to be one Degree, which is nearly equal to the diurnal Motion of the *Sun* in the Ecliptick; for this mean Motion is 59′ 8″. Let A B be an Arch of the Circle of Declination, passing through the *Sun* in A, and intercepted between the Æquator and the Ecliptick. In the right-angular Triangle ♈ B A, having the Side ♈ A 1 Degree, and the Angle A ♈ B, which is the Inclination of the Ecliptick to the Æquator, and is nearly $23\frac{1}{2}$ Degrees, we shall find the Side ♈ B 55′ 1″, almost 5′ less than the Arch ♈ A. Again, suppose the Arch of the Ecliptick ♈ C 89°. From thence we shall find the Arch of the Æquator ♈ D, 88° 54′ 34″; but when the Arch of the Ecliptick ♈ ♋ is 90 Degrees, then A E, the correspondent Arch of the Æquator, is also 90 Degrees. And the Difference of the Arches ♈ C, ♈ D is 1° 5′ 26″. And the Difference of the Arches of the Æquator ♈ B and DE is 10′ 25″, although the Arches of the Ecliptick ♈ A and C ♋, which answers to them, are equal. From which it is evident, that the Arches of the Æquator, answering

The diurnal Arches of the Æquator are not equal to the diurnal Arches of the Ecliptick.

Table XXIII. Fig. 2.

The first Cause of the Inequality of Days.

Lecture XXV. ſwering to equal Arches of the Ecliptick, are unequal: And therefore the diurnal Arches of the Æquator, which paſs through the Meridian, are unequal; but they meaſure the ſolar Days: Wherefore the ſolar Days are unequal.

The ſecond Cauſe of the Inequality of Days.

BUT the Obliquity of the Ecliptick is not the only Cauſe of the Inequality of Days, for the very apparent Motion of the *Sun* in the Ecliptick is unequal; for he proceeds more ſlowly, and ſtays longer in the Northern Signs than in the Southern, by eight intire Days: And therefore, if there were no Obliquity of the Ecliptick, by this Cauſe alone, the diurnal Arches of the Æquator could not be equal. And therefore their Inequality will be much greater, upon Account of theſe two Cauſes concurring together; that is, the unequal Motion of the *Sun*, and the Obliquity of the Ecliptick, which though they are ſometimes contrary to each other, and ſo diminiſh the Inequality; as it happens when the Arches of the Æquator decreaſe, on the Account of the Obliquity of the Ecliptick, but by reaſon of the *Sun*'s approaching the *Perigæon* they increaſe, and on the contrary: Yet ſometimes theſe two Cauſes concur to increaſe the Inequality, and neither of them depend one on the other, but each of them by itſelf has its Effect.

SINCE therefore the apparent Motion of the *Sun* to the Eaſt is unequal, it cannot be a fit Meaſure of *Time*, which ſhould always go on at the ſame Rate. And therefore the natural and apparent Days are no ways to be applied to meaſure the celeſtial Motions, which do not depend upon the Motion of the *Sun:* Therefore the *Aſtronomers* found it neceſſary, inſtead of theſe ſolar Days, to ſubſtitute in their Place others that were equal, and a mean between the ſhorteſt and the longeſt, and by them to diſtinguiſh the celeſtial Motions. And when theſe Motions have been computed according to the *Equal Time*, it is neceſſary to turn that *Time* again into the *Apparent Time*, that theſe Motions may be obſerved by us who meaſure and number our *Times*, by the apparent

apparent Motion of the *Sun*. And on the contrary, if any Appearance in the Heavens, as for Example, an Eclipse were observed according to the *Apparent Time*, and according to its *Astronomical* Tables were to be examined, to see if they did agree with it or not; it will be necessary to turn the *Apparent Time* into *Equal Time*, otherwise the observed *Phænomenon* will differ from that which is found by Computation.

Because we know no Body in Nature, which preserves constantly a perfect uniform Motion, and yet such a Motion is only proper to measure equal Days and Hours; it is convenient to imagine some Body or Star, which moves in the Æquator Eastward, and which never quickens or slackens its Pace, but goes through the Æquator, in precisely the same Time as the *Sun* finishes his Period in the Ecliptick. The Motion of such a Star will rightly represent equal Time, and its diurnal Motion in the Æquator will be daily 59′ 8″, the same as is the mean Motion of the *Sun* in the Ecliptick: And therefore the equal and middle Day is to be determined by the Arrival of this Star to the Meridian, and is equal to the Time in which the whole Circumference of the Æquator, or 360 Degrees, passes the Meridian; and besides that 59′ 8″: And because this Addition of 59′ 8″ always remains the same, all these mean Days will be constantly equal to each other.

The equal and middle Day determined.

Since the *Sun* goes unequally Eastward according to the Æquator, sometimes it will come to the Meridian sooner than this imaginary Star, and sometimes he will touch it later than it does; and the Difference is that which is between the *True* and *Apparent Time*. And this Difference is known by having the Place of the imaginary Star in the Æquator, and the Point of the Æquator, which comes to the Meridian with the *Sun*; for the Arch intercepted between them, being converted into *Time*, shews the Difference between *Equal* and *Apparent Time*, which is called the *Equation of Time*. And it is the *Time* that flows, while

The Equation of Time.

Lecture XXV.

while the Arch of the Æquator intercepted between the Point determining the Right Ascension of the *Sun*, and the Place of the imaginary Star, passes the Meridian.

When the Apparent Time *is faster than the true*;

Let ÆQ be a Portion of the Equinoctial, EC of the Ecliptick, in which imagine S to be the Place of the *Sun*, SA a Circle of Declination passing thro' the *Sun*, and meeting with the Æquator in A, which will be the Point which comes to the Meridian with the *Sun*. Suppose *m* to be the Place of the imaginary Star, which performs its Period in the Æquator, with an equal Motion: And when the *Sun* has arrived at the Meridian, our imaginary Star will be distant from it, by the Arch *m* A; and if the Point *m* be Eastward of the Point A, it will come later to the Meridian than it, and the *Apparent Time* will be faster than the mean; but if the Place of the Star *m*, be more Westerly than A, it will sooner arrive at the Meridian, and the *Apparent Time* is slower than the Mean. And the Arch of the Æquator A *m*, converted into *Time*, is the Equation; which being added to, or subtracted from the *Apparent Time*, gives the true, according as the Point *m* is to the East or West of the Point A; and then we have the true *Time*. For to know the Position of the Point A, in respect of *m*, and the Quantity of the Arch A *m*, take in the Æquator the Arch ♈ s or ♎ *s*, equal to the ♈ S or ♎ S in the Ecliptick; and then the Arch *s m*, will be the Distance between the true and mean Place of the *Sun*, which therefore is given by the *Sun*'s *Anomaly*; but the Arch A *s* is the Difference between the *Hypothenuse* ♈ S, of the right-angled Triangle ♈ S A, and its Side or Base ♈ A, which may be found by *Trigonometry*. Moreover, the Arch A *m* is equal to either the Sum or Difference of the Arches A *s* and *s m*; and therefore when they are known, the Arch A *m* will be likewise known.

Table XXIII. Fig. 3, 4. *When slower than the true.*

The Equation of Time *consists of two Parts.*

Moreover we must observe, that in the first and third Quadrant of the Ecliptick, the Point *s* falls upon the East-side of the Point A: And therefore

fore the Arch A s, being turned into *Time*, is to be subtracted, because the Point S comes later to the Meridian than the Point A does. But in the second and fourth Quadrants, the Point s is more Westerly than A, and arrives at the Meridian before it: And therefore the Arch A s, turned into *Time*, is to be added, to get the *Time* when the Point S arrives at the Meridian. Suppose, for Example, A s be two Degrees, as it is when the *Sun* is in the 20th Degree of ♈; this Arch, turned into *Time*, is 8 Minutes: And therefore we must add 8′ to the *Apparent Time*, to have the *Time* when the Point S comes to the Meridian.

The Effect of the two Parts severally explained.

MOREOVER, in the first Semicircle of *Anomaly*, that is, while the *Sun* in this present Age, goes from the seventh Degree of ♋, to the seventh of ♑, the mean Motion of the *Sun* is greater than its true Motion; therefore, then the middle Place precedes the true Place. And therefore in all that Semicircle the Point m will be to the East of the Point s; and the Arch $m\,s$, turned into *Time*, is to be subtracted from the *Time* that the Point S reaches the Meridian: But in the other Semicircle of *Anomaly*, after the *Sun* has left the *Perigæon*, or rather the *Earth* the *Perihelion*, the mean Motion is less than the true, and then the mean Place follows the true; and consequently the Point m is to the West of the Point S, and comes sooner to the Meridian than it does: And therefore the Arch $m\,s$, turned into *Time*, is to be added to the *Time* in which S touches the Meridian. Having now the Distance of *Time*, between the Coming of the Point m to the Meridian, and the Coming of the Point S to the same; as also the Distance of *Time*, between the Arrivals of the Points s and A to the Meridian, we shall have the Distance of *Time*, between the Coming of the Point m to the Meridian, and of the Point A's Arrival to the same; that is, we

shall

Lecture XXV.

shall have the Difference between the *Apparent and True Time*, which is the *Equation of Time*.

Two Tables for equating Time.

FOR the equating of *Time*, the *Astronomers* compose two Tables; one for the Arch *s m*, which is to be entered into with the *Anomaly* of the *Sun:* And if the Point *m* be Westward of the Point *s*, they mark it with + or the Sign of Addition; but if it be on the other Side, they place — or of Subtraction, The other Table is for the Arch *s* A, which is the Difference between the Place of the *Sun* in the Ecliptick, and his Right Ascension. And the Equations of this Table likewise marked with the Signs of Addition or Subtraction, as the Point *s* is to the West or East of the Point A. The Sum of these two Equations, if they be of the same Kind, that is, both to be added, or both to be subtracted; or their Difference, if their Affections be of different Kinds, make up the absolute *Equation of Time*.

The Temporary Table.

THE Artists likewise make a Table composed of both the former; but it will only serve for a Time: Yet it may be used for a whole Age without any sensible Error; for the same Degree of *Anomaly* keeps nearly in the same Degree of the Ecliptick, the Space of an Age: And therefore for the Space of 50 Years these two Equations may be joined in one. But because of the Precession of the Equinoxes, the Sun's *Apogæan*, in Process of Time, changes its Place in the Ecliptick, and goes Easterly with the Fixed Stars: And therefore, in different Ages, the same Degrees of *Anomaly* will fall upon different Degrees of the Ecliptick: And therefore one Table will not serve for all Ages.

When the solar Days begin to be longer than the mean.

THE imaginary Star, by whose equal Motion we measure Time, goes constantly and uniformly forward to the East: But the Point A, which determines the Right Ascension of the *Sun*, and marks out the *Apparent Time*, has as it were a Libration, or goes backwards and forwards in respect

respect of *m*: Sometimes it gets to the East of *m*, and sometimes is to the West of it, and sometimes coincides with it. And therefore when the Point A has its relative Motion in respect of *m* Eastward, the Point A is more East than the *Star m*, and then the Days are longer than the mean Days, which are measured by *equal Time*; and the faster the Point A goes to the East, so much the longer are the solar Days: For, besides the Revolution of the whole Heavens, the Arch to be added to make up the solar Day is greater, because the Point A goes a greater Space Eastward. Hence it follows, that as soon as the relative Motion of the Point A begins to be Eastward, the solar Days begin likewise to be longer than the mean Days. I speak of the relative Motion in regard of the Point *m*, for the absolute Motion of A is always Eastward; but when the Point A is gone its farthest Distance from *m* to the East, then it begins to come back again to *m*, and has its relative Motion Westward: But before that, the Point A will be for a while stationary in respect of *m*, in the middle Time between its Recess and Access; and then the solar Days will be equal to the mean Days, and in these Points the Equations will be greatest. When the Motion of the Point A to the East is quickest, there the Days become the longest; and where it is the slowest, that is, where the Motion relative to *m* is Westward, and greatest, there the Days are shortest. In the present Age wherein we live, when the *Sun* is in the 10th Degree of *Scorpio*, the Point is at its farthest Distance from *m* to the West of it; and its Distance then amounts to 4° 2′ 45″, and therefore the greatest *Equation* in Time is 16′ 11″. From thence the Days begin to increase, till the *Sun* comes to 22½ Degrees of *Aquarius*, where it has gone to its farthest Distance from *m* Eastward, it being there removed 3 Deg. 42½ Min. whence the greatest *Equation* of Time is 14′ 50″: And from thence the relative Motion of the Point A begins again to be Westward, till the

Lecture XXV.

When the mean and solar Days are equal.

At what Times of the Year the Equations *are greatest.*

Lecture XXV. *Sun* comes to the 24th Degree of *Taurus*, or the *Bull*; and there the Point A is removed from *m* 1 Deg. $1\frac{1}{2}$ Min. to the West of the Star *m*, and the greatest *Equation* of Time is 4′ 6″. Thence it returns again to *m*, and moves Easterly, till the *Sun* comes into the $3\frac{1}{2}$ Deg. of the *Lion*; where A is distant from *m* 1 Deg. $28\frac{1}{2}$ Min. and the greatest *Equation* of Time is 5′ 53″: And from thence its Motion begins to be to the West, 'till the *Sun* arrives at the 10th of the *Scorpion*, and thereabouts it changes its Course and goes Eastward. It is plain, that when the Points A and *m* coincide, that there the mean and *Apparent Time* must likewise coincide. Hence, if we have a Pendulum Clock, accurately and nicely fitted, and the Motion of the Hand set to equal or *true Time*, the Hand of this Clock will always point out the *Time* different from the solar *Time* shewed by a Sun-Dial; except four Times a Year, which is about the 4th of *April*, the 6th of *June*, the 20th of *August*, and the 13th of *December*. At all other Times the Hour, by the Sun-Dial, will either be before, or later than that shewed by the Hand of the Clock: And about the 23d of *October* the Clock will differ most of all from the *Sun*, where its Motion is slower than the solar *Time* by 16′ 11″.

If you inquire in what Points the *Equations* are the greatest, the eminent Dr. *Edmund Halley*, who for his great Inventions is never to be mentioned by *Astronomers* without Honour, has given us the Solution of this Problem. But to understand it, we must first lay down the following *Lemma*.

LEMMA.

IF any plain Figure be projected orthographically on a Plane, which is done by letting fall, from every Point of it, Perpendiculars on the Plane of Projection, the Area of that Projection of the Figure will be to

 the

the Figure projected, as the Co-sine of the Inclination of the Planes is to the Radius. Lecture XXV.

For any Figure can always be resolved into Parallelograms, or Triangles, whose Bases are parallel to the common Section of the Planes, and therefore they will be parallel to the Plane on which they are projected: Wherefore the Bases of these Parallelograms, or Triangles, and their Projections on the Plane, are always equal to each other, and parallel, as we have shewed in *Lecture* XIII. But the Perpendiculars let fall from the Summits, or Tops of the Triangles, and Parallelograms upon the Bases, are also perpendicular to the common Section of the Planes, by the 29th of the first *El.* And therefore the Inclination of the Perpendiculars to the Plane is equal to the Inclination of the Planes to each other. And consequently the Projections of these Perpendiculars will be to the Perpendiculars themselves, as the Co-sine of the Inclination of the Plane is to the Radius. Wherefore every Parallelogram, or Triangle, is projected into another, whose Base is equal to the Base of the Triangle, or Parallelogram projected; and its Height is to the Height of the Figure projected, as the Co-sine of the Inclination of the Planes is to the Radius. But Triangles and Parallelograms, whose Bases are equal, are as the Perpendiculars let fall from the Tops upon the Bases. The Projection therefore of each Triangle is to the Triangle projected, in a constant and given Proportion; consequently, all the Projections of all the Triangles, or Parallelograms, are to the Figures projected in the same Proportion; that is, the Projection is to the Figure projected, as the Co-sine of the Inclination is to the Radius.

If the Orbit of the *Earth* be orthographically projected on the Plane of the Æquator, by letting fall from each of its Points Perpendiculars, the Projection will be an Ellipse, in whose Perimeter the Extremity of a right Line, let fall from the *Earth* perpendicular to the Plane of the Æquator, will

 constantly

Lecture XXV.

constantly move: And this Point, by its Motion, will mark out the right Ascension of the *Earth*, or its Motion according to the Æquator, as it is to be seen from the *Sun*; to which the right Ascension of the *Sun*, seen from the *Earth*, is always equal. Let ♈ A ♎ C be the Ellipse in which the Orbit of the *Earth* is projected, S the Point of Projection of the *Sun*'s Center, ♈ S ♎ the common Intersection of the Æquator and the Ecliptick, A any Point, where a Perpendicular from the *Earth* meets with the Projection: The Angle ♈ S A will measure the right Ascension of the *Sun*. Now, I say, that this Point A, which marks out the Motion of right Ascension, will so proceed in the Ellipse ♈ A ♎ C, that it will describe about the Point S, elliptick Areas proportional to the Times. For, in a given Time, let A move thro' the elliptick Arch A B; draw the Lines A S, and BS; and the trilineal Figure ASB will be the Projection of the correspondent Area, which the *Earth* describes in the Plane of the Ecliptick, in the same Time round the *Sun*: And therefore the Projection ASB, will be to the correspondent Area in the *Earth*'s elliptick Orbit, as the Co-sine of the Inclination of the Æquator, and the Ecliptick, is to the Radius. But in the same Proportion is the whole elliptick Area ♈ A ♎ C, to the whole Area of the *Earth*'s Orbit: Therefore, by Permutation of Proportion, the trilineal Figure ASB will be to the whole elliptick Area ♈ A ♎ C, as the Area, described in the *Earth*'s Orbit round the *Sun*, is to the whole Area of the *Earth*'s Orbit; that is, as the Time in which that Area in the Orbit of the *Earth*, or the Area A S B in the Projection, as described, is to the whole periodical Time. Therefore, the Point A moves in the Perimeter of the Ellipse at such a Rate, that it describes, about S, Areas that are continually proportional to the Times.

Plate XXIII. Fig. 5.

THE same Things being laid down; at the Center S, and Distance S A, which is a mean Proportional between half the greater and half the lesser Axis of the Ellipse, describe a Circle; this Circle will be equal

equal to the whole elliptick Area; as it is eaſy to demonſtrate from the Doctrine of the *Conick Sections*: This Circle will cut the Ellipſis in four Points E, F, G, H: The Points of Interſection ſhew the right Aſcenſions of the *Sun*, where the Equations are greateſt. Imagine a Point M, to move uniformly in the Periphery of the Circle; its Motion will then repreſent the Motion of our imaginary Star *m*, and will deſcribe about the Point S, circular Sectors that are proportional to the Time: And becauſe the Area of the whole Circle, and the Area of the Ellipſe, are equal, the Areas of the elliptick Sectors, and of the circular Sectors, deſcribed in the ſame Time, will be conſtantly equal. Let us now ſuppoſe, that the Point M, in the Periphery of the Circle, and the Point that marks out the *Sun*'s right Aſcenſion in the Ellipſe, be placed at the ſame Time, both in the right Line SLM. Let theſe Points afterwards be in *m* and A, then the elliptick Area LSA will be equal to the circular Sector MS*m*: And becauſe the Arch M*m* is without the Ellipſe, the Angle MS*m* will be leſs than the Angle MSA, and the Difference of the Angles meaſured by the Arch *m*A, which is the *Equation of Time*. When the Point, which marks out the right Aſcenſion of the *Sun*, comes to the Interſection F of the Circle and Ellipſe; there its angular Motion round the *Sun* will be equal to the angular Motion of the Point *m*; for the Areas *m*S*n*, and ASF, are equal, they being both deſcribed in the ſame Moment of Time, and at the ſame Diſtance from S; and conſequently the Arch *q*F is equal to the Arch *mn*. In the Point therefore, F, the Motion of Right Aſcenſion is equal to the Motion of the imaginary *Star* or equal to the mean Motion. The ſame Thing may be ſhewed at G, H, and E: But it was ſhewed before, that where the Motion of Right Aſcenſion was equal to the Motion of the Point *m*, that there the Equations are greateſt. Wherefore, in the Points F, G, H, and E, are the Equations greateſt.

Lecture XXV.

Plate XXIII. Fig. 6.

Lecture XXV.

Which are the Points of the Ecliptick where the Days are longest, and where shortest.

Plate XXIII. Fig. 7.

If you inquire in what Points the Days are longest or shortest, the same eminent Dr. *Halley* has also given us a geometrical Solution of this *Problem.* Let ♈ ♋ ♎ ♑ be the Ellipse into which the *Earth*'s Orbit is projected and S the Point of the *Sun*'s Center, K the Center of the Ellipse. Produce K S on each Hand, so that K G and S H may be to K S (which is the Projection of the Eccentricity) as the Square of the Radius is to the Square of the Sine of the Obliquity of the Ecliptick: Thro' K draw ♈ ♎ parallel to the common Section of the Æquator and Ecliptick, and cut it at right Angles with the Line ♋ K ♑: Thro' G draw G F, and thro' H draw F H, parallel to the Lines ♋ ♑ and ♈ ♎; and thro' S and K describe the Hyperbola A B, whose Asymtots are F G, F H. This Hyperbola, and its opposite C D, will cut the Ellipse in the Points that are required; that is, when the *Sun* is in the Points of the Ecliptick which correspond with B and D, then the Days are the longest, and in B the Days are longer than in D: But the Points of the Ecliptick, which answer to A and C, give us the Places where the Days are shortest. The Demonstration of this depends on the Motion of the Point that marks the right Ascension of the *Sun* round S; for it describes about it Areas that are proportional to the Times. Therefore, the angular Velocity is every where reciprocally as the Square of the Distance from S; consequently, the Velocities must be greatest where these Distances are least; that is, where the least Lines than can be drawn from S fall upon the Ellipse; and the Velocities are the least, where the Lines drawn from S to the Ellipse are the greatest: But by the Construction, and the 62d *Prop. Lib.* V. of *Apollonius*'s *Conicks*, it is evident, that the Hyperbolas will cut the Ellipsis in the Points A and C; where the right Lines S A and SC are the greatest; and in the Points B and D, where the right Lines S B and S D are the least: For in these Points the Lines S A, S B, S C, S D, are perpendicular to the Curve.

Curve. Hence the Motion of the *Sun*, according to his Right Afcenfion, will be quickeft in B and D; and therefore the Days will be then the longeft; and the Motion being floweft in C and A, the Days will be there the fhorteft.

Lecture XXVI.

LECTURE XXVI.

Of the Theories of the other Planets.

The Theories of the Planets attained by the Earth's *Motion.*

AFTER having explained the Theory of the *Earth's* annual Motion, and fhewed the Methods by which the Form of its Orbit, and the Pofition of the *Apfides* are determined, we may then, by the Help of *Aftronomical* Tables, compute for any Time, the Place of the *Earth* in the Ecliptick from the *Sun*, and its oppofite Point in which the *Sun* appears to be, as he is obferved by us. We will now come to explain the Theories of the other Planets, the Knowledge of which cannot be attained without the *Earth's* Motion being perfectly known.

The heliocentrick and geocentrick Place of a Planet that is in Oppofition to the Sun *coincide.*

But the Periods of the Planets, or the Times they take to complete their Circulations, are to be found out in the firft Place. For which Purpofe we muft obferve, that when any fuperior Planet comes to an Oppofition with the *Sun*, they then appear in the fame Point of the Ecliptick, feen

Lecture XXVI.

from the *Earth*, as they would do if the Eye were in the *Sun*, and obſerved them from thence; as alſo the inferior Planets, when they are in Conjunction with the *Sun*, and are obſerved in the *Sun*'s Diſk: If there were an Obſerver in the *Sun*, he would ſee them in the oppoſite Point of the Ecliptick, in which we behold them: And therefore, whenever a ſuperior Planet is in Oppoſition to the *Sun*, his heliocentrick and geocentrick Places coincide: But when an inferior Planet is in Conjunction with the *Sun*, and is ſeen in his Diſk, the heliocentrick and geocentrick Places are oppoſite to each other. Moreover, in the inferior Planets, when they are at their greateſt Elongations from the *Sun*, the Angle at the *Sun*'s Center, contained between the right Lines drawn to the *Earth* and Planet, is nearly the Complement of the Elongation: For in Orbits which are nearly circular, a Line touching the Orbit is almoſt perpendicular to the Line drawn from the *Sun* to the Point of Contact: And therefore that Angle will be given. But we have the Point of the Ecliptick, in which the *Earth* is at that Time, ſeen from the *Sun*; and conſequently, the Point of the Ecliptick in which the Planet is, ſeen from the *Sun*. And therefore, in theſe Poſitions, we have the heliocentrick Places of the Planets.

How to find nearly the periodical Time of a Planet.

If then any ſuperior Planet, as for Example *Jupiter*, were obſerved when he is in Oppoſition to the *Sun*: And again, if he were likewiſe obſerved when he comes next in Oppoſition to the *Sun*; we ſhall have that Arch which the Planet, ſeen from the *Sun*, has in the mean Time deſcribed. Say, as that Arch is to the whole Circumference, ſo is the Time between the Obſervations to a Fourth, which will be nearly the periodical Time of the Planet. After the ſame Manner by obſerving two heliocentrick Places of an inferior Planet, we may nearly collect its periodical Time. I ſay nearly, and not exactly; for the

23

front. p. 328.
Plate XXIII

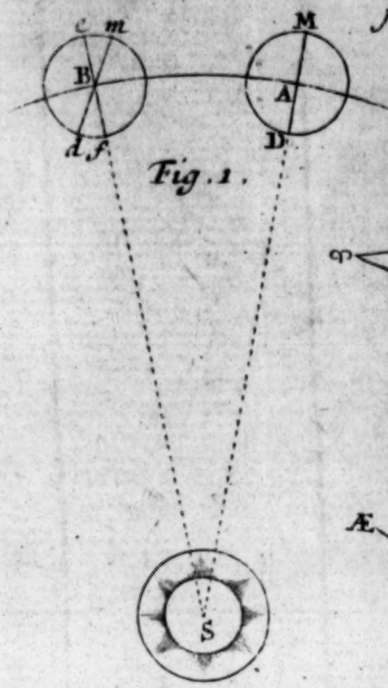


Fig. 1.

Fig. 2.

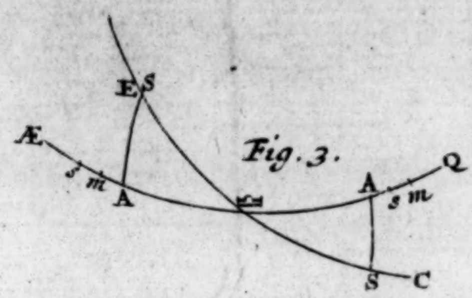


Fig. 3.

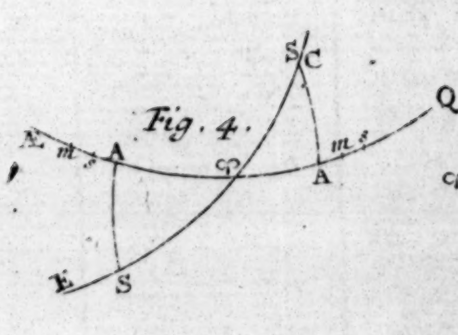


Fig. 4.

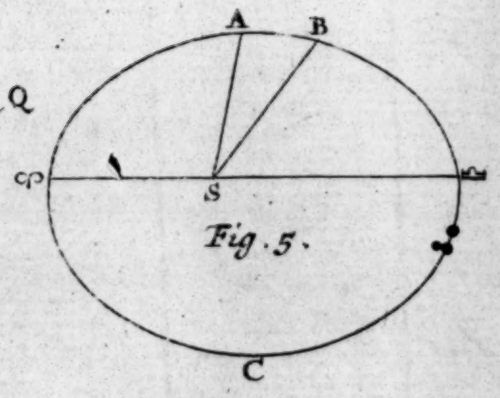


Fig. 5.

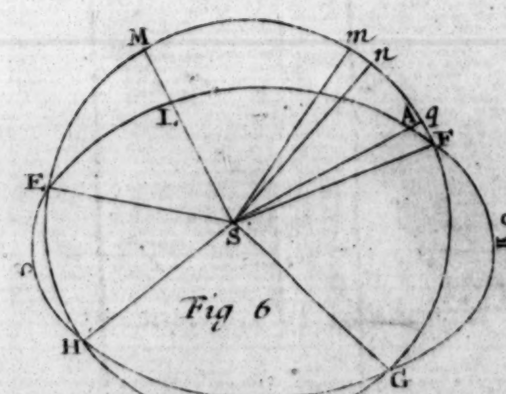


Fig 6

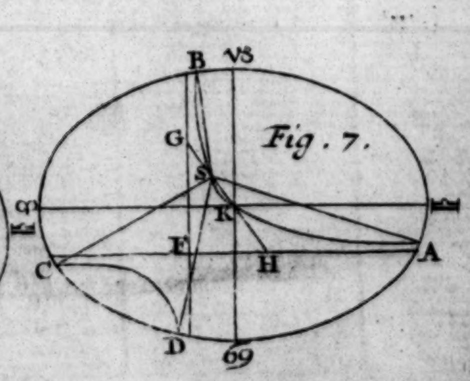


Fig. 7.

the Calculation supposes, that the Motion of the Planet is in a Circle, and that thro' the whole Period it is uniform, which is not true.

Lecture XXVI.

A more accurate Method of finding the same.

The following Method for finding the periodical Time is more accurate. Observe a Planet twice successively in the same Node; that is, let there be two Observations, when the Planet about the same Part of its Orbit, has no Latitude, which can only happen when the Planet has really no Latitude, and is placed in its Node: The Time between the two Observations will be equal to the periodical Time of the Planet. For, because all the Planets move in Orbits, whose Planes are different from the Plane of the Ecliptick, and the *Sun* is in the common *Focus* of all the Orbits, these Planes will all cut the Ecliptick, in Lines which pass through the *Sun*, and in the Intersections of these Lines with the Ecliptick are the Nodes; and the Planet in the whole Time of its Period, can never be observed but once, in one and the same Node. Now the Nodes are either at rest, or they have a very slow Motion, so that for the Space of one Period, they may be esteemed as at rest. And therefore, if we know the Time between two Arrivals of a Planet at the same Node, immediately following each other, we shall likewise know the Time of a Planet's Period.

By the very same Observations, if we have the Theory of the *Earth*'s Motion, we can find the Position of the Line of the Nodes or the Points of the Ecliptick, in which that Line does intersect it. Let A T B, be the Orbit of the *Earth*, C N D the Orbit of the Planet, N S *n* the Line of the Nodes; and in the first Observation, suppose the *Earth* in T, and the Planet to be observed, in N: And because the Place of the Planet, seen from the *Earth*, is known by Observation, and the apparent Place of the *Sun*, at the Time, is known by the Theory of the *Earth*'s Motion, we have the Arch of the Ecliptick, between the two Places, or the

Table XXIV. Fig. 1.

Lecture XXVI. the Measure of the Angle N T S. In the second Observation, let the *Earth* be in *t*, and the Planet in the same Node N, and we shall have, by the same Way, the Angle N *t* S. In the right-lin'd Triangle T S *t*, we have the Sides T S, *t* S, and the Angle T S *t*, which is known by the Theory of the *Earth*; therefore, by *Trigonometry*, we can find the Angles S T *t*, and S *t* T, as likewise the Side *t* T: Therefore, from the known Angles S T *t*, take away the known Angle S T N, and we shall have the Angle N T *t*. To the known Angle *n t* S, add the Angle S *t* T, which was found out, and we have the Angle N *t* T. Then, in the Triangle N *t* T, we have all the Angles, and the Side *t* T; consequently, we shall have the Side N T, the Distance of the Planet from the *Earth*, at the first Observation. Lastly, in the Triangle N T S, we have the Sides N T and T S, and the Angle N T S, which was known by Observation; consequently we shall find the Side N S, the Distance of the Planet from the *Sun*, when he is in the Node, and the Angle T S N, which shews the Positions of the Line of the Nodes; for that Point of the Ecliptick which the *Earth* is in, seen from the *Sun* in the Time of the Observation, is known; and the Angle T S N is likewise known: Therefore we have the Point of the Ecliptick N, in which the Node is placed, seen from the *Sun*; and the Point *n* opposite to it will be the Place of the other Node. And therefore the Position of the Nodes is found.

How to find the Position of the Nodes.

To find the Inclination of the Planet's Orb to the Ecliptick.

THE Places of the Nodes being once determined, we shall easily find the Inclination of the Plane of the Planet's Orbit to the Ecliptick: For having the Places of the Nodes, we can find the Time when the *Earth*, seen from the *Sun*, is in one of them. At the same Time, observe the *geocentrick* Latitude of the Planet, and his Distance from the opposite Node: Then the *geocentrick* Latitude of the Planet is equal to the *heliocentrick* Latitude it will have, when seen from the *Sun*, at the same Distance from the *Node*;

Plate XXIV. Fig. 2.

Node; that is, make the Angle nSp, equal to the Angle nNP, and the Latitude of the Planet at p, observed from S, will be equal to its Latitude at P, observed from N. For let CPD be the Orbit of the Planet, NSn the Line of the Nodes, BNT a Portion of the Orbit of the *Earth*, in which suppose the *Earth* to be at N, in the Line of the Nodes, and the Planet in its Orbit to be at P; and then the *Sun Earth* and Planet, will be all in the Plane of the Planet's Orbit. From the Point P, on the Ecliptick's Plane, let fall the Perpendicular PE, and in the Plane of the Ecliptick draw the Line NE; then the Plane of the Triangle NPE will be perpendicular to the Ecliptick, and the Angle PNE will be the visible Latitude of the Planet, seen from the *Earth*. Through S, draw Spf parallel to NP, and pe parallel to PE; and the Plane through Sp, and pe, will be parallel to the Plane NPE, and consequently perpendicular to the Plane of the Ecliptick; and therefore Se, the common Section of the Ecliptick, with this Plane, will be parallel to NE. And because NP and NE are parallel to Sp and Se, the Angle pSe, the *heliocentrick* Latitude, will be equal to the Angle PNE, the Latitude of the Planet observed from the *Earth*, in the Node N. Lecture XXVI.

LET nf be a Portion of the Planet's Orbit, extended to the Heavens, nh a Portion of the Ecliptick, fh a Circle of Latitude passing through the *heliocentrick* Place of the Planet. In the right-angled spherical Triangle nfh, having nh, the Distance of the Planet from the Node, equal to what was observed when the *Earth* was at N, and the Latitude fh, equal also to what was observed at N, we can find from thence the Angle hnf, the Inclination of the Plane of the Orbit of the Ecliptick, which was to be found out.

HAVING once found out this Inclination, by Observation we can find out the *heliocentrick* Place of the Planet, and his Distance from the *Sun*, when ever he comes in Opposition to the *Sun*.

LET

Lecture XXVI.

How to find the heliocentrick Place and Distance from the Sun, when the Planet is in Opposition to the Sun. Plate XXIV. Fig. 3.

LET ATB be the Orbit of the *Earth*, DPE the Orbit of the Planet: Suppose the Planet in P, and the *Earth* in T, and NS*n* the Line of Nodes, in which the *Sun* is in S. The Planet being in Opposition to the *Sun*, its Place, reduced to the Ecliptick, will be in the Line ST, which passes through the *Earth*. Observe the Angle PTE, the Planet's *geocentrick* Latitude, and we have the Angle PST, his *heliocentrick* Latitude, because we have the Planet's Distance from the Node. Likewise by the Theory of the *Earth*'s Motion, we have ST, its Distance from the *Sun*. And therefore, in the Triangle PST, having all the Angles, and one Side ST, we shall find SP, the Distance of the Planet from the *Sun*; and the Angle PS*n*, the Distance of the Planet from the Node, is found by the *heliocentrick* Latitude; by which Means we have the Place of the Planet, in its own Orbit. In the same Manner, if we have any other two Observations of the same Planet, in a Position or Aspect opposite to the *Sun*, we shall have the Position and Magnitude of three Lines, through whose Extremities the Planet's Orb passes, and the *Sun* is in one of the *Foci* of the Orbit. And therefore, to determine this Orbit, its Form and Position, we must describe an Ellipse, whose *Focus* is given, and which will pass thro' three given Points. The *Geometers* shew the Method of constructing this Problem, and we will likewise give the Solution of it hereafter.

To find the heliocentrick Place and Distance, when the Planet is in any other Aspect. Plate XXIV. Fig. 4.

THOUGH a Planet were not in an achronical Position, but in any other Aspect besides, in Opposition to the *Sun*; yet, by one Observation, its Distance from the *Sun*, and *heliocentrick* Place may be found. Suppose PAE the Orbit of the Planet, TGH the Orbit of the *Earth*. Let the *Earth* be in T, and the Planet in P, and the *Sun* in S, in the Line of the Nodes NS. From P let fall the Perpendicular PB, on the Plane of the Ecliptick: Draw BT, and produce it, till it meets with the Line of the Nodes in N, and the Plane of the Triangle NPB will be perpendicular to the Plane of the Ecliptick.

ecliptick. Let the Line CT, be alſo perpendicular to it, and meet with the Plane of the Planet's Orb in C. From T, upon the Line SN, let fall the Perpendicular TD; join the Points D and C, and the Angle TDC will be the Inclination of the Planet's Orbit to the Ecliptick, which Angle is therefore given. Obſerve the Angle BTP, the Planet's *geocentrick* Latitude, and the Angle BTS, its Elongation from the *Sun*, according to the Ecliptick. In the Triangle NTS, from the Theory of the *Earth* we have the Side ST, the Diſtance of the *Earth* from the *Sun*, and the Angle TSN, which is known by the Place of the *Earth* and Node, in the Ecliptick, as alſo the Angle STN, the Complement of the Angle STB to two Right Angles: Hence we ſhall have NT. In the right-angled Triangle TSD having TS, and the Angle TSD, we ſhall find TD; and therefore, in the right-angled Triangle TDC, having TD, and the Angle TDC, the Inclination of the Orbit to the Ecliptick, we ſhall find the Side TC. In the right-angled Triangle TCN, we have TC and NT; and therefore we can find the Angle TNC. And again, in the Triangle NTP, we have all the Angles; for the Angle NTP is the Latitude of the Planet found by Obſervation, or its Complement to two Right Angles, and the Angle PNT juſt now found, as alſo the Side NT; wherefore we ſhall find the Side TP. In the Triangle PTB, rectangular at B, we have the Side TP, and the Angle PTB, which is the Latitude obſerved; wherefore, we ſhall have the Sides BT and BP. And in the Triangle TSB, having TB and TS, and the Angle BTS, we ſhall have the Side SB, which is called the *curtate Diſtance* of the Planet from the *Sun*, as alſo the Angle TSB, and conſequently, the *heliocentrick* Place of the Planet reduced to the Ecliptick. Laſtly, In the Triangle PBS, we have the Sides PB, BS, and by them we ſhall have SP, the Diſtance of the Planet from the *Sun*, and the Angle PSB, the *heliocentrick* Latitude of the Planet. But having the *heliocentrick* Latitude, and the

Lecture XXVI.

Lecture XXVI.

the Inclination of the planetary Orbit to the Ecliptick, we can find the Planet's Distance from the Node, and consequently, his Distance from the Node, in his proper Orbit, and his centrick Place, seen from the *Sun*. If, by this Method, we find out two other *heliocentrick* Places of the Planet, and the Distances from the *Sun*, having likewise the *Focus* of the Orbit, which is the Center of the *Sun*, we may describe an Ellipse, which shall pass through the given Points, and be the Orbit of the Planet.

Dr. Halley*'s Method of determining the* heliocentrick *Place and Distance from the* Sun. Plate XXIV. Fig. 5.

THE most ingenious Dr. *Halley* contrived another Method for finding out the centrical Places of the Planet, and its Distances from the *Sun*; which supposes only, that the periodical Time of the Planet is known. Let K L B be the Orbit of the *Earth*, S the *Sun*, P the Planet, or rather the Point in the Plane of the Ecliptick on which the Perpendicular let fall from the Planet meets that Plane. And first, when the *Earth*, is in K, observe the *geocentrick* Longitude of the Planet, and having the Theory of the *Earth*, we have the apparent Longitude of the *Sun*; wherefore we have the Angle P K S. The Planet, after it has compleated an intire Revolution, does again return to the same Point P; at which Time suppose the *Earth* in L, and there again let the Planet be observed, and find the Angle P L S, the Elongation of the Planet from the *Sun*. Having the Times of the Observations, we have the Places of the *Earth* in the Ecliptick, or the Points K and L, and consequently, the Angle L S K, and the Sides L S and S K; wherefore we shall have the Angles S K L and S L K, and the Side L K. From the known Angles S K P and S L P, take away the known Angles S K L and S L K, and we shall have the Angles, P K L and P L K known; therefore, in the Triangle P L K, having all the three Angles, and the Side L K, we shall find the Side P L; and, in the Triangle P L S, having the Sides P L and L S, and the intercepted Angle P L S, we shall have the Angle L S P, which determines the *heliocentrick* Place,

and

and its Diſtance from the Node, according to the Ecliptick, as alſo the Side S P. But, as the Tangent of the *geocentrick* Latitude is to the Tangent of the *Heliocentrick*, ſo is the *curtate Diſtance* of the Planet from the *Sun* to its *curtate Diſtance* from the *Earth*. But, by Obſervations, we have the Planet's *geocentrick* Latitude, wherefore its *heliocentrick* Latitude will alſo be found. By which, and the *curtate Diſtance* of the Planet from the *Sun*, we can find out the true Diſtance which was wanted. If, by this Means, we acquire three *heliocentrick* Places of the Planet, and the correſpondent Diſtances from the *Sun*, we ſhall have the Form of the Orbit, and the Poſition of the *Apſides*, by deſcribing an Ellipſe, which ſhall paſs through three given Points. This *Ellipſis* is determined by the following Method.

To deſcribe a Planet's Orbit, or to find its Poſition and Eccentricity. Plate XXIV. Fig. 6.

LET S D, S C and S B, be three Lines given in Magnitude and Poſition. Draw D C and B C, and produce them, ſo that D F may be to C F, as D S is to C S; and alſo C E to B E, as C S is to B S. Join F E, on which, from S let fall the Perpendicular S G, which will give the Poſition of the Axis. Draw the Lines D K, C i and B H; and cut S G in A, and produce it, ſo that G A may be to S A, as K D is to S D; and alſo G a be to S a, in the ſame Proportion; and make $s\,a$ equal to S A. And the Points A and a are the *Vertices* or *Apſides* of the Orbit, whoſe *Foci* are S and s, and greater Axis A a; and if, with theſe *Apſides* and *Foci*, we deſcribe an Ellipſe, it will be of the ſame Form with the Orbit of the Planet. For, becauſe S D is to C S, as F D is to F C, and ſo D K is to C i; by Change of Proportion, D S is to D K, as C S is to C i, and ſo is S B to B H. But as D S is to D K, ſo, by Conſtruction, is S A to G A, and S a to G a; therefore, as S A : G A : : S a—$s\,a$ or S s : a G — A G, or A a. And therefore, we have S D : D K : : S C : C i : : S B : B H : : S s : A a. But this is the Property of an Ellipſe whoſe *Focus* is S, and greater Axis A a,

Lecture XXVI.

A a, as is demonstrated by the Writers of Conicks; and particularly by Mr *Milnes*, in his Treatise of *Conick Sections*, Part IV. *Prop.* 9. It is evident therefore, that the Ellipse described with the *Foci* S s, and Axis A a, will pass through the Points B, C and D.

BECAUSE in *Astronomy* a Calculation is always preferable and more useful than the neatest Construction, the Form and Position of the Ellipse is in this Manner found out by Computation. In the Triangle D S C, having the Sides D S and C S and the Angle D S C, we can find out the Side D C, with the Angles S C D and S D C; and, in the same Manner we can find in the Triangle B S C the Side B C, and the Angles S B C and S C B: And, because we have the *Ratio* of D F to C F, and we have D C, we shall also have C F. In like Manner, because we have the *Ratio* of C E to B E, and we have B C, we shall have C E and B E. But we have the Angle B C D, equal, to the Sum of two known Angles, and therefore, we have the Angle F C E, its Complement to two Right Angles. In the Triangle F C E, we have the Sides F C and C E, and the Angle F C E; wherefore we can find the Angle F E C, and, its, Complement to a Right, the Angle i C E; to which adding the known Angle S C B, we have the whole Angle S C i: And because S a and C i are parallel, the Angle C S a is equal to the Angle S C i: Having therefore the Angle C S a, we have the Position of the Axis. In the right-angled Triangle E C i, having E C and the Angle E, we can find C i; and therefore, we can find the Proportion of C S to C i. In the Triangle C S L, right-angled at L, we have the Angle C S L, the Complement of the Angle C S a to two Right Angles and the Side C S: Therefore, we shall have S L, to which adding L G, equal to C i, we have the whole S G: And because C S and C i are known, their Proportion will be known:

As

As CS is to C*i*, so let SA be to AG, and so let S*a* be to *a*G, and so let S*s* be to A*a*, and we have the *Apsides* of the Ellipse A and *a*, and its *Foci* S and *s*, which were to be found.

Lecture XXVI.

We shewed above, how by an Observation to find the *heliocentrick* Place of a Planet; and we have now shewed how to determine the Position of the *Aphelion*, by which Means we can find the Distance of a Planet from the *Aphelion*, at the Time of the Observation; this Distance from the *Aphelion* is called the *true* or *co-equated Anomaly* of the Planet. But having the Eccentricity of the Orbit, and the periodical Time, we shewed before how to find the Planet's mean *Anomaly*, in the *Lecture* in which we gave the Solution of *Kepler*'s Problem; therefore, at the Time of the Observation, we shall have the mean *Anomaly* of the Planet, or the Place he would have in his Orbit, did he constantly proceed with the same angular Motion round the *Sun*. This being once obtained, we have the Planet's mean Motion for any other Time. For say, as the periodical Time of the Planet is to the Time between the Observation and the Moment of Time for which the mean Place is desired, so is 360 Degrees, or a whole Circle, to a Fourth. This Arch, if the Time preceeded the Observation, being subtracted from the Place before found, or added to it, if it comes after the Observation, will give the mean Place of the Planet for the Time proposed.

How to find a Planet's mean Anomaly.

For the more easily finding the mean Place of a Planet, for any Moment of Time, it will be easiest to take out its Motion from the *Astronomical* Tables, in which is set down the Planet's mean Place or mean *Anomaly*, in the Beginning of any remarkable Æra; such as is from the Birth of our Lord, the Æra of *Nabonasser*, of the Creation, the Building of *Rome*, or the first Year of the *Julian* Period. The Places of the Planets, at these Instants of Time, are found by the preceeding Methods, and are computed according to the *Meridian* of *equal Time*, and

The Radix *or* Epocha.

 not

Lecture XXVI.

not for the *apparent Time*. The Place for that Time is called the *Epocha* or *Radix*, that is, the Root from which, as from an immoveable Foundation, all the Motions are calculated.

How to compute the mean Anomaly.

If the Times be numbered by the Years from the Birth of Christ, or from the Beginning of the *Julian* Period, it will be most convenient that the Year take its Beginning from that Mid-day which preceeded the first of *January*; so that at the Mid-day of the first of *January* there is one Day compleatly finished. Say, as the periodical Time is to a common Year of 365 Days, so is a whole Circle, or 360 Degrees, to a Fourth, which will be the mean Motion of the Planet for one Year. In like Manner say, as the periodical Time is to a Day, so a whole Circle is to a Fourth, and we shall have the mean *Anomaly* pertaining to one Day. And by working with the like Proportion, we shall have the mean *Anomaly* for an Hour, and for every Minute and Second. And if we add the Motion for one Year continually to itself, we shall have the Motion in two, three, and four Years: But because every fourth Year is *Bissextile*, and consists of 366 Days, to the Motion of the fourth Year we must add the Motion of one Day; then, by constantly adding the Motion of one Year, we shall have the Motion of five, six, and seven Years. But the Motion of the eighth Year is to be increased by the Motion of one Day, or rather the Motion of four Years is to be doubled, to have the Motion of eight Years, because that Year is *Bissextile*. From these Motions, so collected by Addition, we must always reject the whole Circles, or 360 Degrees; for after a Planet has completed its Circle, it returns to the same Place. By this Method we have the mean Place of the Planets to twenty Years; and if the Motion of twenty Years be constantly added to itself, we shall have the mean Place for 40, 60, 80, and 100 Years; and to each of them add the Motion of ten Years, and we have the Motions and

Places

Places for 30, 50, 70, and 90 Years. And by the continual Addition of the Motions of 100 Years, rejecting always whole Circles, we ſhall have the Motions, of 200, 300, 400, 500, *&c.* Years, even to 1000: And proceeding in the ſame Way, we ſhall have the Motion of 2000, 3000, 4000, and 5000 Years, and ſo as far as you will.

THESE Motions, being in this Way computed, are to be diſpoſed in Tables, which are called Tables of the mean Motion; or mean *Anomaly*, when they are numbered from the *Aphelion*; and they are found in *Aſtronomical* Tables for each Planet. But if the mean Motions are computed for the Equinoctial Points, inſtead of the periodical Time, you muſt take the Time the Planet revolves in the *Zodiack*, which is ſomewhat leſs than the periodical Time, becauſe of the Preceſſion of the Equinoctіes, by which the Interſection of thoſe Points meet the Planet. If the *Aphelion* of the Planet be ſuppoſed to move, this Motion muſt be conſidered; and the Motion of the Equinoxes, and of the *Aphelia*, (which, for ought we know, are equal but in the *Moon*) are likewiſe to be computed and diſpoſed in Tables, for Years, Tens, Hundreds, and Thouſands of Years, as the other mean Motions are; that for any Time, we may have the Places and Diſtances or the *Aphelia* from the Equinoxes.

THE *Aſtronomers* have ſome other Tables which give the true *Anomaly*, anſwering to every Degree of mean *Anomaly*, which are computed by ſome of the Methods delivered above in the former *Lectures*. If Minutes and Seconds are added to the mean Motions, we muſt take the Difference of the true *Anomalies*, which are one Degree diſtant from each other; and, by proportioning, we muſt add that Part which is to be added to the tabular *Anomaly* which is next leaſt, or is to be ſubtracted from it, if it be next greateſt.

FOR the Motions of the *Sun* and *Moon* they commonly compute the Equations or *Proſthaphæreſis*;

Lecture XXVI.

which are the Differences between the true and mean *Anomaly*; and they, either subtracted or added to the mean *Anomaly*, as the Planet is in the first or second Semicircle of *Anomaly*, give the True.

The Argument of Latitude.

HAVING the Places of the *Aphelion* and *Nodes*, we have their Distance from each other: And therefore, having the true *Anomaly* of the Planet, we have its Distance from the *Node*, which is called the *Argument of Latitude*, by which, and an easy *trigonometrical* Calculation, we can find the centrical Latitude of the Planet, and its *curtate Distance* from the *Sun*, which is the Distance from the *Sun* to that Point, where a Perpendicular, let fall from the Planet, meets with the Ecliptick. And thus we have shewed how to find the centrical Place and Latitude of the Planet, and its *curtate Distance* from the *Sun*. After these are found, we shall likewise be able to find his *geocentrick* Place, or where he will be seen from the *Earth*, in the following Method.

The curtate Distance of a Planet.

FIND first the Planet's *heliocentrick* Place, and *curtate Distance* from the *Sun*, as also the *Earth*'s Place and Distance from the same. Let T C F be the Orbit of the *Earth*, in which the *Earth* is in T; A P E the Orbit of the Planet, and its Place P, the *Sun* S, *n* S N the Line of *Nodes*. From the Place of the Planet let fall, on the Plane of the Ecliptick, the right Line P B; draw S B, and produce it till it meet with the Ecliptick in the Planet's Place reduced to the Ecliptick; which Place is found by the Arch P N, and the Inclination of the Orbit to the Plane of the Ecliptick, which are known: But we have the Place of the *Earth* seen from the *Sun*; and therefore we have the Distance between them, or the Angle T S B, which is called *the Angle of Commutation*. Then, in the Triangle S T B, we have ST, the Distance of the *Earth* from the *Sun*, and S B, the *curtate Distance* of the Planet; wherefore we shall find the Angle S T B, the Elongation of the Planet from the *Sun*, or the Arch of the Ecliptick intercepted between the *Sun*'s Place and the Planet's

Plate XXVI. Fig. 1.

The Angle of Commutation.

Planet's Place, reduced to the Ecliptick; as also TB, the *curtate Distance* of the Planet from the *Earth*. But the Place of the *Sun* is given, for it is opposite to the Place of the *Earth* seen from the *Sun*: Wherefore also we shall have the Place of the Planet in the Ecliptick, as it is seen from the *Earth*. Moreover, in the two Triangles PSB and TPB, that are rectangular at B, the Tangent of the Angle PSB is to the Tangent of the Angle PTB, as TB is to SB: But as TB is to SB, so is the Sine of TSB, the Sine of the Commutation, to the Sine of the Elongation STB. Wherefore say, as the Sine of the Commutation is to the Sine of the Elongation, so is the Tangent of the heliocentrick Latitude to the Tangent of the geocentrick Latitude, which was to be found. And by these Means the *Astronomers* are able to find, for any Instant of Time, the geocentrick Place and Latitude of any Planet.

Lecture XXVI.

How to compute the Planet's geocentrick Place.

COMPARING the Distances of the Planets from the *Sun*, with the Times of the Periods round him, we find that they all observe a wonderful, regular, and elegant Harmony and Law, *viz.*

The Harmony between the Periods and Distances of the Planets.

THE *Squares of the periodical Times are in all of them proportional to the Cubes of their mean Distances from the Sun.* Now their Periods and mean Distances are these, which we here give in the following Table.

	The Periods. D.	h.	′	″	*Mean Distances.*
♄	10759	6	36	26	953800
♃	4332	12	20	35	520110
♂	686	23	27	30	152369
☉	365	6	9	30	100000
♀	224	16	49	24	72333
☿	87	23	15	53	38710

THE illustrious Mathematician Mr. *Huygens* has determined very nicely the Diameters of the Planets, by comparing them with that of the *Sun*, in his

Lecture XXVI.

his *Saturnian System*, which he did by the following Method.

COPERNICUS, by his new and divinely invented System, has shewed us how to find out the Distance of each Planet from the *Sun*, in Proportion to the *Earth*'s Distance from the same: But their apparent Diameters, and how much some are bigger than others, has been discovered by the Help of the Telescope. For by comparing the Proportions of their Distances and apparent Magnitudes, the Proportion of their true Bigness to that of the *Sun* will easily be found, by the Principles we have laid down in our first *Lecture*.

The Diameters and Magnitudes of the Planets estimated.

As for *Saturn*, the Diameter of his Ring, when he is nearest to us, subtends an Angle of 68 Seconds. And because this least Distance of *Saturn* is near eight Times the mean Distance of the *Sun* from us, it follows, that if *Saturn* were as near us as the *Sun* is, the Diameter of the Ring would appear eight Times bigger than now it does, that is, it would be 9′ 4″. But the Diameter of the *Sun* in his mean Distance is 30′ 30″. Therefore the Proportion of the Diameter of the Ring to the Diameter of the *Sun*, is that of 9′ 4″ to 30′ 30″, which is near the Proportion of 11 to 37. But the Diameter of *Saturn*'s Body is to the Diameter of the Ring, as 4 to 9; that is, nearly as 5 to 11: And therefore it is to the Diameter of the *Sun*, as 5 to 37.

JUPITER's Diameter, when he is next to us, is 64 Seconds; and then his Distance is to the mean Distance of the *Sun*, as 26 to 5; say, as 5 is to 26, so is 64 Seconds to a Fourth, which will be 5′ 35″; which is the Bigness of the Angle that *Jupiter*'s Diameter would subtend, were he as near as the *Sun* is: But the *Sun* is seen under an Angle of 30′ 30″; therefore the Proportion of *Jupiter*'s Diameter to that of the *Sun*, is, as 5′ 35″ to 30′ 30″; that is, a little more than 1 to $5\frac{1}{2}$.

VENUS, when she is nearest the *Earth*, subtends an Angle of 85 Seconds, and then her *Perigæon* Distance

26

Plate XXIV. front. p. 342

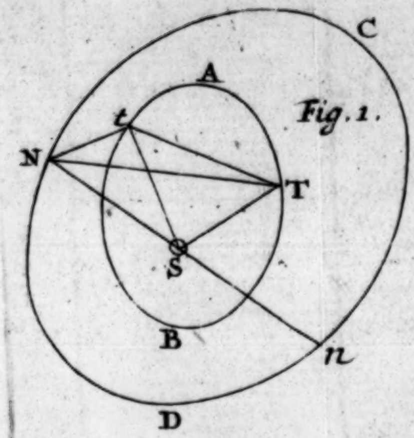


Fig. 1.

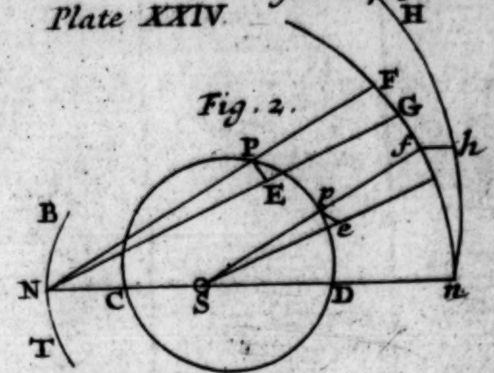


Fig. 2.

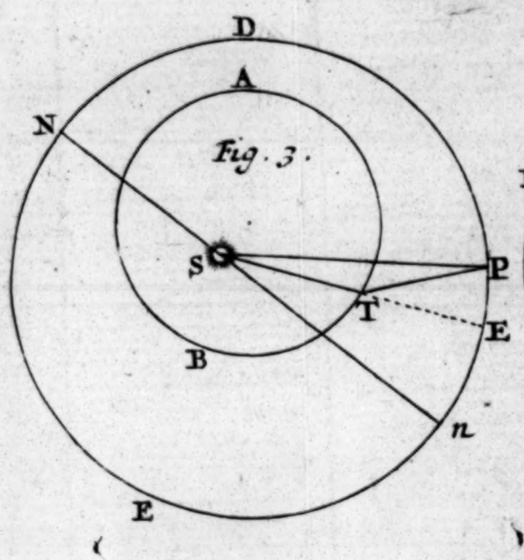


Fig. 3.

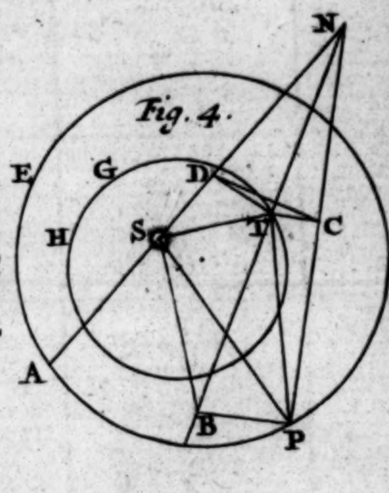


Fig. 4.

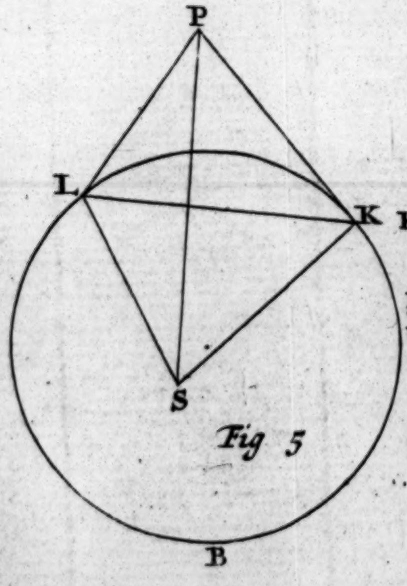


Fig 5

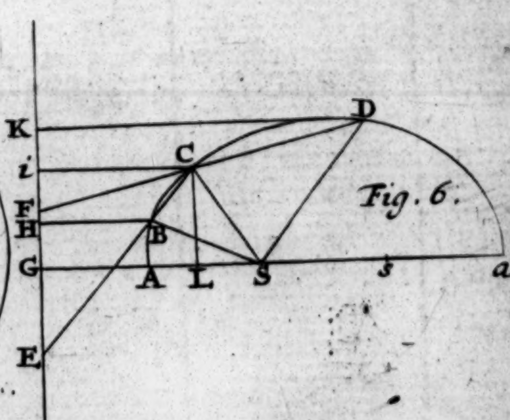


Fig. 6.

Diſtance is to the mean Diſtance of the *Sun* nearly, as 21 is to 82: And therefore, if *Venus* were removed to the Diſtance of the *Sun*, her Diameter would be only 21″ 46‴: And therefore we know that the Diameter of *Venus* is to that of the *Sun*, as 21″ 46‴ to 30″; that is, as 1 to 84.

But the Diameter of *Mars*, when neareſt to the *Earth*, is not greater than 30″: And therefore, ſince the leaſt Diſtance of *Mars* is to the *Sun*'s mean Diſtance, as 15 to 41; the Proportion of *Mars*'s Diameter will be to the *Sun*'s Diameter, as 1 is to 166: Therefore *Mars* is but half the Bigneſs of *Venus* in its Diameter. *Hevelius*, by Obſervations, found, that *Mercury*'s Diameter was to that of the *Sun*, as 1 is to 290.

The Magnitude of the *Earth*, in Compariſon of that of the *Sun*, is variouſly eſtimated by the *Aſtronomers:* They who ſuppoſe the horizontal Parallax of the *Sun* to be 15 Seconds, muſt make the Diſtance of the *Sun* from us, to be but 13750 Semidiameters of the *Earth*; and the Diameter of the *Earth* will be to the Diameter of the *Sun*, as 1 to 61: But we have a probable Argument, which proves the Diſproportion or Inequality greater. For, becauſe the Diameter of the *Moon* is ſomewhat more than a fourth Part of the Diameter of the *Earth*, if the Parallax of the *Sun* were 15 Seconds, then the Body of the *Moon* would be greater than that of *Mercury*; but it ſeems incongruous that a ſecondary Planet ſhould be greater than a primary Planet. Let us ſuppoſe therefore, that the Semidiameter of the *Earth*, ſeen from the *Sun*, be 11 Seconds, as it was lately collected from the Parallax of *Mars*, obſerved by Dr. *Halley* and Mr. *Pound:* And then the *Earth*'s Diſtance will be nearly 20000 Semidiameters of the *Earth*, and the *Moon* will be leſs than *Mercury*. And the Proportion of the Diameter of the *Earth* to the *Sun* will be that of 1 to 83; to which Proportion we may give our Aſſent, till by an Obſervation of *Venus* in the *Sun*'s Body, which will happen

Lecture XXVI.

in 1761, we may be made more certain of the *Sun's* Parallax. Therefore the Diameter of the *Sun* is to the Planets Diameters, nearly in the Proportions represented in the following Table.

The Diameter of the *Sun* is to the Diameter of	*Saturn*	as 1000 is to	137
	Jupiter		181
	Mars		6
	the *Earth*		12
	Venus		12
	Mercury		4

And because Spheres are to one another, as the Cubes of their Diameters,

The *Sun* will be to	*Saturn*	as 1000000000 is to	2571353
	Jupiter		5929741
	Mars		216
	the *Earth*		1728
	Venus		1728
	Mercury		64

Hence it follows, that the *Sun* is a hundred and sixteen Times bigger than all the Planets put together. *Saturn* is 400 Times less than the *Sun*: But for Quantity of Matter it is 2400 Times less than the Matter of the *Sun*. *Jupiter*, the biggest of all the Planets, is 160 Times less than the *Sun*, and his Matter that composes his Body, is 1033 Times less than the Matter of the *Sun*. But our *Earth*, if it be compared with the *Sun*, is but of a very small Magnitude, and not bigger than a physical Point; for it is 500000 Times less than it. Besides, comparing the Planets with one another, we find that *Jupiter* is bigger than all the rest of the Planets put together; and that he is above 3000 Times bigger than the *Earth*; but *Venus* is of the same Bigness with the *Earth*. And yet there are two of the six Planets, *viz.* *Mercury* and *Mars*, less than the *Earth*.

Jupiter *is bigger than all the Planets put together.*

LECTURE XXVII.

Of the Stations of the Planets.

IF the *Earth* were at Reſt, and the Planets alone turned round the *Sun*, an inferior Planet would ſeem to be ſtationary in that Point of its Orbit, where a Line drawn from the *Earth* touched the Orbit; for when the Planet is near that Point, if the *Earth* ſtood ſtill, it would directly approach the *Earth*, and would have no viſible Motion, or at leaſt, its viſible Motion would be the leaſt of all. In like manner, if a ſuperior Planet were at reſt, when it is viewed from the *Earth*, it would appear to ſtand in that Part of its Orbit, where a Line drawn from the Planet touches the *Earth*'s Orbit. But becauſe both the *Earth* and Planets move round the *Sun*, when an inferior Planet, is in the fore-mentioned Tangent, then the Motion of the *Earth*, will make the Planet appear to change its Place, and the Planet will not be come to its apparent Station. And, upon the ſame Account, when the *Earth* is in the Line which touches its Orbit, and paſſes to a Planet, the proper Motion of the ſuperior Planet, will change its viſible Place. And therefore it happens, that neither an inferior Planet ſeems to reſt, when it and the *Earth* are in a Line which touches its Orbit; nor is a ſuperior Planet ſtationary, when the *Earth* and it, are in a Line touching the Orbit of the *Earth*.

An inferior Planet is not ſtationary, when it is ſeen in a right Line touching its Orbit.

Nor is a ſuperior ſtationary when it is ſeen in a Line touching the Earth'*s Orbit.*

BUT

Lecture XXVII.

When a Planet appears stationary.

But since all the Planets do sometimes appear to move forward, and afterwards to return backwards; between the progressive Motion and the regressive, there will be some Point, where it will appear to stand and continue in the same Situation in the Heavens. Now it seems to keep the same Station in the Heavens, when the Line that joins the *Earth* and Planet's Center is constantly directed to the same Point in the Heavens; that is, when it keeps parallel to itself: For all right Lines, drawn from whatever Point of the *Earth's* Orbit, parallel to one another, do all point to the same *Star*; because the Distance of these Lines is not sensible in Comparison of the great Distance of the fixed *Stars*.

Therefore, to find out the Points of Station, we must inquire the Position of the Line, which, joining the *Earth* and Planet, keeps parallel, to itself: For which Purpose we must observe, that if the Centers of the *Sun*, *Earth* and Planet be joined by right Lines, they will form a Triangle, whose two Sides, drawn from the *Sun*, are always equal to the Distances of the *Earth* and Planet from the *Sun*; and the Base is a right Line joining the *Earth* and *Planet*: And because the Sides of this Triangle, in circular concentrick Orbits, do keep always the same Magnitude, the Proportion of the Sines of the Angles at the Base, will be constantly the same: For the Sines are as the opposite Sides.

Table XXV. Fig. 1.

Let the Circle BDG, be the Orbit of the Planet, and A H K, the Orbit of the *Earth* concentrical to it. And let us suppose the *Earth* in A, the Planet in B, and the *Sun* in the Center S. In the Triangle A S B, the Sines of the Angles A and B, at the Base, are as the opposite Sides S B, S A.

Let us suppose then, that in a small Particle of Time, the *Earth* is moved through the small Arch A C, and the Planet at the same Time through the Arch of its Orbit B D; their angular Motions

tions at the *Sun*, made in the ſame or equal Times, are reciprocally as their periodical Times; for how much the greater is the periodical Time, ſo much leſs is the Angle it deſcribes, round the *Sun*, in any given Time. Therefore, the Angle A S C, the angular Motion of the *Earth*, is to the Angle B S D, the angular Motion of the Planet, as the periodical Time of the Planet is to the periodical Time of the *Earth*; that is, the angular Motions in the ſame Time are in a conſtant Proportion. Lecture XXVII.

LET the Center of the *Earth* in C, and of the Planet in D, be joined by the Line C D, which is parallel to the former Line A B: And in that Caſe, as we have ſhewed, the Planet will appear ſtationary. Let S A cut C D in M, and S D, produced, cut A B in E: And becauſe A B and C D are parallel, by the 29th of *El.* I. the Angle S M D will be equal to the Angle A; but by the 32d of the firſt *El.* the Angle S M D is equal to the Angles C and M S C; wherefore the Angle C is equal to the Angle A, bating the Angle M S C or C S A. Likewiſe, becauſe of the Parallels A B and C D, the Angle S D C is equal to the Angle S E A, which is equal to the Angles S B A and B S E; wherefore that Angle is equal to the Sum of the two Angles S B A and B S E. Therefore the momentaneous Increaſe of the Angle S B A is equal to the angular Motion of the Planet at the *Sun*. And before it was ſhewed, that the Decrement of the Angle A, was equal to the Angle A S C, or to the angular Motion of the *Earth*; but theſe angular Motions are conſtantly in a given Proportion, which is reciprocal to their periodical Times.

Angular Motions are in a given Proportion reciprocal to their periodical Times.

A PLANET therefore appears ſtationary when the momentaneous Change of the Angle at the *Earth* is to the momentaneous Change of the Angle at the Planet, as the periodical Time of the Planet is to the periodical Time of the *Earth*.

LET there be two Arches or Angles, whoſe Sines are to one another in a conſtant Proportion. Table XXV. Fig. 2.

Lecture XXVII. tion. I say, that their Co-sines, or the Sines of the Complements of these Arches, are in a Proportion compounded of the direct Proportion of the Sines of these Arches, and a reciprocal Proportion of the momentaneous Changes of the Arches or Angles. For Example, Suppose the two Arches AM and CM, whose Sines are AB and CD, and their Co-sines SB, SD. Let the Arches AM and CM decrease into EM and GM; so that the Sines EK, GL, may be proportional to the former AB and CD. Let AE and CG be the Decrements of the Arches, which being infinitely small, may be taken for Right Lines. Draw FE and GH, parallel to SM; the Triangles AFE and ASB are equiangular: For the Angles B, and AFE are both right, and the Angle EAF is equal to the Angle ASB, the Angle SAB being the Complement of both to a Right Angle. After the same Way it may be proved, that the Triangles CHG and CSD are equiangular. Therefore CG : CH :: CS : SD; and AF : AE :: SB : AS or CS: Wherefore multiplying the Antecedents together, and the Consequents together, we have the Proportion CG × AF : CH × AE :: CS × SB : SD × AS or CS × SD :: that is, SB is to SD in a Proportion compounded of AF to CH and of CG to AE. But the *Ratio* of AF to CH, is the same with the *Ratio* of the Sines AB and CD; and the *Ratio* of CG to AE is the *Ratio* of the momentary Decrements of the Arches CM and AM. Therefore SB, the Co-sine of the Arch AM, is to SD, the Co-sine of the Arch CM, in a Proportion, compounded of the direct Proportion of the Sines of these Arches, and a reciprocal Proportion of the instantaneous Decrements of the Arches, that is of CG to AE.

This applied to Planets when stationary.

HENCE, if the Centers of the *Sun*, of the *Earth*, and of a Planet that is stationary, be joined with Lines, the Co-sine of the Angle at the *Earth*, will be to the Co-sine of the Angle at the Planet, in a Proportion composed of the Sines of the Angles A and

A and B, and a reciprocal Proportion of the momentary Decrease of those Angles A and B. But the Proportion of the Sines is the same with the Proportion of the Distances of the *Earth* and Planet from the *Sun*; and the *Ratio* of the momentary Decrease of the Angles A and B is the Proportion of the periodical Times of the Planet and the *Earth*, as has been proved. Let the periodical Times be called t and T; and then the Co-sine of the Angle A will be to the Co-sine of the Angle B, when the Planet is stationary, as T × S B is to t × S A; that is, the Co-sine of the Angle at the *Earth* is to the Co-sine of the Angle at the Planet, in a Proportion, compounded of the periodical Times directly, and a reciprocal Proportion of the Distances from the *Sun*. Hence the Points of Stations are easily determined by the following Construction.

Lecture XXVII.

Table XXV. Fig. 3. *Construction to determine the Points of Stations.*

LET A H be a Portion of the Orbit of the *Earth*, G B K a Portion of the Planet's Orbit, and let the *Sun* S be in the Center of both Orbits. Cut S A in E, so that S A may be to S E, as the periodical Time of the *Earth* is to the periodical Time of the Planet. Upon the Diameter A E, describe a Semicircle cutting the Orb of the Planet in B, the Angle S A B will be the Elongation of the Planet from the *Sun* when it appears stationary. Draw the Lines A B F, E B, and S F, parallel to E B, and the Angle A B E in a Semicircle, is a Right Angle: And therefore, A F S parallel to it, must be right likewise. Moreover, A S : A F : : Rad. : Co-sine of A; and also, B F : S B : : Co-sine S B F : Rad. Therefore multiplying the Antecedents together, and the Consequents together, we shall have A S × B F to A F × S B, as the Co-sine of S B F is to the Co-sine of the Angle A. Therefore, the *Ratio* of the Co-sine of the Angle A, to the Co-sine of the Angle S B F, is compounded of the *Ratio* of A F to B F, and of S B to A S. But the *Ratio* of A F to B F is equal to the *Ratio* of A S to S E, or of T to t. Therefore, the *Ratio* of the Co-sine of the

Lecture XXVII.

the Angle A, to the Co-ſine of the Angle SBF, is compounded of the *Ratio* of T to *t*, and of SB to SA. But it was ſhewed, that when the Co-ſines of the Angles had this Proportion, then the Planet ſhould be ſtationary. Therefore it is evident, that when the Planet is in B, and the *Earth* at A, the Planet will appear to be ſtationary.

When a Planet is ſeen from the Earth *to be ſtationary, the* Earth *ſeen from the Planet will appear ſtationary.*

HENCE we ſee, that when an inferior Planet is ſeen from the *Earth* to be ſtationary, the *Earth*, alſo, viewed from the inferior, will likewiſe appear to be ſtationary, and will ſeem to remain in the ſame Place. For the *Earth* is ſeen ſtationary, when the Line that joins the Centers of the Planet and the *Earth* keeps parallel to itſelf; and ſo long as this Line keeps a Paralleliſm, it will always be directed to the ſame Point in the Heavens.

BY the ſame Method, we find the Poſitions of the other ſuperior Planets, in reſpect of the *Earth* and *Sun*, when they are to be obſerved by us to be ſtationary, *viz.* by inquiring where the *Earth*, conſidered as an inferior Planet, will appear ſtationary, ſeen from a ſuperior Planet.

IF the periodical Times were proportional to the Diſtances from the *Sun*, the Points E and B would coincide with G, and the Planet would be ſtationary, when the Angle A was nothing; that is, when the inferior Planet was in Conjunction with the *Sun*: But if SE bore a greater Proportion to SA than SG did to SA, the Circle ABE could cut the Planet's Orb in no Point at all; and the Planet could no where be ſtationary, but would appear conſtantly to move directly forward: But neither of theſe Caſes obtain Place in the Planets; for SE is always leſs than SG or SB, which I thus demonſtrate.

CALL the Diſtance of the *Earth* from the *Sun* p, the Diſtance of the Planet SG or SB call q, the periodical Times T and t. And, by the univerſal Law above explained and obſerved in all the Planets, we have, $T^2 : t^2 :: p^3 : q^3$, and

and T : t : : $p^{\frac{3}{2}} : q^{\frac{3}{2}} :: p \times p^{\frac{1}{2}} : q \times q^{\frac{1}{2}}$; but as T is to t, ſo is S A : S E : : p : S E. Say therefore, as $p \times p^{\frac{1}{2}} : q \times q^{\frac{1}{2}} :: p$ to $\frac{q \times q^{\frac{1}{2}}}{p^{\frac{1}{2}}}$; which is therefore equal to S E : And becauſe p is greater than q; therefore, $q \times p^{\frac{1}{2}}$ is greater than $q \times q^{\frac{1}{2}}$; and dividing all by $p^{\frac{1}{2}}$, we have q greater than $\frac{q \times q^{\frac{1}{2}}}{p^{\frac{1}{2}}}$ or than S E. And therefore, ſince S B or S G is greater than S E, the Circle upon the Diameter A E, will cut the Orbit of the Planet: And therefore we on the *Earth*, in ſome certain Poſitions, may ſee each of the Planets ſtationary.

To find the Stations by Calculation.

If you deſire to uſe a Calculation, the Angle at the *Earth* or the Elongation of the Planet from the *Sun*, is defined in this Manner. Let the Radius be z, and the Sine of the Angle at the *Earth* $q\,x$; the Sine of the Angle at the Planet, will be $p\,x$, ſuppoſing that p is to q in the Proportion of the *Earth* and Planet's Diſtances from the *Sun*: And becauſe the Sine of the Angle at the *Earth* is $q\,x$, its Co-ſine is $\sqrt{z^2 - q^2 x^2}$; and the Co-ſine of the Ang. at the Planet will be $\sqrt{z^2 - p^2 x^2}$: And therefore $\sqrt{z^2 - q^2 x^2} : \sqrt{z^2 - p^2 x^2} ::$ T $\times\, q : t \times p$: And ſquaring the Terms of the Analogy $z^2 - q^2 x^2 : z^2 - p^2 x^2 ::$ T$^2 \times q^2 : t^2 \times p^2$; but T$^2 : t^2 :: p^3 : q^3$: Wherefore, inſtead of T^2 and t^2, put p^3 and q^3, which are proportional to them, and we ſhall have $z^2 - q^2 x^2 : z^2 - p^2 x^2 :: p^3 \times q^2 : q^3 \times p^2 :: p : q$: And therefore we have the Equation $q\, z^2 - q^3 x^2 = p\, z^2 - p^3 x^2$, and $p^3 x^2 - q^3 x^2 = z^2 \times \overline{p - q}$; and $x = z \times \frac{\sqrt{p - q}}{\sqrt{p^3 - q^3}}$

and

Lecture XXVII.

and $q\,x$, the Sine of the Angle at the *Earth*, $=$ $q\,z \times \frac{\sqrt{p-q}}{\sqrt{p^3-q^3}}$, which is equal to $\frac{qz}{\sqrt{p^2+pq+q^2}}$; therefore the Square of the Co-sine of this Angle is $z^2 - \frac{q^2z^2}{p^2+pq+q^2} = \frac{z^2p^2+z^2pq}{p^2+pq+q^2}$; and therefore the Co-sine is $z \times \frac{\sqrt{p^2+pq}}{\sqrt{p^2+pq+q^2}}$: The Co-sine is therefore to the Sine, as $z \times \frac{\sqrt{p^2+pq}}{\sqrt{p^2+pq+q^2}}$ to $\frac{qz}{\sqrt{p^2+pq+q^2}}$ as $\sqrt{p^2+pq}$ is to q. But, as the Co-sine is to the Sine, so is the Radius to the Tangent; therefore say, as $\sqrt{p^2+pq}$ is to q, so z is to $\frac{zq}{\sqrt{p^2+pq}}$, which is the Tangent of the Angle at the *Earth:* And by this Analogy the Angle is easily found. For if half the Sum of the Logarithms of p and $p+q$ be subtracted from the Logarithm of q, there will remain the logarithmick Tangent of the Angle at the *Earth.* From this Value of the Tangent we have the following geometrical Construction, which determines the Angle. Let H A Q be a Portion of the *Earth*'s Orbit, G B D the Orbit of the inferior Planet, and the *Sun* in S the common Center of the Orbits. Produce A S, 'till it meets with the inferior Orbit in D; upon the Diameter A D, describe the Semicircle A C D, and at S, erect upon A D the Perpendicular S C, cutting the Semicircle in C; join A C, in which take A F $=$ S D; and from F upon A S, let fall the Perpendicular F E; in S C take S L $=$ A E, and draw A L: Then S A L will be the Angle required; and B, where A L cuts the inferior Orbit, will be the Point of the Station: For the Square of

Table XXV. Fig. 4. *Another more easy Construction of the Problem.*

of S C is equal to the Rectangle A S D $= p q$, and A C square $=$ A S square $+$ S C square $= p^2 + p q$: But A C : A F :: A S : A E :: A S : S L :: the Radius : Tangent of the Angle S A L, that is $\sqrt{p^2 + pq} : q :: z : \frac{zq}{\sqrt{p^2 + pq}}$, which is therefore the Tangent of the Angle required.

Lecture XXVII.

The above Calculation and Construction agrees not with eccentrick Orbits and Ellipses.

THESE Calculations and Constructions would sufficiently determine the Points of Station, if the Planets Orbits were concentrical Circles: But since they are eccentrick and Ellipses, both the Angles at the *Sun* and Planet will be different and changeable, according to the different Places of the Planets in their Orbits, at the Points of Station. And therefore because in this Case, according to the infinite Varieties of Position of the *Earth* and Planets in their Orbits, there will be likewise an infinite Variety of Angles, they cannot be defined by any algebraical Equation; neither can the *Problem* be universally constructed by any algebraical Curve of any Kind, although some Mathematicians have undertaken to do it. Yet, if we have the Position of a Planet in its proper Orbit, we may find the Position of the *Earth* in its Orbit, from whence the Planet in that Point will appear stationary.

FOR this is a determined *Problem*, and admits of two Answers, for the two Roots of the Equation that involves the Nature of the *Problem*. The most industrious Dr. *Halley*, hath communicated to me the following Solution of this *Problem*; for the understanding of which, we give the following *Lemma*.

WHATEVER the Form of the *Earth* and Planets Orbits may be, if from their Places in the Times of Station there be drawn Tangents to the Orbits, and produced till they meet, the Portions of those Tangents, intercepted by their mutual Concourse, are proportional to the absolute Velocities of the *Earth* and Planets.

LET F G and A H be two Portions of the Orbits which the *Earth* and Planet describe; A B and

Lecture XXVII.

CD two infinitely small Portions of them, described in the same Time, when the Planet is stationary: Draw the Lines CE and AE touching the Orbits, and meeting in E. And because the Planet is stationary, BD and AC will be parallel: And therefore by the 2d *Prop. El.* VI. CD : AB :: CE : AE. But CD and AB, since they are Portions of the Orbits described in the same Time, are as the Velocities of the Earth and Planet: Therefore the Tangents CE and AE are as the Velocities. This *Theorem* is Mr. *John Bernoulli*'s, and is published in the *Berline* Acts, and flows immediately from the Parallelism of the Lines AC, BD. But Mr. *Bernoulli* has not given us from thence any Solution of the Problem. Dr. *Halley*'s Solution is this.

Table XXV. Fig. 5.

PROBLEM.

To find the Place of the Earth, from which a Planet, seen in a given Point of its Orbit, would appear stationary.

Table XXV. Fig. 6.

SUPPOSE the Sun at S, ΠKLA the Orbit of the Earth, which we may here suppose to be circular, $\pi P \alpha$ the Orbit of the Planet, P its given Place. Draw VPQ, touching the Orbit in P, and intersecting the Orbit of the Earth in V and Q; bisect VQ in R, and on it erect the Perpendicular PB, which may be to VR or RQ, as the Velocity of the Planet is to the Velocity of the Earth: At the Center R, upon the Diameter VQ, describe the Semicircle V*bd*Q, to which draw the Tangents B*b*, B*d*, and produce them till they meet with VQ produced in Σ and T; draw R*b*, R*d*; take ΣK equal *b*Σ, and TL equal to *d*T. Then I say the Points K and L are what are to be found. For, because of the equiangular Triangles R*b*Σ and BPΣ,

$BP\Sigma$, $\Sigma P : PB :: \Sigma K : Rb$ or RV, and by alternation of Proportion, $\Sigma P : \Sigma K :: PB : RV$; but by Construction, PB is to RV, as the Velocity of the Planet is to the Velocity of the Earth; and Σb touches the Semicircle in b; wherefore its Square is equal to the Rectangle $V\Sigma Q$, by *Prop.* 36 *Elem.* III. And ΣK is equal to Σb; therefore ΣK will touch the Orbit of the Earth in the Point K, by 37 *Prop. Elem.* III; therefore the Tangents of both Orbits ΣP and ΣK are as the Velocities, and therefore a Planet in P will appear stationary when the Earth is in K. In the same Manner it may be shewed, that the right Lines PT and LT are, as the Velocities, and that LT touches the Orbit of the Earth in L. Lastly, the Lines SK SL being drawn will shew the Places of the Earth seen from the *Sun*, and the Angles KSP LSP are the Commutations. And if the Line SA be the Line of the *Apsides* of the Earth's Orbit, KSA and LSA, will be the Angles of the true *Anomaly*; and consequently, if any Error be committed in supposing the Earth's Velocity, it can be most accurately corrected by having the true *Anomaly*.

Lecture XXVII.

It is a *Problem* of a very different Kind to define the Time when a Planet is to be stationary, and its Solution cannot be had from common *Geometry*; yet the aforesaid Dr. *Halley*, by an indirect Method, and an Approximation has shewed how to find it. It is as follows.

WHEN the Time of a Station is to be accurately determined.

Having by the former Construction, or a rude Calculation, or even from an *Ephemeris*, found out the Day of the Station, find out by the most perfect *Astronomical* Tables, for the Meridian of that Day, the Place of the Sun; as also the Planets *heliocentrick* and *geocentrick* Places, and the *Logarithms* of their Distances from the Sun; and to reduce their Motions to the same Plane, *curtate* the Di-

Lecture XXVII.

stance of the Planet, and we have the Triangle S T P, from the Principles of *Astronomy*, where S represents the *Sun*, T the *Earth*, and P the Planet. Draw T Q a Tangent to the *Earth*'s Orbit, and P Q a Tangent to the Planet's Orbit, which meet in Q. Now, if the real Velocities of the Planet and *Earth* are as P Q and T Q, or as the Sines of the Angles P T Q and TPQ, it is plain that the Planet is then in the Situation required; that is, it will be there stationary.

Table XXV. Fig. 8.

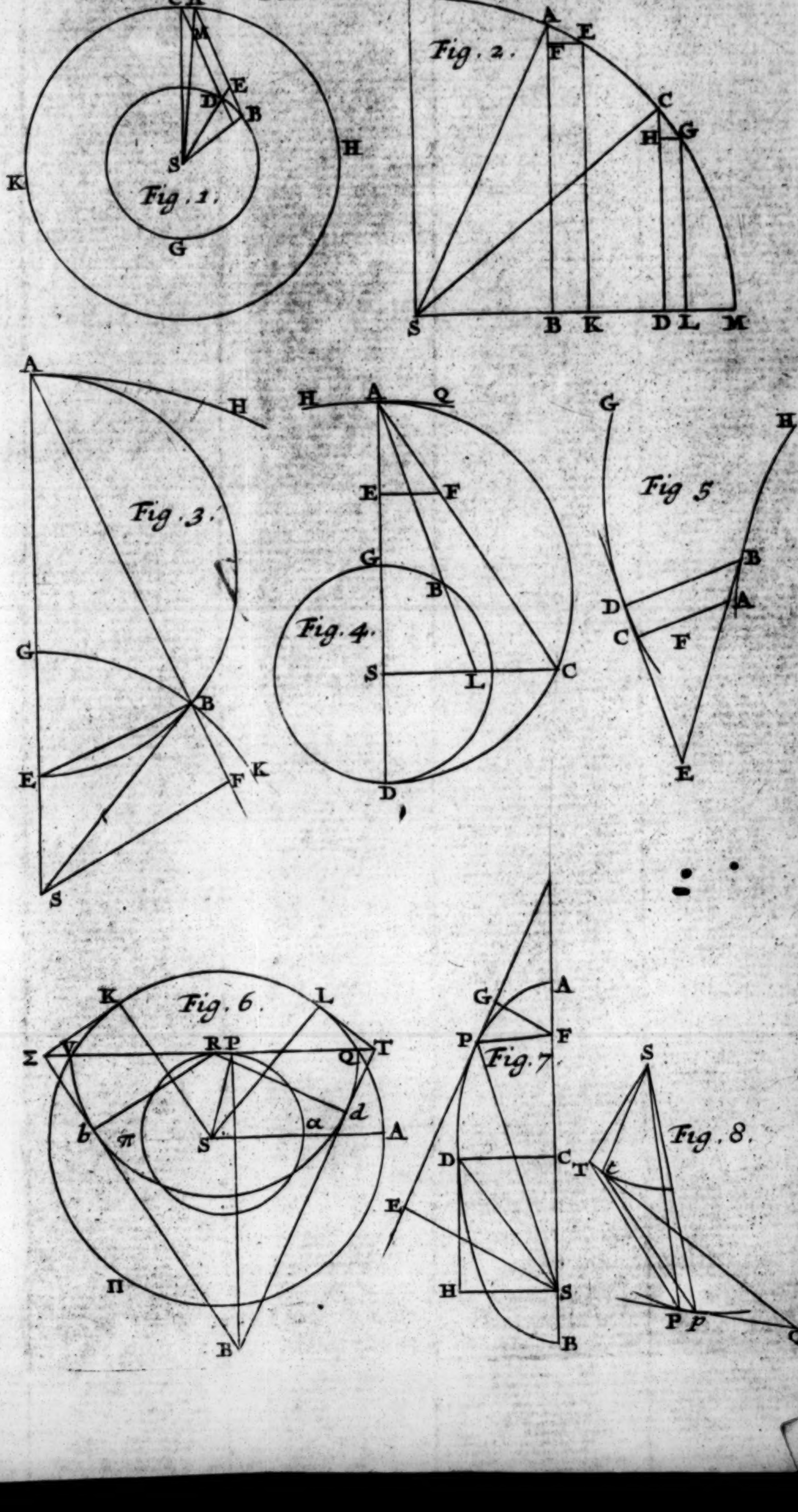
HAVING now the Distances S T, S P, we have the *Ratio* of their real Velocities, or of T *t* and P *p*. For the real mean Velocities of different Planets, that is, those Velocities with which at Distances equal to half the transverse Axes of the Orbits, they would describe Circles, are in a reciprocal subduplicate Proportion of the Axes. And * the mean Velocity of any Planet is to its Velocity in any other Point P or T, in a subduplicate Proportion of its Distance from the *Sun*, to its Distance from the other *Focus* of the Ellipse, which call respectively P F and T F: and putting R for half the tranverse Axe of the superior Ellipse, and *r* for half the transverse Axe of the inferior, and then compounding the Ratios, the Velocity of the inferior Planet will be to that of the superior, or T*t* to P*p* as $\sqrt{R \times SP \times TF}$ is to $\sqrt{r \times ST \times PF}$; and therefore we must have ready the Logarithm of this Ratio, reduced to the Plane of the Ecliptick.

* *See the Theory.* Lecture XXIII.

FROM the same Distances we have likewise the Angles S T Q and S P Q; for the Radius is to the Sine of the Angle S T Q, as $\sqrt{ST \times TF}$ is to half the conjugate Axis of the Orbit; and likewise the Radius is to the Sine of S P Q, as $\sqrt{SP \times PF}$ to half the conjugate Axis of the Planet's Orbit; or, which is more readily perform'd, say, as the Distance of the Planet, in its *Aphelion*, is to its Distance, in the *Perihelion*, so is the Tangent of half the Angle, by which it is distant from the *Perihelion*, to the Tangent of an Angle; which being subtracted

27

Plate XXV *front. p. 358.*



ſubtracted from the foreſaid half, leaves the Complement of the Angle S P Q to a Right, or its Exceſs above a Right, as it happens that this Angle is acute or obtuſe. Reduce this Angle, if it is needful, to the Plane of the Ecliptick; and theſe Things being done, ſubtract the Angle S T Q from the Angle S T P, and to the Angle S P Q add the Angle S P T, and we ſhall have the Angles Q T P and Q P T; and if the Sines of theſe Angles have the ſame Proportion that the real Velocities have in the Points T and P, the Eſtimation is right: But if not, take the Difference of the Logarithms of each, or the Error of the firſt Poſition. And if the Ratio of the Velocities be leſs than the Ratio of the Sines, we muſt diminiſh the Angle T S P, by adding or ſubtracting a known mean Angle, ſuch as will agree to one Day's Motion; and the contrary is to be done, if the Ratio of the Velocities be greater. And by a Calculation, juſt like the former, ſeek the Logarithms of the aforeſaid *Ratios* for the Noon of the preceeding or following Day, as the Caſe requires. Then compare the Differences of the foreſaid Logarithms, or the Error of the firſt Poſition with the Error of the ſecond; and the Sum of the Errors, if they be of ſeveral Kinds, or their Difference if they be of the ſame Sort, will be to 24 Hours, as either of the Errors to the Time between the Point of Station, and the Noon on which the aſſumed Error was found. This is plain to thoſe who underſtand the Rule of Falſe.

In this Manner the Stations of the Planets may be obtained within a few Minutes. But to take away the ſmall Errors which may ariſe, by Reaſon the Logarithms do not uniformly increaſe as the Time, if any one pleaſes, he may renew the Calculation, for the Time juſt found out, and which is very near the Truth, and ſo bring out the true Time to a Minute; but there is no need of this Correction but in *Mercury* or *Mars*.

For to make this plainer, we will add an Example of the Station of *Jupiter*, which lately happened on *November* the 9th 1717.

	November 9th at Noon.				*Nov.* 10th at N.			
	S.	gr.	′	″	S.	gr.	′	″
The mean Anom. of ♃	9	10	00	00	9	10	5	00
Mean Motion of ⊙	7	00	7	00	7	1	6	8
♃ Helio. from the 1 of ♈	2	25	11	00	2	25	15	53
⊙ Place from the 1 of ♈	6	28	53	17	6	29	54	00
Log. Dist. of ♃ from the Sun	5,720650				5,720680			
Log. Dist. of ⊙ from the Sun	4,994267				4,994186			
♃ Geocentrick Place	3	5	4	28	3	5	4	27
The Angle STP		113	48	49		114	49	33
The Angle SPT		9	53	28		9	48	34
The Angle STQ		89	23	54		89	23	54
The Angle SPQ		92	41	20		92	41	14
The Angle PTQ		24	25	42		25	25	39
The Angle TPQ		102	34	48		102	29	48
Log. of the Ratio of Velocit.	0,368210				0,368321			
Log. of the Ratio of the Sines	0,372912				0,356757			
The Error of the 1st Posit.	0,004702				0,011564			
The Error of the 2d Posit.	0,011564							

And because one of these Errors does exceed the Truth, and the other is deficient from it, say, as 16266, the Sum of the Errors, is to 4702, so is 24 Hours to 6 Hours 56 Minutes. Hence we conclude the Time of the Station of *Jupiter* to have been *November* the 9th, six Hours and 56 Minutes after Noon.

LECTURE XXVIII.

Of the Division of TIME, *and its Parts.*

THE Parts of *Time* are known to all Men, being Days, Hours, Weeks, Months, and Years. A natural Day is determined by the apparent Motion of the *Sun* from East to West, and is that Space of *Time* that flows while the *Sun* goes from any Meridian, or horary Circle, till it arrives to the same again. It is called natural, to distinguish it from that Signification of the Word Day, which is opposed to Night, and which is called the artificial Day.

A natural Day.

ALL nations do not begin their Day alike. The *Babylonians* counted the Beginning of their Day from the Sun-rising. The *Jews* formerly, and the *Athenians*, from Sun-setting, which the *Italians*, *Austrians*, and *Bohemians*, do at this Time; so that when the *Sun* comes to the Western Horizon, they count the twenty-fourth Hour; and the Hour after the *Sun* is set, they call the first Hour.

The different Beginnings of the Day.

THEY who begin their Day at Sun-rising have this Advantage, that their Hours tell them how much Time is already gone since Sun-rising; and they who reckon their Hours from Sun-setting have this Use of it, that they know how long it is to Sun-setting, that they may proportion their Journies, and Labours

for that Time. But both of them have this Inconvenience, that they cannot immediately tell by their Hours the Times of Mid-day or Mid-night, but they must compute by the Length of the Day-time, or the Season of the Year; for in different Seasons the Time of Mid-day is reckoned by different Hours. The *Egyptians* antiently began their Day at Mid-night, from whom *Hipparchus*, that antient and famous *Astronomer*, brought that Way of reckoning into *Astronomy*. And *Copernicus*, and some other *Astronomers*, have followed him therein. But the much greater Part of the *Astronomers* have thought it better to begin their Day from Noon. Yet the Method of Beginning from Mid-night is received in *Britain*, *France*, *Spain*, and most of the Nations in *Europe*.

Hours equal and unequal.

There are two Sorts of Hours, equal and unequal. An equal Hour is the twenty-fourth Part of the natural Day. Besides the Division of Hours received by the Vulgar into half Hours, Quarters, and half Quarters, we now generally follow the *Astronomical* Division, and reckon every Hour 60 Minutes, in every Minute 60 Seconds, and in every Second 60 Thirds, *&c.*

An unequal Hour is the twelfth Part of the artificial Day, or the twelfth Part of the Night; and it is called the *temporary Hour*, because at different Seasons of the Year it is of a different Length. For a diurnal Hour in the Summer is longer than one in the Winter; a Night Hour is shorter. But in the equinoctial Day, the Hours in the Day and Night are equal to each other, and therefore the equal Hours are called *Equinoctial*. The *Jews* and *Romans* formerly used these Hours, and the *Turks* reckon by them at this Day, and their Noon always falls upon the sixth Hour of the Day. These Hours are also called *planetary Hours*, because in every Hour they supposed one of the seven Planets to preside over the World, and they took it by Turns; so that the first Hour after Sun-rising fell on *Sunday* to the *Sun*,

Sun, the next to *Venus*, the third to *Mercury*, and the rest in order to the *Moon*, *Saturn*, *Jupiter*, and *Mars*. By this Means, on the first Hour of the next Day the *Moon* presided, and on that Accout gave the Name to that Day; and the Days of the Week by this Method have had their Names from the Planet that governed the first Hour, till the End of the Week. Lecture XXVIII.

A Week is a System of seven Days, in which each Day is distinguished by a different Name. The Christian Church called the first Day of the Week the *Lord's Day*, the Vulgar term it *Sunday*, and none but the *Fanaticks* of our Time ever called it *Sabbath-day*. The rest of the Days of the Week were called *Feriæ*, *Monday* the second *Feria*, *Tuesday* the third *Feria*, &c. and *Saturday* they also called the *Sabbath-day*. But the common People use the same Names that were given by the *Romans*, each Day being denominated from a Planet. *A Week.*

A Month is properly that Space of Time the *Moon* takes to perform its Course in the *Zodiack*, which in the Space of a Year it runs over twelve Times. There is another Month nearly equal to it, which is measured by the Motion of the *Sun*, and is that Space of Time in which the *Sun* moves through one Sign, or twelfth Part of the Ecliptick: These Months are properly *Astronomical*. A Civil Month is different from them, and consists of a certain Number of Days, fewer or more according to the Laws and Ordinances of the Kingdom or Republick in which they are observed. The *Egyptians* made each Month to consist of 30 Days and the Year consisting of 5 Days more than 12 Months, they added them to the End of the Year, and called them *Epogamenæ*. *A Month.*

The Year is either *Astronomical* or Civil; both Kinds of the *Astronomical* Years, *viz.* the *Tropical* and *Periodical*, we have already defined in our XXII[d] *Lecture*. The Civil Year is the same with the Political Year, established by the Laws of a Country, and is of two Kinds, Lunar or Solar, according as it is designed *The Civil Year of two Sorts.*

Lecture XXVIII.

designed to be regulated by the Motions of the *Moon* or *Sun*. There are two Sorts of Lunar Years, the one moveable, the other fixed: The moveable Year consists of twelve synodick Months, or of twelve Lunations which are completed in 354 Days, and after that Time the Year begins again. This Year is less than the Solar Year, which brings back all the Seasons, by eleven Days. And therefore the Beginnings of such Years move through all the Seasons, and that in the Space of 32 Years: This Form of a Year is observed by the *Turks* and *Mahometans*.

The moveable Lunar Year.

The fixed Lunar Year.

SINCE twelve Lunations is less than a Solar Year by eleven Days, three Lunar Years are less than three Solar Years by 33 Days; and therefore, to keep the Months in the same Seasons and Times of the Solar Year, to the third Year there is a whole Month added, and it consists of 13 Months: And this is done as often as is needful to keep the Beginning of the Year always in the same Season. And the Month added is called an *Embolimæan* or Intercalary Month. In 19 Years there are seven such Months, and this Kind of Lunar Year is called Fixed, and was observed by the *Greeks*, whom the *Romans* followed in this Matter till the Times of *Julius Cæsar*.

The Solar Year.

THE Solar Year, which is made conformable to the Motion of the *Sun*, is likewise of two Kinds, moveable and immoveable. The moveable is called the *Egyptian* Year, because observed in that Country: And it consists of 365 Days, and is less than the tropical Year, in which the *Sun* runs his Course in the Ecliptick by almost six Hours. By the neglecting of these six Hours, it happens, that four such Years are less than four tropical Solar Years by a whole Day. And therefore, in four Times 365 Years, that is, in 1460 Years, the Beginning of the Year moves through all the Seasons of the Year.

SINCE therefore an *Egyptian Year* is less than a true Solar Year, by almost six Hours; that all the Years may go on according to the *Sun*'s Motion, a Regard must be had to these six Hours. But it is requisite

requisite also that the Political Year have always the same Beginning, and that it commence with the Day; for it would be inconvenient to have the Year begin sometimes at one Hour of the Day, sometimes at another, which would necessarily fall out if we added to every Year six Hours. Now these Hours, amounting in three Years to eighteen Hours, if they be added to the six Hours of the fourth Year, will make a whole Day: Therefore this Day being added to the fourth Year, will reduce it again to be even with the Motion of the *Sun*. *Julius Cæsar* perceiving this, ordered that every fourth Year should have an Intercalary Day, which therefore consists of 366 Days, and the Day added is put in the Month of *February*. And because in the common Year the 24th of *February*, in the *Roman* Way of Reckoning, was the sixth of the *Kalends* of *March*, or the sixth Day before the *Kalends* of *March*, *Cæsar* ordered that for that Year there should be two Sixths, or that the Sixth of the *Kalends* of *March* should be twice reckoned; upon which Account the Year was called *Bissextile*, which we name the Leap-Year. This Form of a Year was instituted by *Julius Cæsar*, who was then High-Priest among the *Romans*, and was called the *Julian* Year; whose Nature is, that every fourth Year consists of 366 Days, and the other three only of 365.

Lecture XXVIII.

The Julian *Year.*

But it must be acknowledged, that the Time appointed by *Julius Cæsar* for the Solar Year is too much; for the *Sun* finishes his Course in the Ecliptick in 365 Days, 5 Hours and 49 Minutes, and therefore he begins again his Round eleven Minutes before the Civil Year is ended. So that if the *Sun* in any Year has entered the Equinox upon the 20th of *March* at Noon-day, after 4 Years he will arrive at the Equinox 44 Minutes before Noon, and the fourth Year after that he will be there 1 Hour 28 Min. before Noon; and so every Year 11 Minutes sooner than by this Reckoning, so that in 131 Years he will anticipate or enter the Equinox a whole

Day

Lecture XXVIII. Day before the 20th of *March*: And therefore the celestial Equinox will not always fall upon the same Day of the Month, but by Degrees it will move towards the Beginning of the Year; and this so sensibly, in Compass of Time, that it cannot be doubted.

HENCE in the Time of the Council of *Nice*, (about which Time the Terms were settled for observing *Easter*) the vernal Equinox fell upon the twenty-first of *March*; but the Equinox continually falling backwards in the Year, in the Year of *Christ* 1582, when the *Kalendar* was corrected, it was found that the *Sun* entered the equinoctial Circle on the 11th of *March*, and was departed ten whole Days from its former Place in the Year: And therefore when *Gregory*, the XIIIth of that Name, Bishop of *Rome*, designed to place the Equinocties in their former Situation, in respect of the Year; he took those ten Days out of the *Kalendar* that Year, and ordered that the eleventh of *March* should be reckoned as the twenty-first: And to prevent the Seasons of the Year from going backwards as they did before, he ordained that every hundredth Year, which in the *Julian* Form was to be a *Bissextile*, should be a common Year, and consist only of 365 Days; but because that was too much, every four hundredth was to remain *Bissextile*. This new Form of the Year being established by the Authority of the Bishop of *Rome*, *Gregory* XIII. is called by his Name the *Gregorian* Year, and is received in *France*, *Spain*, *Italy*, *Germany*, and in all the Countries where the *Pope*'s Authority is acknowledged, as likewise lately in several where the reformed Religion is observed. Yet in *Britain*, and other Northern Countries, the *Julian* Form of the Year is still retained.

The Gregorian *Year.*

THE *Persians* observe the *Egyptian* Form of the Years to this Day, whence it is that Equinocties remain not in the same Month, but move through them all; and after a Period of about 1460 Years, the Beginning of their Year falls in with the same Time of the true Solar Year. This Time or Period

riod is called the *Great Canicular Year*, or the *Sothiacal* Period, because it takes its Beginning on the first Day of the Month *Thoth* or the first Day of that Year, when the Dog-Star rises heliacally, for the Word *Sothis* in the *Egyptian* Language signifies the Dog, which in *Greek* is called ἀστροκύων, that is, the Dog-Star, which the *Astronomers* name *Sirius*. Lecture XXVIII. *The* Great Canicular Year.

THE Antients not only distinguished the Times by Years, but by several Revolutions or Collections of Years; such was the *Jubilee* of 49 or 50 Years; an Age consisted of an hundred Years. But among the *Greeks*, the *Olympiads* were esteemed the most famous, each of them containing the Space of four Years.

The Christian Æra.

AS in the Heavens there are certain Points, from which the *Astronomers* begin their Computations of the Planets Motions; so also there must be certain Points, or Instants of Time, from which, as from Roots, all Calculations must begin: And all memorable Actions are disposed and recorded, according to the Series of Years which follow from that Root. These Roots are called *Epochs* or *ÆRAs*, from which we generally count our Years and Times. The most famous, best known, and most used by us, is that which is reckoned from the Nativity of our Lord *Jesus*, which begins at the *Kalends* of *January*, that immediately followed his Birth.

NOW, although this *Epoch* is generally received by Christians, yet the *English* and *Irish* have an *Epoch* a whole Year posterior to it, which they commonly use in all Publick and Ecclesiastical Affairs: For they do not begin their Year with the first of *January* that follows the Nativity, but with the Feast of the Conception or Incarnation, which is observed on the 25th of *March*; and therefore it is, that the *English* reckon from the Feast of *Lady-Day* 1718, that there are compleated 1717 Years; but from the Birth of our Lord, to the Feast of the Nativity of the Year 1717, they number only 1716 Years elapsed

Lecture XXVIII. elapsed; whereas all the rest of the Christian World count 1717 Years.

The Author of the Æra. In this Affair they exactly agree with *Dionysius*, surnamed *The less*, according to whom *Christ* is supposed to have been conceived the 8th of the Kalends of *April*, in the first Year of this *ÆRA*; and was born the Winter following, at the End of the 46th Year from the Reformation of the Kalendar by *Julius Cæsar*. This Way of computing was at first universally received; but afterwards, by Degrees and tacitly, all Nations receded from it; so that it does only now take Place in *England*, and the Dominions thereof; and the common Opinion is, that *Christ* was born the Winter preceding the Time that *Dionysius* reckoned the Conception to have been; and by this means they make *Christ* to have been a Year before *Dionysius*, the Author of the *Æra*, supposed he was born.

But yet for all this the *English*, for the greatest Part of the Year, design it by the same Number that the rest of the Christian World does: But for three Months, *viz.* from the Kalends of *January* to the 8th of the Kalends of *April*, they write one less.

There is likewise the *Epocha* of the *Creation*, which is much noted, yet about it there are great Controversies among the *Chronologists*; some affirming, that the World was created 3950 Years before the Birth of *Christ*; others again say, that at the Birth of *Christ* the Age of the World was 3983 Years. The *Greek* Church, and the Emperors of the East, used an *Epocha* of the Creation, which was much more antient, and makes the World to have been created 5509 Years before the Coming of *Christ*.

Among the prophane Authors the antientest *Epocha* is that of the *Olympiads*, which begun at the Summer of the Year 777, preceding the Birth of *Christ*, and on the Kalends of *July*.

The

THE *Epoch* of *Rome*, or of the Building of the City, is not long after the *Olympiads*, and there are two of them, the *Varronian* and *Capitolian*; according to the first, the City was built the Year before *Christ* 753; according to the other, it was in the Year 752.

THE *Æra* of *Nabonasser* has always been famous among the *Astronomers*, and begun on the 6th of *February* of the *Julian* Year carried backward, and before *Christ* 747. And because that Day was then the first of the *Egyptian* Year, *Ptolemy*, and after him *Copernicus*, computed the Motion of the Stars, according to that *Æra*, by *Egyptian* Years; for the *Egyptian* Year is very convenient for *Astronomical* Calculations, it being interrupted by no intercalary Days.

AFTER this, we have another *Epocha* of the Death of *Alexander* the Great, the three hundred and twenty-fourth Year before *Christ*, on the 12th of *November*, which was then the first Day of the *Egyptian* Year. *Theon*, *Albategnius*, and some others, have computed from thence according to the *Egyptian* Year. Between the two *Æras* of *Nabonasser*, and of the Death of *Alexander*, there are precisely 424 *Egyptian* Years. The *Abissines* reckon by another *Æra*, which is called the Æra *of the Martyrs or of* Dioclesian. The *Turks* and *Arabians* reckon by an *Æra*, which they call the *Hegira*, which takes its Beginning from the Flight of *Mahomet*. The *Persians* have likewise an *Epoch*, which they call *Jesdegird*; all which are explained by the *Chronologists*. But the *Julian* Period seems to be the most useful and convenient of all, it including almost all other *Æras* within it, and it is a Period of 7980 Years; which Number is composed by the Multiplication of the three Numbers, 15, 19, and 28. The first is the *Cycle* of the *Indiction*, the second is the *Metonick Cycle* of the Moon, and the third is the *Cycle* of the *Sun*. And the first Year of this Period was that wherein all these three *Cycles* began together. I will

here add a Table, which gives the first Year of the several *Æras*, and reduces them to the Years of the *Julian* Period, and to the Years before or after the Birth of *Christ*.

	Years before Christ.	*Years of the* Julian *Period.*
The Creation of the World according to the *Greek* Emperors,	5508	
The Common *Æra* of the Creation,	3950	765
The Beginning of the *Olympiads*,	776	3938
The Building of *Rome*, according to *Varro*,	753	3961
The Building of *Rome*, according to the Registers of the Capitol,	752	3962
The *Æra* of *Nabonassar*,	747	3967
The Death of *Alexander* the Great,	324	4390
	Years after Christ.	
The common Christian *Epoch*,	1	4714
The *Dioclesian Æra*,	284	4997
The *Hegira* of the *Turks*,	622	5335
The *Jesdegird* of the *Persians*,	632	5345

LECTURE XXIX.

Of the Kalendars. *Of* Cycles *and* Periods.

THE *Kalendar* is a Table, in which are set down all the Days of the Year in a regular Disposition, according to their Months, with a Distribution of them into Weeks. The Vigils, Holy-days and Law-days, together with the *University* Terms, are likewise annexed. The Distribution of Days into Weeks is made by the seven first Letters of the Alphabet, ABCDEFG. Beginning at the first of *January*, to which is placed the Letter A; to the second of *January* B is joined; to the third C, and so on to the seventh, where G is figured: And then again beginning with A, which is placed at the eighth Day, B will be at the ninth, C at the tenth; and so continually repeating the Series of these seven Letters, each Day of the Year has one of those Letters in the *Kalendar*. By this Means the last of *December* has the Letter A joined to it. For if the 365 Days which are in a Year be divided by seven, we shall have 52 Weeks and one Day over. If there had been no Day over, all the Years would constantly begin on the same Day of the Week, and each Day of a Month would constantly have fallen upon the same Day of the Week. But now, because, besides the 52 Weeks in the Year, there is one Day more, from thence it happens, that on whatever Day of the Week the Year begins, it ends upon the same Day, and the next Year begins with the following Day. For Example, in a common

Lecture XXIX.

Year of 365 Days if the Year begin on a *Sunday*, it will end on a *Sunday*, and the first Day of the next Year will be *Monday*.

The Letters being ranked in this Order, that Letter which answers to the first *Sunday* of *January*, in a common Year, will shew all the *Sundays* throughout the Year; and to whatever Days in the rest of the Months that Letter is put, these Days are all *Sundays*; and therefore that Letter is called the *Dominical* or *Sunday* Letter of that Year. So also whatever Letter is joined to the first *Monday* in *January*, the same, as often as it is repeated in the *Kalendar*, shews the *Mondays* throughout the Year.

If the first Day of *January* be a *Sunday*, the last Day of the Year, as I have said, will likewise be a *Sunday*; and therefore the next Year will begin on *Monday*, and the *Sunday* will fall on the seventh Day, to which is annexed the Letter G, which therefore will be the *Sunday* Letter for all that Year: And since the Year began on *Monday*, it will also end on that Day; and the following Year will begin on a *Tuesday*, and the first *Sunday* will fall upon the sixth of *January*, to which Day is adjoined the Letter F, which is the *Sunday* Letter for that Year. And in the same Manner for the Year next following, the *Dominical* Letter will be E: By this Means the *Sunday* Letters will go in a retrograde Order by G F E D C B A. In the yearly *Kalendars*, which we call *Almanacks*, which is an *Arabick* Word, the *Dominical* Letter, to distinguish it the better, is denoted by a Capital, and generally printed in Red, and all the rest are of a smaller Form. By this Means, at one View, we shall see all the *Sundays* in the Year.

If all the Years were *Egyptian* Years of 365 Days, after a Period of seven Years, the same Days of the Month would return to fall on the same Days of the Week. But we observing the *Gregorian* Year, where every fourth is *Bissextile*, or consists of 366 Days, in which, besides the 52 Weeks, there are two Days over; if that Year should begin with a *Sunday*, it will

will end on a *Monday*, and the next Year will begin on *Tuesday*; and the first *Sunday* of that Year will be on the sixth of *January*, to which is annexed the Letter F, which will be the *Dominical* Letter for the Year following that Leap-year, whose *Dominical* Letter was A. Lecture XXIX.

By this Means the *Bissextile* Year returning every fourth Year, the Series of the *Dominical* Letters, succeeding each other, is interrupted, and does not return in Order till after four times seven, or twenty-eight Years. Hence ariseth the *Cycle* of 28 Years, which is called the *Cycle* of the *Sun*; which being compleated, the Days of the Months return in the same Order to the same Days of the Week. In this *Cycle* all the *Bissextile* Years have two *Dominical* Letters, the first of which takes Place till the 24th or 25th of *February*, and the other serves for all the rest of the Year; for in the *Bissextile* Year the 24th and 25th of *February* are esteemed as one and the same Day, and both of them have the same Letter F annexed to them, and by this the Order of the *Sunday* Letter is interrupted. For Example, If in the Beginning of the Year the *Sunday* Letter is E, the 24th of *February* will fall upon a *Monday*, and the 25th on a *Tuesday*, both which Days are marked with the Letter F; and therefore the following Letter G, which shewed the *Tuesdays* before, will now point out the *Wednesdays*; and the next *Sunday* will fall upon the sixth of *March*, to which, in the *Kalendar*, is annexed the Letter D, which will point out the *Sundays* for all the rest of the Year, and then becomes the *Dominical* Letter.

The Cycle *of the* Sun.

THE first Year of the *Cycle* of the *Sun* is a *Bissextile*, and the *Dominical* Letters, answering to it, are D and C. For the second Year the *Sunday* Letter is B, for the third A, the fourth G; and again, the fifth Year of the *Cycle* being *Bissextile*, has two *Dominical* Letters F and E; and so in the rest. The following small Table shews what *Dominical* Letter belongs to each Year of the *Cycle*.

Lecture XXIX.

1	DC	5	FE	9	AG	13	CB	17	ED	21	GF	25	BA
2	B	6	D	10	F	14	A	18	C	22	E	26	G
3	A	7	C	11	E	15	G	19	B	23	D	27	F
4	G	8	B	12	D	16	F	20	A	24	C	28	E

To *find the Year of the* Cycle *of the Sun for any Year of the Christian* Æra; To the current Year of *Christ* add 9, becaufe from the Beginning of the *Cycle*, till the firft Year of *Christ*, there were 9 Years paft: Divide the Sum by 28, the Quotient fhews the Number of *Cycles*, that have revolved fince the firft Year before *Christ*, till the current Year; and the Remainder, if there be any, is the current Year of the *Cycle*; but if there be no Remainder, then 28 is the current Year of the folar *Cycle*.

The moveable Feafts.

BESIDES the fixed and fettled Feafts, which are always on certain and determined Days of the Year, there are other Feafts and Holydays which are moveable, and in different Years fall upon different Days of the Month, and fometimes in different Months: Thefe Holydays are not regulated by the Motion of the *Sun*, but by that of the Moon: Such was the Feaft of the *Paffover*, inftituted by God himfelf, for the *Jews* to obferve, and is fucceeded by the Chriftian *Eafter*, in Memory of our *Saviour*'s Refurrection.

The Feaft of Eafter.

God ordained that the *Paffover* fhould be celebrated in the firft Month, the 14th Day of the Month at Even. *See* Leviticus, *Chap.* XIII. Now the *Jewifh* Year was a Lunar Year, and fo ordered by intercalary Months, that the Month, whofe fourteenth Day, or whofe Full Moon, fell either on the Vernal Equinox, or next after it, was reckoned the firft Month of the Year. The Chriftian Church was willing to obferve the fame Method in celebrating of *Eafter*. But yet, to diftinguifh it from the *Jewifh* Paffover, would not keep the Feaft on the 14th Day, but on the

the *Sunday* after, because our Lord rose upon the *Sunday* after the *Jewish* Passover. Lecture XXIX.

THEREFORE for determining the Time for the Celebration of *Easter*, we must define the Time of the Equinox, which was believed to be fixed to the twenty-first of *March*, and the Fathers thought it could never happen on any other Day, and therefore they made their *Kalendar* upon that Supposition. Then they called that the *Paschal* or first Month, whose 14th Day or Full Moon, fell on the equinoctial Day, that is, on the 21st of *March*, or next followed the 21st of *March*. But because the *Jewish* Months were Lunar Months, the 14th Day immediately preceeded the Full Moon; therefore, for the Time of the Observation of *Easter*, we must have a Regard to the Motion of the Moon, and the Times of New Moons and Full Moons must be found. The *Jews* had no other Way of finding the New Moon, but by observing it; and when the Moon first appeared to emerge out of the *Sun*'s Rays, or to rise in the Evening heliacally, that Day they called the first Day of the Moon or Month. The Christian Church computed their Lunations by the *Metonick Cycle* of 19 Years; and therefore they inserted this *Cycle* in their *Kalendars*, and called its Numbers the *Primes* or *Gold-numbers*, by which they determined the Times of the Lunations.

The Cycle *of the Moon, or the* Metonick Cycle.

THE *Metonick Cycle*, called so from its Inventor *Meton*, is also termed the *Cycle* of the Moon, and is a Period of 19 Years, which when they are compleated, the New Moons and Full Moons return on the same Days of the Month; so that on whatever Days the New and Full Moons fall this Year, 19 Years hence they will happen on the very same Days of the Months, as *Meton* and the Fathers of the Primitive Church thought. And therefore at the Time of the Council of *Nice*, when the Way of settling the Time for observing the Feast of *Easter* was established, the Numbers of the Lunar *Cycle* were inserted in the *Kalendar*, which upon the Account of

Lecture XXIX. their excellent Use were set in Golden Letters, and the Year of the *Cycle* for any Year was called the Golden Number of that Year.

THE Golden Numbers were placed in the *Kalendar* according to the following Method; taking any Year for the Beginning of the *Cycle*, which they reckoned by the Number 1, and observing every Month the Days on which the New Moons happened that Year, just by those Days they joined the Number 1. And because in that Year the New Moons happened on the twenty-third of *January*, the twenty-first of *February*, the twenty-third of *March*, the twenty-first of *April*, the twenty-first of *May*, the nineteenth of *June*, and so in the rest, just by those Days in the Column of the Golden Number they put the Number. 1. The second Year observing the New Moons, to the Days on which they happened they inscribed the Number 2, *viz.* to the twelfth of *January*, the tenth of *February*, the twelfth of *March*, and tenth of *April*, and so on in the rest of the Months. The same Thing was done the third Year, annexing the Number 3 to all the Days the Moon changed in that Year; and so on in the following Years till the whole Period of 19 Years was compleated. But the most accurate Disposition of the Golden Numbers is by the mean Lunations, as they are set down in the *Astronomical* Tables, for every Month and each Year of the Lunar *Cycle*; and fixing, according to that Computation, the Character of the Year to each Day on which the Lunation happens.

BECAUSE a lunar *Astronomical* Month consists of 29 Days, 12 Hours, 44 Minutes and 3 Seconds; the common People, who cannot distinguish the small Particles of Time, make the Lunar Months to consist of intire Days without Fractions, and on that Account they alternately put one Month of 30 Days, and the next of 29 Days. These are called Hollow or Cave, that is, deficient Months, the others Full; the 12 Hours over the 29 Days, requiring this

this Alternation: But because there are 44 Minutes besides, which is almost three Quarters of an Hour, in every Lunation; in 32 Lunations these Minutes will make up a whole Day, which is to be added to a hollow Month; and by this Means the Lunations of the *Kalendar* will nearly agree with those of the Heavens.

Lecture XXIX.

If we know the Year of the Lunar *Cycle*, which in our *Liturgy* is called the *Prime*, we have the Days of the Moon's Change throughout the Year; for in each Month, the Golden Number or *Prime* is set to the Day the Change happens on; and adding to that 14 Days, we shall have the Day of Full Moon.

The Antients imagined that the *Cycle* of 19 Years did exactly compleat 235 Lunations; and therefore, after the Revolution of that Time, that the New Moons not only fell on the same Days of the Month, but likewise on the same Hours of the Day, which is not true; for in 19 *Gregorian* Years there are 6939 Days, and 18 Hours. But if to every Lunation we allow 29 Days, 12 Hours, 44 Minutes, 3 Seconds, which the Motion of the Moon requires; 235 Lunations will make 6939 Days, 16 Hours, 31 Minutes, and 45 Seconds. Therefore 235 Lunations are not equal to 19 *Gregorian* Years, but are less by one Hour and an half. And consequently the New Moons, after 19 Years, will not return to the same Hour, but will be an Hour and a half sooner, and in 304 Years they will anticipate a whole Day. And therefore the Golden Number will, precisely enough for common Use, shew the Lunations in the Space of three Centuries, without the Error of one Day. In the Council of *Nice*, when the *Cycle* of the Moon was fitted to the *Kalendar*, and some Centuries afterwards, it did nearly enough give the Time of the New Moons. But the Lunations in every 304 Years anticipating a whole Day, they lately happened almost five Days sooner than they should do, according to the Rule of

Lecture XXIX.

the Golden Number: And for this reaſon the *Kalendar* was altered in the Year 1752. But the Computation by Golden Numbers retained, with proper Corrections; by which the Eccleſiaſtical and Aſtronomical Full Moons nearly agree, which were very different before theſe Corrections took place: And the general Table or Rule for finding *Eaſter* for ever, which is now in our Liturgy, by the Golden Number and Dominical Letter, is made according to the corrected Diſpoſition of the Golden Number.

In the firſt Year of the Chriſtian *Æra*, 2 was the Golden Number; and therefore, if to the current Year of *Chriſt* we add 1, and divide the Sum by 19, the Remainder, after the Quotient, will give the Golden Number.

From the *Cycles* of the Sun and Moon multiplied into one another, ariſes a third Period of 532 Years, which is called the *Victorian* or *Dionyſian* Period: And after the Completion of this Period, not only the New and Full Moons return to the ſame Days of the Month, but alſo the Days of the Months return to the ſame Days of the Week; and therefore the *Dominical* Letters and the moveable Feaſts return again in the ſame Order. Hence this *Cycle* is called the great *Paſchal Cycle.*

To find the Year of the *Dionyſian* Period for any Year of *Chriſt*, to the current Year add the Number 457, and divide the Sum by 532, the Remainder, after the Quotient, is the Year of the Period.

It is a *Problem* of another Kind: *Having the* Cycles *of the Sun and Moon, to find the Year of the* Dionyſian *Period.* For Example, ſuppoſe the Year of the *Cycle* of the Moon to be 17, and of the Sun 21: To ſolve this *Problem*, it is required to find a Number, which, when it is divided by 19, leaves 17, and when it is divided by 28, leaves 21. To find this let it be required to find two Numbers, one of which will divide exactly by 28, without a Remainder, but when it is divided by 19 leaves 17; and another Number, which will

will divide exactly, without a Remainder by 19, but when it is divided by 28, leaves 21: It is plain, that the Sum of these two Numbers will answer the Question. Lecture XXIX.

WE will here shew the analytical Investigation of these two Numbers. We will suppose the first Number to be $28x$, for it is a Multiple of 28; and because this Number, divided by 19, leaves 17; if we take 17 from it, the Remainder, divided by 19, will be a Number; since then 19 divides $28x - 17$, and it also divides $19x$, therefore it will divide their Difference, which is $9x - 17$; consequently $\frac{9x - 17}{19}$ is a Number. Call that Number n, consequently $9x - 17 = 19n$, and $9x = 19n + 17$, and $x = \frac{19n + 17}{9}$ and because x is an intire Number 9 must divide $19n + 17$, but 9 divides $18n + 9$; therefore it will divide the Remainder $n + 8$, and therefore $\frac{n + 8}{9}$ is a Number. Let that Number be 1, and n will be $= 1$, and $x = 4$; consequently, $28x = 112 =$ to the first Number to be found. Suppose the second Number to be $19y$; for it must be a Multiple of 19; therefore $\frac{19y - 21}{28}$ is an integer Number; suppose it to be n; then $19y - 21 = 28n$, and $y = \frac{28n + 21}{19}$, which must likewise be an intire Number; and therefore $28n + 21$, will be divided by 19: But 19 divides $19n + 19$; wherefore it will divide the Remainder $9n + 2$, and $\frac{9n + 2}{19}$ is an intire Number; suppose it $= p$; then is $9n + 2 = 19p$, and $n = \frac{19p - 2}{9}$

therefore

Lecture XXIX. therefore 9 must divide $19\,p - 2$; but it divides $18\,p$, therefore it must also divide $p - 2$, and $\frac{p-2}{9}$ is an Integer or 0: Let it be equal to 0, then is $p = 2$, and $\frac{19p-2}{9} = n = 4$, and $19\,y = 28\,n + 21 = 133$; therefore one of the Numbers is 112, and the other is 133, whose Sum is 245, the Number requir'd: And whenever the *Cycle* of the *Sun* is 21, and that of the *Moon* 17, the Year of the *Dionysian* Period is 245.

The same Problem may be otherwise solved, by stated and constant Multiplicators, which are such, that one of them can be divided by 28, without any Remainder; but if it be divided by 19, there remains 1: The other will divide by 19 without a Remainder; but when it is divided by 28, there is a Remainder of 1: Such Numbers are found in the same Way, as the preceding Numbers were; *viz.* I suppose the first to be $28\,x$, and the other $19\,y$; wherefore 19 will divide the Number $28\,x - 1$, without any Remainder; and therefore it will divide $9\,x - 1$ without a Remainder, and $\frac{9x-1}{19}$ will be a whole Number: Suppose it equal to n; then $x = \frac{19n+1}{9}$; therefore $\frac{n+1}{9}$ is an intire Number, and the least Number that can be put for n is 8: Let therefore $n = 8$, and x, being $= \frac{19\,n + 1}{9}$, must be 17; therefore the first Number, being $28\,x$, will be 476. Again, let $\frac{19\,y - 1}{28} = n$, then $y = \frac{28\,n + 1}{19}$; suppose $\frac{9\,n + 1}{19} = p$; then is $n = \frac{19p-1}{9}$, and $\frac{p-1}{9}$ is either a whole Number or nothing; let $p - 1 = 0$, and $p = 1$; then $n\ \frac{19\,p - 1}{9} = 2$, and $19\,y = 28\,n$

+

+1 = 57. Therefore the two Numbers that were to be found are 476 and 57. And because the Number 476, divided by 19, leaves 1, if it be multiplied by any Number less than 19, and the Product be divided by 19, there will remain the Number which multiplies it. In like Manner, because 57, divided by 28, leaves 1, if 57 be multiplied by any Number less than 28, and the Product be divided by 28, there will be left that Number which multiplied 57. Hence we draw this general Rule for finding the Year of the *Dionysian* Period. Multiply the *Cycle* of the *Sun* by 57, and the *Cycle* of the Moon by 476; divide the Sum of the Products by 532, the Remainder, after the Quotient, will be the Year of the *Dionysian* Period.

The Cycle *of Indictions.*

BESIDES the *Cycles* of the Sun and Moon, there is another Period of Years, which is called the *Cycle* of Indictions, which the *Romans* used, and has no Connection with the celestial Motions, but is a Revolution of 15 Years, which being compleated, it begins again. It is frequently mentioned in the *Imperial* and *Pontifical Diplomas*. The Year before the Birth of *Christ* the Indiction was 3; and therefore, if to the Year of *Christ* you add 3, and divide the Sum by 15, the Remainder shews the Year of the Indiction. If there be no Remainder, the Indiction is 15.

The Julian *Period.*

OF these three *Cycles* of the Sun, Moon and Indiction, by their mutual Multiplication, the *Julian* Period of 7980 Years is composed. This Period had its Beginning 764 Years before the Creation, and is not yet compleated; and therefore it comprehends all other *Periods*, *Cycles* and *Epoches*, and the Times of all memorable Actions and Historians. There is but one Year in the whole Period which has the same Numbers for the three *Cycles* of which it is made up; and therefore, if the Historians had remarked in their Annals the *Cycles* of each Year, there had been no Dispute about the Time of any Action.

THE

Lecture XXIX.

From the Year of Christ *to find the* Julian *Period.*

THE Year before the Birth of *Christ* was the 4713th Year of the *Julian* Period. And therefore, if to the current Year of *Christ* we add 4713, the Sum will be the Year of the *Julian* Period: On the contrary, from the Year of the *Julian* Period subtract 4713, there will remain the Year of the Christian *Æra*.

From the Cycles *to find the Year of the* Julian *Period.*

HAVING the Years of the *Cycle* of the Sun, Moon, and Indiction, to find the Year of the *Julian* Period.

THIS *Problem* may be solved in the same Manner as we shewed in the like Case about the *Dionysian* Period, *viz.* by finding 3 Numbers, such as the first is a Multiple of 19 and 15, or of their Product 285; but being divided by 28, leaves the Number of the *Cycle* of the Sun for a Residue. The second Number must be a Multiple of 28 and 15, or of their Product 420; but being divided by 19, leaves the Year of the *Cycle* of the Moon. *Lastly*, The third must be a Multiple of 28 and 19; but being divided by 15, leaves the Year of the Indiction: The Sum of these Numbers, if less than 7980, is the Year of the *Julian* Period. But if the Sum be bigger, divide by 7980, and the Remainder will be the Year of the Period required.

THE *Problem* may likewise be solved by constant and stated Multiplicators, the first of which is a Multiple of the Number 285; but divided by 28, leaves 1. The second is a Multiple of 420; but divided by 19, leaves 1 for a Residue. The third is a Multiple of 532; but being divided by 15, leaves for a Remainder 1. These Numbers are to be found by the same Method we shewed in our former *Problem* concerning the *Dionysian* Period, and are 4845, 4200, 6916; which being once found, the Canon, or Rule for finding the Year of the *Julian* Period from the Years of the *Cycles*, is this; multiply the Number 4845 by the Year of the *Cycle* of the *Sun*, and the Number 4200 by the Year of the *Cycle* of the Moon, likewise the Number 6916 by the Year of the Indiction. Divide the Sum of these Products by

by 7980, neglecting the Quotient, the Remainder will be the Year of the *Julian* Period required. Example: In the Year 1719 the Year of the *Cycle* of the Sun, is 19, of the Moon 9, and of the Indiction 11; multiply 4845 by 19, the Product is 92055; and 4200 being multiplied by 9, the Product is 37800; and lastly, 6916 being multiplied by 11, the Product is 76076. The Sum of the Products is 205931; which, being divided by 7080, will have a Remainder of 6431 Years, which is the Year of the *Julian* Period.

Lecture XXX.

LECTURE XXX.

An Appendix, containing a Description and Use of both the Globes; *together with some Spherical* Problems *that are to be solved by a* Trigonometrical *Calculation.*

OF the Things which pertain to the *Globes*, some are common to both *Globes*, some are peculiar to one of the two: And those Things that are common, are either without the Surface, or painted on the Surface. Without the Surface of the *Globes* are to be seen,

First, The two *Poles*, about which the *Globes* revolve: The one is the *Arctick* Pole, named so from the two *Bears* that are nigh to it, and is called likewise the *Septentrional* or *North Pole*, from the *Septem Triones*, or the seven *Stars* of *Charles*'s Wain: The other opposite to it is called the *Antarctick* Pole.

Secondly, The *Brazen Meridian*, and one Side of it only, which is distinguished and divided into Degrees, and which, passing through the Poles, represents the true Meridian; and this Side is always to be turned to the East, and the *North Pole* to the North,

Lecture XXX.

North. The *Meridian* is divided into four Quadrants of 90 Degrees, two of which are reckoned from that Part of the Equinoctial that is above the Horizon, towards each of the Poles; the other two Quadrants have their Divisions of 90 Degrees, beginning at the Poles, and ending in the Æquator.

THIRDLY, The wooden Horizon, whose upper Side does only represent the Horizon, and is divided into several Circles, the innermost of which contains the twelve Signs of the Zodiack, distinguished by their Names and Characters, and each Sign is divided into 30 Degrees. Next to this are joined two Circles, with the *Julian* and *Gregorian* Kalendars, disposed according to their Months and Days. The outermost is a Circle with all the Points of the Compass, and the Winds, as they are denominated by the Seamen.

FOURTHLY, A Brass Quadrant of Altitude, whose Edge is divided into Degrees, and is to be fastened to the Meridian at the 90th Degree from the Horizon; from which the Degrees are numbered upon it upwards to the Zenith.

FIFTHLY, The Horary Circle divided into twice twelve Hours: The 12th Hour at Noon is upon the upper Part of it at the Meridian, and the 12th at Night is on the Meridian at the lower Side towards the Horizon. The Pole carries round the Hand which shews the Hour, and is in the Center of the Circle: The Hours upon the East Side of the Meridian are the Morning Hours; these on the West Side are the Hours after Noon.

SIXTHLY, the Mariner's Compass is fixed upon the Pediment or wooden Frame, which contains the Globe, and by it the Globe is put in a right Position in respect of the Points of the Heavens.

SEVENTHLY, The Semicircle of Position, whose Extremeties are fixed to the Points of North and South, so that the Semicircle can be moved freely from the Horizon to the Meridian, and may be raised to any Position. These Things we have described are

are without the Globe. But on the Surfaces are delineated the Things following.

FIRST, The Equinoctial Circle divided into 360 Degrees; the Numbers begin at the vernal Intersection, or at first, of ♈, and are continded till they return to the same.

SECONDLY, The Ecliptick, divided into 12 Signs, and each Sign into 30 Degrees, their Names, Characters and Order are to be learned; they are,

♈ *Aries*, or the *Ram*. ♉ *Taurus*, or the *Bull*. ♊ *Gemini*, or the *Twins*. ♋ *Cancer*, or the *Crab*. ♌ *Leo*, the *Lion*. ♍ *Virgo*, the *Virgin*. ♎ *Libra*, the *Balance*. ♏ *Scorpius*, the *Scorpion*. ♐ *Sagittarius*, the *Archer*. ♑ *Capricornus*, the *Goat*. ♒ *Aquarius*, the *Water-bearer*. ♓ *Pisces*, the *Fishes*.

THE *Sun* in his annual Motion passes through the Ecliptick, and if we add to it a broad Space of about eight Degrees on each Side, we have the Zodiack, in which are the 12 *Asterisms* or Constellations, the most of which have the Likeness of some living Creature, upon which Account the Zodiack has its Name. In this broad Circle the Moon and all the Planets perform their Motions: The Ecliptick is to be distinguished from the Equinoctial Circle by this, that the Equinoctial, while the Globe is turned, does always cut the Meridian and the Horizon in the same Points. But the Ecliptick constantly changes its Position; sometimes while the *Globe* is turning it is high, sometimes it is low; sometimes it cuts the Æquator and the Horizon in one Degree, sometimes in another.

THIRDLY, There are two Tropicks of ♋ and ♑, which are the Limits or Boundaries of the *Sun's* Deviations from the Equinoctial, either towards North or South, including between them the oblique Course of

Lecture XXX. of the *Sun*, that is, the Ecliptick, and may be called the outermost of the *Sun*'s Parallels. For because the *Sun* every Day passes to a different Degree of the Ecliptick in its annual Motion, that Degree with the *Sun* in it, being carried round the Earth, by the diurnal Revolution of the Heavens, will describe a Circle parallel to the Æquator; and then there must be so many Parallels, as there are Days from the longest to the shortest; though the *Sun* does not remain for a Day in one Point of the Ecliptick, but is continually advancing forward; and therefore does not describe a perfect Parallel, but rather a spiral Line: But the Distance between each Spire being but very small, especially near the Tropicks, we may well suppose the single Revolutions, but especially the outermost, to be Parallels; which is sufficient for common Use, and is most convenient.

FOURTHLY, The two polar Circles, the Arctick and Antarctick, which have been explained in our VII and XVIII *Lectures*. These Things we have here mentioned, are common to both *Globes*, though the Ecliptick and the Semicircle of Position do properly belong only to the celestial *Globe*; yet they are put upon the terrestrial *Globe* also, that the *Phænomena*, which depend upon the Motion of the *Sun*, and the Points or Cusps of the Houses, may be thereby explained if needful.

THOSE Things which are peculiar or proper to one Sort of *Globe*, are partly some Circles or curve Lines; as in the celestial *Globe*, the two *Colures*, and the *Circles* of *Latitude*; in the terrestrial, the Meridians, Parallels and Rhumbs; partly the Representations in the terrestrial *Globe*, of Seas, Islands and Countries, which we leave to the Geographers to describe. In the celestial *Globe* the Figures of the Constellations are painted, and the *Stars* represented in the same Order, Magnitude and Position they have in the Heavens. These we have enumerated in our VI *Lecture*.

HAVING

HAVING described the *Globes*, we come now to shew their Use, which is manifold; but for our present Purpose, it it chiefly contained in the following *Problems*. Lecture XXX.

PROBLEM I. *Having a Place in the terrestrial* Globe, *to find its Longitude and Latitude.*

TURN the *Globe* till the Place comes to the Meridian (I mean to the Eastern Side of the Brass Meridian); and the Degree of the Æquator, which is then under the Meridian, whatever Number it is mark'd by, shews the Longitude of the Place; then upon the Meridian, count up from the Æquator the Degrees mark'd, till you come to the Place, and you have the Latitude; which will be North Latitude, if the Place be on the North Side; or Southern if it lies upon the Southern Side of the Æquator.

PROBLEM II. *Having the Longitude and Latitude, to find out the Place on the terrestrial* Globe, *to which they belong:* Seek in the Æquator the Degree of Longitude that is given, and bring it to the Meridian; then count from the Æquator on the Meridian, the Degree of Latitude given, towards the Arctick or Antarctick Pole, according as the Latitude is either North or South, and under that Degree of Latitude lies the Place that was to be found.

PROBLEM III. *To rectify both* Globes, *and set them to a given Latitude or Elevation of the Pole; and to apply the Quadrant of Altitude to the Vertical Point; and to place the* Globes *according to the Points of the Compass, by Help of the Needle.* If the Latitude of the Place be North, raise the Arctick Pole above the Horizon; but for a South Latitude, you must raise the Antarctick: Then from the elevated Pole, count upon the Meridian towards the Horizon, the Degrees of the Pole's Elevation; and that Point where the Reckoning ends, bring to the Horizon, and then the *Globe* is adjusted to a due Elevation. Count from the Æquator on the Meridian the Latitude required, and there will be the Vertex of the Place, or the Zenith. To this Point of the Meridian fasten the Quadrant of Altitude, with the Screw

Lecture XXX. that is at the End of it, ſo that the Edge of the Quadrant, which is divided into Degrees, may be join'd to this Point. *Laſtly*, Turn the whole Frame in which the *Globe* is with its Pediment, till the magnetick Needle lie in the Plane of the Meridian, ſo that the North Point of the Horizon of the *Globe* be turn'd Northwards; then will the reſt of the Points on the Horizon of the *Globe* agree with the correſponding Points of the Horizon of the Place.

PROBLEM IV. *To find for any Day of the Year the Degree or Place of the* Sun *in the Ecliptick, by the Help of the Kalendar, and the Circle of Signs adjoin'd; and then to mark it upon the Ecliptick.* Seek in the wooden Horizon, the Month and Day given; but take care to diſtinguiſh the *Julian* and *Gregorian* Kalendars, that you may not miſtake the one for the other: Then in the innermoſt Circle, which is the Circle of Signs, over-againſt the Day, you will ſee the Degree and Sign in which the *Sun* is that Day. In the Ecliptick which is drawn upon the Surface of the *Globe*, ſeek firſt the Sign, and then the Degree of the *Sun*'s Place. The Place of the *Sun* is more accurately found out by an *Ephemeris*, which is made for each Year, or elſe it may be calculated by *Aſtronomical* Tables.

PROBLEM V. *To find the right Aſcenſion and Declination of the* Sun, *or any* Star; *and by that Means to adjuſt the Hand which points the Hours to the twelfth Hour.* Bring the *Sun*'s Place in the Ecliptick, found by the laſt *Problem*, to the Meridian, and mark the Degree of the Æquator which is then under the Meridian, that will be the right Aſcenſion of the *Sun*: Then compute from the Equinoctial on the Meridian, the Number of Degrees to the Place of the *Sun*; they will ſhew the *Sun*'s Declination, which will either be North or South, as the *Sun* is on this or the other Side of the Æquator. When the Place of the *Sun* is in the Meridian, turn the Hour-hand upwards, till it comes to the twelfth Hour at Noon. After the ſame Manner bring the Place of any fixed *Star* to the Meridian, and you ſhall find its right Aſcenſion on the Æquator,

Æquator, and its Declination on the Meridian. Having the Place of the *Sun*, we ſhew'd how to find its right Aſcenſion and Declination by *Trigonometry*, in our XIXth *Lecture*. Lecture XXX.

PROBLEM VI. *To find the Meridian Altitude of the* Sun, *or any fixed* Star, *by a Quadrant, or any other Inſtrument fit for the Purpoſe.* We ſhew'd the Method of obſerving the *Sun*'s or a *Star*'s Altitude in *Lecture* XIX.

PROBLEM VII. *Having the Declination and Meridian Altitude of the* Sun, *or of a fixed* Star, *to find the Latitude of the Place or Height of the Pole above the Horizon.* The Method of finding the Latitude by Obſervation was likewiſe explain'd in *Lecture* XIX.

PROBLEM VIII. *Having the right Aſcenſion of the* Sun, *and of a fixed* Star, *to find the Time when the* Star *culminates or comes to the Meridian.*

SUBTRACT the right Aſcenſion of the *Sun* from the right Aſcenſion of the *Star*, adding, if needful, 360 Degrees; and there will remain the Arch of the Æquator, that has paſſed the Meridian between Noon and the Time of the Culmination: Turn this Arch into Time by dividing the Degrees by 15, and the Quotient gives the Hours; then multiply the remaining Degrees by 4, and you have the Minutes; and likewiſe divide the Minutes, which are the Parts of Degrees, by 15, and the Quotient gives the horary Minutes: And if there be any Minutes of Degrees remaining after the Diviſion, multiply them by 4, and you have the horary Seconds: The Time made up of theſe Hours, Minutes and Seconds, ſhews the Moment of the Culmination of the *Star*.

PROBLEM IX. *Having the Place of the* Sun, *or any* Star, *to find its oblique Aſcenſion and Deſcenſion; as alſo its Eaſtern and Weſtern Amplitude.* Bring the Place of the *Sun* or *Star* to the Horizon in the Eaſt, and mark the Point of the Æquator that riſeth with it; it will be its oblique Aſcenſion: Then count from the Point of Eaſt upon the Horizon, to the Place of the *Sun* or *Star*; the Degrees intercepted will be the

Lecture XXX. Eastern Amplitude. If you bring the Place of the *Sun* or *Star* to the Western Side of the Horizon, the Degree of the Æquator which goes down with it, is the oblique Descension; and the Arch of the Horizon between the West Point, and the Place of the *Sun* or *Star*, is the Western Amplitude.

Table XXVI. Fig. 2. THE *Trigonometrical* Solution of the *Problem* is this: Let H P O P be the Meridian, Æ Q the Æquator, P the Pole, S the *Sun*, or *Star* in the Horizon, whose Declination is S R; *o r* the Point of East or West. In the right-angled Triangle *o r* R S, he have the Side R S the Declination of the *Sun* or *Star*, and the Angle R *o r* S, which the Æquator makes with the Horizon, and is equal to the Complement of the Latitude; whence we shall find the Arch *o r* R, which is the Ascensional Difference of the *Sun* or *Star*; which, added to the Right Ascension, or subtracted from it, according as the *Sun* or *Star* is towards the depressed or elevated Pole, gives the oblique Ascension. We shall have moreover in the same Triangle, the Arch *o r* S, the Amplitude of the *Sun* or *Star*. The Ascensional Difference added to a Quadrant, or subtracted from it, according as the *Sun* or *Star* is towards the elevated or depressed Pole, gives the Semidiurnal Arch; which being turned into Time, shews the Time of half the Stay of the *Sun* or *Star* above the Horizon.

PROBLEM X. *Having the Ascensions of the* Sun *or* Star, *both Right and Oblique, to find the half Time of their Stay above the Horizon: As also the Length of Day and Night, and the Time of* Sun *rising and setting.* Take the Difference of the Oblique and Right Ascension, and we shall have the Ascensional Difference: Convert it into Time, as we shew'd in the VIIIth *Problem*, which, when the *Sun* or *Star* declines to the elevated Pole, is to be added to six Hours; but if to the depressed *Pole*, it is to be subtracted from six Hours; and we shall have half the Time of the Stay of the *Sun* or *Star* above the Horizon: And its Complement to 12 Hours, is half the Time it abides under the Horizon. Half the

Time of the *Sun's* Stay above the Horizon, being computed from Noon, gives the Time of *Sun* setting; and half the Time of the *Sun's* Stay under the Horizon, computed from Midnight, gives the Hour of the *Sun's* rising; and the half Time of the *Sun's* Stay above the Horizon being doubled, gives the Length of the Day; and the half Time of the Stay below the Horizon doubled, gives the Length of the Night. If you put the Hour-hand to the 12th Hour, when the Place of the *Sun* is in the Meridian, and turn the *Globe* round, till the Place of the *Sun* comes to the Eastern Side of the Horizon, the Hand will point out the Hour of Sun-rising: Bring it to the Western Side of the Horizon, and the Hand will shew the Time of Sun-seting; from which it is easy to compute the Length of Day and Night.

PROBLEM XI. *Having the Time of the Culmination of a* Star, *and of its half Stay above the Horizon; to find the Hour of its rising and setting.* If from the Time of the *Star's* Culmination, you subtract the Time of the half Stay above the Horizon, you will have the Hour wherein the *Star* riseth: If you add that Time to the Time of the Culmination, you shall have the Time of the *Star's* setting, which in both Cases is computed from Mid-Day. Or if, when the Place of the *Sun* culminates, you bring the Hour-hand to the 12th Hour, and then turn round the *Globe*, 'till the *Star* comes to the Eastern or Western Side of the Horizon, the Hand will point to the Hour of the Rising or Setting of the *Star*.

PROBLEM XII. *To find the Degree of the Ecliptick, which rises or sets with a given* Star; *and from thence to determine its Cosmical and Achronical rising and setting.* Bring the given *Star* to the Eastern and Western Side of the Horizon, and mark what Degree of the Ecliptick rises or sets with it; then in the wooden Horizon look for that Sign and Degree which rose or set with the *Star*; and over-against it in the Kalendar, you will find the

Lecture XXX. Month and Day of the cosmical Rising of the *Star*. And if you look in the same Horizon, the Point opposite to the rising Point; over-against it in the Kalendar, you shall have the Month and Day of its cosmical Setting. So likewise, over-against the Degree that sets with the *Star*, you will find the Day and Month of the achronical Setting; and the opposite Degree will shew in the Kalendar the Day and Month of the *Star's* achronical Rising.

Table XXVI. Fig. 3. THE *Trigonometrical* Solution of the *Problem* is this: Let HO be the Horizon, HZO the Meridian, ÆQ the Æquator, EC the Ecliptick, ♈ the Point of Intersection of the Equator and Ecliptick; A the Point of the Ecliptick which rises with the *Star*; and the Point of the Æquator which rises with the *Star*, suppose to be *o r*. In the Triangle ♈ *o r* A, we have ♈ *o r* the oblique Ascension of the *Star*; and the Angle ♈, the Inclination of the Æquator and Ecliptick; and the Angle ♈ *o r* A, the Height of the Æquator above the Horizon, or its Complement to two right Angles. Hence we shall find the Arch of the Ecliptick ♈ A, and the Point of it A, which rises with the *Star*. But by the Kalendar, or an *Ephemeris*, we have the Time when the *Sun* is in that Point; and therefore, we have the Time when the *Star* rises cosmically; we have likewise the Angle ♈ A *o r* the Angle of the Ecliptick and the Horizon at the rising Point. When the *Sun* is in the Point opposite to the Point A, then the *Star* rises achronically: And by a like Calculation we shall find the Time when the *Star* sets cosmically or achronically.

PROBLEM XIII. *Having the Latitude of the Place, and the Degree of the Ecliptick which rises or sets with the* Star, *to determine the heliacal Rising or Setting of a* Star. Bring the *Star* to the Eastern Side of the Horizon, and turn the Quadrant of Altitude round to the Western Side, till it cut the Ecliptick in the twelfth Degree from the Horizon, on the Quadrant of Altitude, if the *Star* be of the first Magnitude. Then mark the Point of the Ecliptick, where

where the Quadrant interſects it; that Point, when the *Star* riſes, is twelve Degrees high above the Weſtern Horizon, but at the ſame time the oppoſite Point is twelve Degrees below the Eaſtern Horizon: Look that Point in the wooden Horizon, and over-againſt it you will find the Month and Day when the *Sun* enters that Point of the Ecliptick, which is the Day of the *Star's* riſing heliacally; when it begins to get ſo far from the *Sun's* Beams, that it may be ſeen in the Morning before Sun-riſing: but if you would know the heliacal Setting, bring the *Star* to the Weſt-ſide of the Horizon, and turn the Quadrant of Altitude to the Eaſt-ſide, 'till the twelfth Degree of it from the Horizon cuts the Ecliptick, and mark that Point where it interſects the Ecliptick; the Point oppoſite to this, is ſo many Degrees depreſs'd under the Horizon at the Weſtern Side; and if we find, in the wooden Horizon, the Month and Day when the *Sun* comes to that Point, we ſhall have the Time of the heliacal Setting. Lecture XXX.

By *Trigonometry* the *Problem* is thus to be ſolved: In the Figure of the preceeding *Problem*, let A be the Point of the Ecliptick, which riſes with the *Star*: And ſuppoſe the *Sun* in the Ecliptick at ☉, ſo that the Arch ☉R of the Circle of Depreſſion may be 12 or 13 Degrees, according as the *Star* is of the firſt or ſecond Magnitude. In the right-angled Triangle A R ☉, we have the Angle R A ☉, the Angle of the Ecliptick and Horizon, and the Side R ☉, which is 12 or 13 Degrees: Hence we ſhall have the Side A ☉, which, added to ♈ A, gives the Arch ♈☉, and the Point ☉, which the *Sun* muſt be in when the *Star* riſes heliacally. In the ſame Manner we may find the Time of the heliacal Setting. Table XXVI. Fig. 3.

PROBLEM XIV. *Having the Latitude of the Place, and the Place of the* Sun *in the Ecliptick; to find the Beginning and End of Twilight.* Rectify the *Globe* for the Latitude of the Place by *Problem* third, and put the Hour-hand to the twelfth Hour, the *Sun's* Place being in the Meridian; then take

the Point of the Eclíptick opposite to the *Sun's* Place, and turn the *Globe* Westward, as also the Quadrant of Altitude, 'till the Point opposite to the *Sun's* Place, cut the Quadrant of Altitude in the 18th Degree above the Horizon: The Hour-hand will shew the Time of Dawning in the Morning. But if you take the Point opposite to the *Sun*, and bring it to the Eastern Hemisphere, and turn it 'till it meets with the Quadrant of Altitude in the 18th Degree, the Hand will shew the Hour when Twilight ends in the Evening. The *Trigonometrical* Solution of this *Problem* is in *Lecture* XX.

PROBLEM XV. *Having the Latitude of the Place, and the Place of the* Sun, *if we have besides any one of the three following Things, viz. The Hour of the Day or Night, or the Altitude or Azimuth of the* Sun *or a* Star; *to find the other two.* Rectify the *Globe* for the Latitude given, and bring the Place of the *Sun* to the Meridian; and the Hour-hand to the Twelve o'Clock Hour: Then if the Hour be given, turn round the *Globe* 'till the Hand points to it, and bring the Quadrant of Altitude to the Place of the *Sun* or *Star*; you will see in the graduated Edge, the Degree of Altitude; and where the Quadrant intersects the Horizon, there you will find its Azimuth, to be counted from the Intersection of the Meridian and Horizon: But if the Altitude be given, turn the *Globe* with one Hand, and the Quadrant of Altitude with the other, 'till the Place of the *Sun* meets with the Quadrant at the given Altitude; then the Hand will point to the Hour; and the Intersection of the Quadrant and Horizon will shew the Azimuth. But if the Azimuth be given, turn the Quadrant 'till it intersects the Horizon at the given Azimuth, and there keep it fixed; but turn the *Globe*, 'till the Place of the *Sun* or *Star* meets with the Quadrant, and the Degrees upon the Quadrant will give the Altitude, and the Hour-hand will point to the Hour.

The *Problem* is solved *Trigonometrically* thus: Let HO be the Horizon, HPO the Meridian, ÆQ the Æquator, Z the Vertex or Zenith, P the Pole, S the *Sun* or *Star*, whose Distance from the Vertex is ZS, and SP the Complement of Declination, or its Distance from the Pole. Because we have the Right Ascension of the *Sun* or *Star*, we have the Difference of their Right Ascensions, which being turned into Time will shew the Time of the Culmination of the *Star*; and the Arch which measures the Angle ÆPS, being turned into Time, will give the Hour of the Night. Now in the Triangle ZPS, having ZP the Distance of the Zenith from the Pole, and PS the Complement of the Declination of the *Star*; if I have, besides these two, the Angle P, which the Hour gives, we can from them find the Angle Z, which shews the *Star*'s Azimuth, and the Arch ZS the *Star*'s Zenith Distance, which will shew the Altitude. Or if we have the Arch ZS, we shall find the Angle P, and by that the Hour of the Night, as also the Angle PZS the Azimuth. Or if we have the Angle PZS, we can from thence find ZS the Complement of the Altitude, and the Angle ZPS, which will give the Hour. By the same Method having the Altitude of the *Sun*, which we take by an Observation, and his Declination, which is known by Tables, from his Place in the Ecliptick, we can find the Angle ÆPS, which being turned into Time, will give the Honour of the Day.

Lecture XXX.

Table XXVI. Fig. 4.

PROBLEM XVI. *To find the Distance between two Places on the Surface of the Terrestrial Globe.*

Let us, for Distinction-sake, call one of the Places the first, and the other the second. Rectify the *Globe*, for the Latitude of the first Place, and bring it to the Meridian, and there fix the *Globe* with the Quadrant of Altitude to the Vertex, and turn the Quadrant till its graduated Edge pass through the second Place: Then count the Degrees of Distance, from

Lecture XXX. from the Vertex to the second Place, and the Arch of the Horizon intercepted between the Meridian and Quadrant will give you the Angle of Position.

Table XXVI. Fig. 5. By *Trigonometry* we thus proceed: Let ÆQ be the Æquator, P the Pole, S s two Places, whose Complements of Latitude are PS and P s; and because their Longitudes are given, we have their Difference of Longitude, which is measured by the Angle SP s: Therefore in the Triangle SP s, we have the Sides SP, s P, with the Angle SP s; by them we can find S s in Degrees and Minutes, which being converted into Miles, allowing 69 *English* Miles for each Degree, we shall have their Distance in Miles. We can also find the Angles PS s and P s S, which are the Angles of Position.

In the same Manner in the Heavens, if we have the Right Ascensions and Declinations of two *Stars*, or their Longitudes and Latitudes, we shall find their Distances.

PROBLEM XVII. *For any Time and Place to erect the Theme or Scheme of the Heavens.* Rectify the celestial *Globe* for the Latitude of the Place. If you have not a celestial *Globe*, a terrestrial will do, Take the Place of the *Sun* for the given Time, and bring it to the Meridian, and the Hour-hand to the twelfth Hour; then turn the *Globe* till the Hand shews the given Hour: Or, if you like to be more accurate in your Work, to the Right Ascension of the *Sun* add so many Degrees and Minutes, as the Time from Mid-day requires, for every Hour counting 15 Degrees, and for every four Minutes a Degree, rejecting, if it exceeds it, 360 Degrees; so by this you will have the Right Ascension of Mid-heaven, or the Degree of the Equinoctial, which then culminates, which is to be placed under the Meridian. Then fasten the Semicircle of Position to the Meridian, at the Points of South and North in the Horizon. From the Point of the Æquator culminating, count on the Æquator 30 Degrees Eastward, and bring

bring the Semicircle of Poſition to the 30th Degree, and obſerve in what Degree this Semicircle cuts the Ecliptick; that will be the Cuſp of the eleventh Houſe, which muſt be ſet down on Paper. Again, move the Semicircle of Poſition to the 60th Degree of the Equinoctial from the culminating Point, and mark where it cuts the Ecliptick, and you have the Cuſp of the twelfth Houſe, which is likewiſe to be writ down. Bring the Semicircle of Poſition to the Weſtern Side, and count thirty Degrees from the culminating Point, and letting the Semicircle paſs through that Point, obſerve where the Semicircle cuts the Ecliptick, that will be the Cuſp of the ninth Houſe. Then count from the culminating Point again Weſtward 60 Degrees, and the Semicircle of Poſition, paſſing through that Point, will cut the Ecliptick in the Cuſp of the eighth Houſe: And the Meridian cuts the Ecliptick in the Cuſp of the tenth Houſe. And the Place where the Horizon Eaſtward cuts the Ecliptick is the Cuſp of the firſt Houſe, or the *Horoſcope*; and the Weſtern Side of the Horizon ſhews in the ſame Ecliptick, where it cuts it, the Cuſp of the ſeventh Houſe: And as it is diametrically oppoſite to the firſt, ſo is the ſecond to the eighth, and the third to the ninth, the fifth to the eleventh, and the ſixth to the twelfth.

Lecture XXX.

PROBLEM XVIII. *Having erected the Theme, to direct any Point to any other Point.* To a Planet or Aſpect aſſign its Place in the Zodiack, according to its Longitude and Latitude; and chuſe any Planet or Degree of the Ecliptick, which you would direct, and which, for Diſtinction-ſake, we will call the firſt Place; and the Place to which you would direct this firſt Place, call the ſecond; then through the firſt Place, which uſed to be called the Significator, draw the Semicircle of Poſition, and mark that Degree in which it cuts the Æquator; then, keeping the Semicircle in the ſame Poſition,

Lecture XXX. sition, turn the Globe Westward, till the second Place arrive at it, and then again observe where the Equinoctial is cut by the said Semicircle. Subtract the Degree first observed, from the Degree observed in the second; adding, if need be, 360 Degrees; the Remainder is the Arch of Direction, which was to be found.

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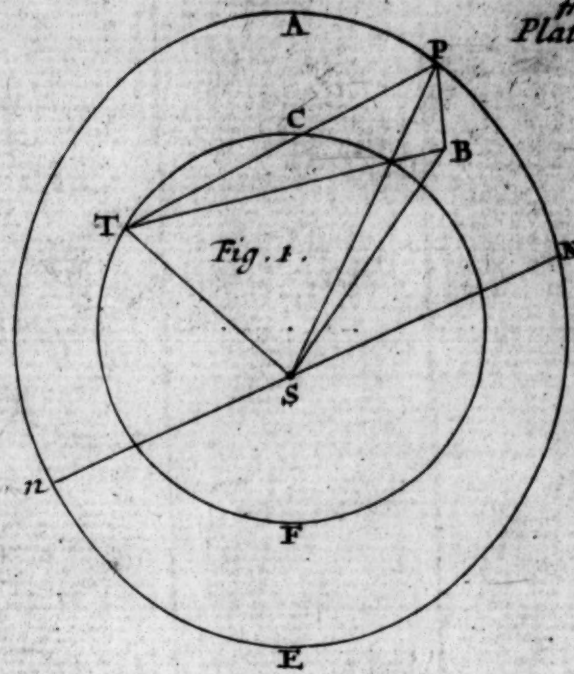
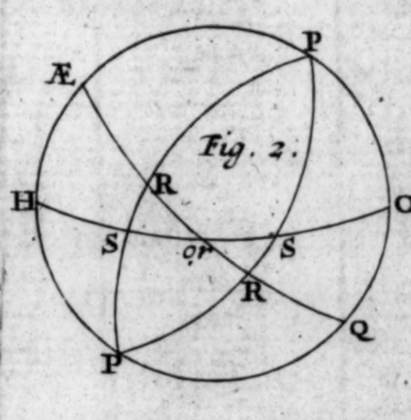
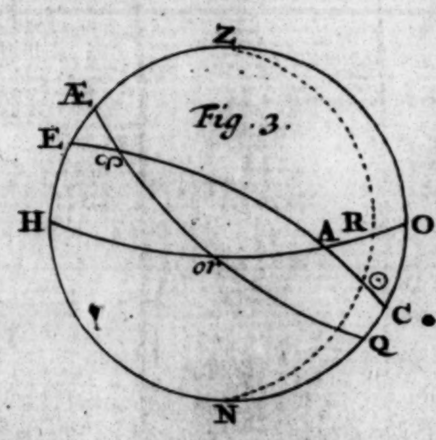
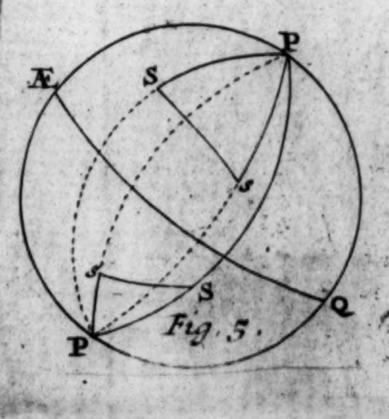
25

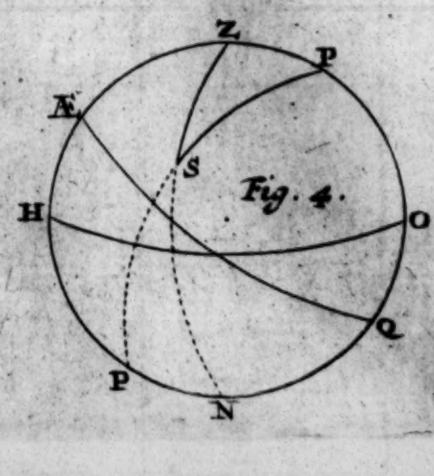
front. p. 396
Plate XXVI











THE

INDEX,

Which sheweth where the Terms and Words used in *Astronomy* are explained in this BOOK.

A.

A BSIDS, See *Apsides*,
Achronical rising, Pag. 222
Æquation, what, 285
Where greatest, ibid.
Æquation of Time, 317
Æquation of Time when greatest, 322
The Æquator or Equinoctial, 63,208
Æquator's Secondaries, 74,209
Æquinocties or Equinoctial Points, 269
Æra, or Epocha, 365
Table of remarkable Æras, 368
Altitude of the Pole, 217
Of a Star, 6,223
Of the Earth's Shadow, 120
Of the Moon's Shadow. ibid.
Amphiscii, 212
Amplitude, Eastern and Western, 214
Amplitude of the World, 40
Anastrous Signs, 81
Angles their Measure, 4

Method

Method of observing, Pag. 5
The Angle under which the Sun would appear, seen from a fixed Star, 37
The Angle of Commutation, 340
The Angle of the Æquator and the Ecliptick, 209
Of the Ecliptick and Meridian, 226
Of the Ecliptick and the Horizon, 267
Of the Ecliptick and Vertical Circle, or the Parallactick Angle, ibid.
Anomaly, the Mean, 86
The True. 87
Of the Eccentrick, 288
Anomalistical Year, 272
The Antarctick Circle, 70,209
The Antecedentia, *a Motion,* 79
Antipodes, 211
Antœci, ibid.
Aphelion, 85
Apogeon, 100
Its Motion, 102
Apparent Diameters, 6
Of the Sun, 82,277
Apparent Diameters of the Moon's Shadow and Penumbra, 125
Of perpetual Apparition, the Circle, 220
Apsides, 85
The Arctick Circle, 70,209
Areas or Elliptick Spaces equably described, 86, 278,280
Argument of Latitude, 340
Aristarchus, *his Problem to find the Sun's Distance,* 261
Ascension right, 210
Ascension oblique, 220
Ascensional Difference, ibid.
Ascii, *or Shadowless,* 213
Aspect, Quadrate, or Quadrature, 93
Astronomical Tables, 339
Asterisms or Constellations, 48
Atmosphere, its Advantages, 232
Its Height, 236
The Cause of Twilight, 233
Of the Refraction, 240

Axis of the Earth, Pag. 63
Its Parallelism, 66
Axis of the Ecliptick, 73
Constellations, their Names, 48,49,50
Azimuths, 214

B.

Bissextile Year, 363
Bullialdus, *his Correction of* Ward's *Hypothesis*, 306

C.

The Calculation of the Geocentrick Place of a Planet, 340
Cardinal Points, 67
The Circle terminating Twilight, 236
Circles, their Division into Degrees and Minutes, 4
The Circle bounding Light and Darkness, 65
Great Circles in the Sphere, 206
Less Circles, ibid.
The Circle of perpetual Occultation, 220
Of Vision, 92
Climates, 220
Colure of the Equinoctıes, 210
Of the Solstices, ibid.
Comets, a Sort of Planets, 189
Their Course in the Heavens, 197
Their Figure, 195
Their Motion, 198
They demonstrate the Necessity of a Vacuum, 203
The true Form of their Orbits, 198
Their Parallaxes, 192
Their Tails, 203
Commutation, 340
Conjunction of the Sun and Moon, 93
The Conical Shadow, 111,116
Its Height, 120
Its Angle, 116
Constellations, their Names,, &c, 48,49,50
Copernicus, *his Prophecy*, 163
Cosmical rising, 222
Crepusculum, *or Twilight*, 233
Where shortest, &c. 239
Its different Durations, 238
Its Beginning and Ending, 240

Culmi-

Culmination, what, Pag. 214
Curtate Distance, 340
Cycle of the Moon, or the Metonick Cycle, 373
Cycle of the Sun, 371
Cycle of Indictions, 379

D.

Days Natural and Artificial, 359
Days being longer than Nights the Cause of hot Weather 88
The Inequality of Days, 314
The longest and shortest Days when they happen, 326
Declination, 201
The Declination of the Sun how observed, 225
Delineation of the Moon's Phasis, 94
Diameters apparent, 6
Of the fixed Stars, 37
Of the Sun greater in Winter than in Summer, 82
Diameter of the Moon's Shadow, 225
Of the Penumbra, ibid.
Of the Earth's Shadow, ibid.
Diameters of the Planets, 344
Dichotomy of the Moon, 93
Dionysian *Period,* 376
Disk of the Earth, 128
Distance of the Sun how to be found, 259
Diurnal Motion of the Sun in the Ecliptick, 272
Dominical Letters, 369
Dragon's Head and Tail, 99

E.

Earth, its Annual Motion, 60
Poles, 63
Distance from the Sun, 266
Rotation round its Axis, 64
Eclipses explained, 109
Eclipses of the Moon, when they happen, 111
Of the Sun, when, 112
Eclipses total, central, partial, 113
Eclipses annular, 121
Eclipses of the Earth, ibid.
Eclipses, their Limits, 124, 132
Ecliptick, 60, 207
Its Secondaries, 73, 207

Obliquity,

Mean Distance, Pag. 85
Mercury, *one of the Planets*, 155
Meridian, a Circle of the Sphere, 210
Universal Meridian, 215
Meridian Line, how drawn, 224
Difference of Meridians, or of Longitude, 187
Midheaven, 215
Milky-way, 50
Month, 361
Months, synodical and periodical, 97
Moon, an Attendant on the Earth, 90
Its Apogeon and Perigeon, 100
Diurnal Motion from the Sun, 98
Elongation from the Sun, 94
Its Face as drawn by Astronomers, 108
Its Spots, Mountains and Cavities, 107, 108
Its Libration, 103
Illumination, and its Quantity, 95
Its Light in total Eclipses, 154
Its Nodes, 99
Shadow, 118
Its Distance from the Earth, 121
Variation, 102
Motion, how visible, 3
Motion of the Apogeon, 102
Motions equable, why they appear unequal, 9
Motion of a Ball falling in a Ship, 12
Motion in Longitude, 87
Mean Motion, ibid.
Motion of the Planets round their Axes, 45

N.

Nadir, *or the Vertical Points*, 213
New Moon, 93
Nodes, *and the Lines of* Nodes, 99
Nonagesimal Degree of the Ecliptick, 215
Northern Hemisphere, 208

O.

Oblique Ascension, 220
Optical Inequality, 10
Opposition, 92
Orthographical Projection, 128

P.

Parallel Circles, Pag. 206
Parallels and Climates, 221
Parallelism of the Earth's Axis, 66
Parallax of the Stars, 245
Parallax of the Moon, 123, 150, 257
Parallax of Altitude, 246
Of Longitude, 249
Of Latitude, 250
Parallax of the Annual Orb, 183
Parallax of the Sun, 258
Penumbra, 118
Penumbra, *its Dimensions*, 119
Perigeon, 100
Perihelion, 85
Periods of the Planets, 341
Periscii, 213
Periœci, 211
Periods, Dionysian, Julian, *and* Sothaical, 376, 379, 365
The Phases of the Moon, 92
The Phases of Venus, 161
Place Geocentrick and Heliocentrick, 341
Place, its Position on the Earth's Disk for any given Time, 141
Place of a Star reduced to the Ecliptick, 207
Planets or Wanderers, 21
Planets, their Order, 22
Opake Bodies, 23
They move round the Sun, 21
Inferior, 155
Superior, 171
Planets, Direct, Retrograde, and Stationary, 169, 178
Planets, their Distances from the Sun, compared with their Periods, 341
Planets Motions, observed from the Earth, are irregular, 177
Secondary Planets, 22
Pole of the Ecliptick or Heavens, 73
Poles of the Horizon, 213
Of the World, 63
Polar Circles, 70, 209

The INDEX.

Precession of Æquinocties, Pag. 80
Projection of the Moon's Shadow, 131
Prosthaphæresis, 277

Q.

Quadrature, or Quadrate Aspect, 93
Quantity of the Year, 271

R.

Radix, or Epocha, 365
Refraction, 240
How to be observed, 242
Retrogradation of the Nodes, 99
  Of the Planets, 169, 179

S.

Saturn, *a Planet*, 21
Saturn, *his Ring*, 25
  His Satellits, ibid.
Signs Twelve, 60
Selenographers, 109
The Sun in the Center of our System, 23
  His Apparent Motion, 61
  His Apparent Motion unequal, 272
Sun's Right Ascension, Declination, and Longitude how to be found, 225
The Sun turns round his Axis, 42
Sun's Axis inclined to the Plane of the Ecliptick, 44
Sun's Spots, 43
Spots of Jupiter, 44
Solstices, 210, 269
The Setting of the Stars Cosmically, Achronically, and Heliacally, 221
A Spectator is in the Center of his own View or Prospect, 19
Sphere Right, Oblique, and Parallel, 218, 219, 220
Stars are Suns, 39
  Their Order and Bigness, 47
New Stars, 56
Stars which appear Periodically, ibid.
Stars uniformed, 50
Stars their Catalogues, 51
  Their Number, 54
Stations, 169, 178, 345

T.

The INDEX.

T.

Telescopes, their Advantages, Pag. 8
Theory of the Earth's Motion, 268
Theory of the Planets, 327
Time, its Equation, 312
Its Parts, 359
Tropicks, 69, 209

V.

Venus, *a Planet,* 21, 155
Venus, *seen in the Sun,* 29, 266
Venus, *her Phases,* 161
Venus, *her Brightness,* 165
Her greatest Elongation, 158
How two Conjunctions with the Sun, ibid.
Vision, *how it is made,* 2
Vertical Circles, 213
Vortices *in the Heavens, there are none,* 201

Y.

A Year, Astronomical, *Civil, Tropical or Periodical,* 361
Lunar, moveable or fixed, 362
Year, Ægyptian, ibid.
Gregorian, 364
Julian, 363
The great Canicular Year, 365
The great Year, consisting of 25920 Julian *Years,* 81

Z.

Zenith, 213
Zodiack, 207
Its Breadth, ibid.
Zones, *five in Number,* 212

FINIS.

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